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## ALSEP TRANSIENT ANALYSIS

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Unscheduled ATM presenting results of the ALSEP Transient Analysis program.

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PAGE 2 OF 14 PAGEScope

This ATM describes the transient analysis of ALSEP (E-1 Model) when subject to the LEM-ALSEP trapezoidal pulse shown in Figure 1, and defined in Paragraph B of GAEC Interface Document LIS 360-22302. The g level of 10.852 (computed in ATM No. 94) has been used in this study. Figure 1 shows the input pulse and Figure 2 shows the mathematical model.

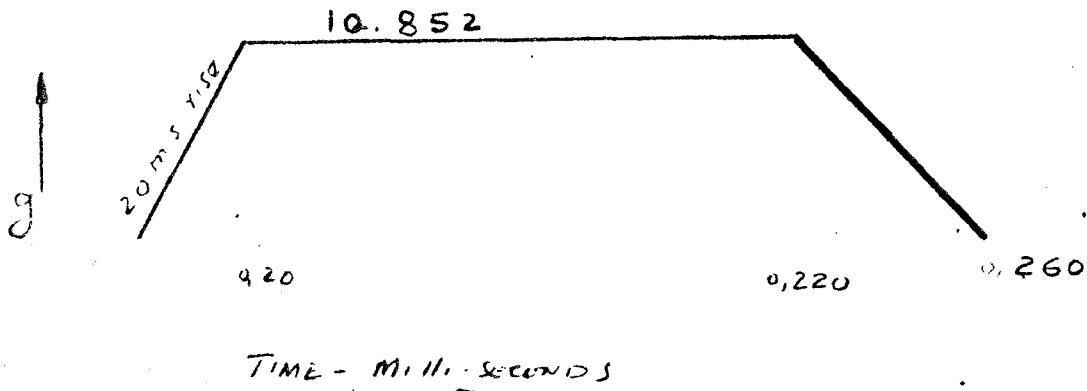


Figure 1 ALSEP Lunar Landing Shock Pulse

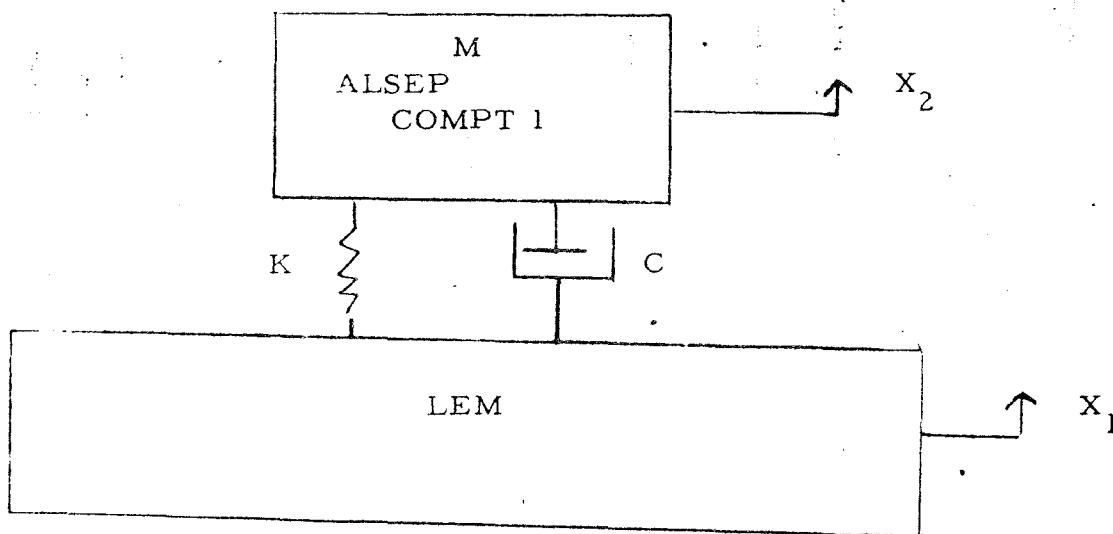


Figure 2 LEM - ALSEP Mathematical Model



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A differential equation of motion describing the transient response of ALSEP with respect to LEM. The motion of the LEM was described for two exciting functions:

- a. ramp type step function (combination of two ramp functions shown in Figure 6)
- b. step function excitation shown in Figure 7

Digital computer time response program 17A was used to compute the acceleration time history of ALSEP response shown in Figures 3, 4, and 5. A viscous damping factor of 0.05 was used in the analysis, Array A weight of 126.5 was assumed and the natural frequency of 35 cps was used from the model E-1 test data.

For relative comparison of various pulses, a shock spectrum analysis was run using the shock spectrum analysis program (DO-3 by M. Bahn/S. Pohnert). The results of maximum response and the residual response for four shock pulses are shown in the appendix. This analysis is a "standard" shock response spectrum analysis and is generally performed on aero-space structures instead of the time history response analysis explained in this ATM. The design shock pulse shown in Figure 5 is valid for all the experiments in Compartment 1.

Results

After preliminary analysis, it was found that the ramp shock pulse shown in Figure 2 was much more representative of the lunar landing pulse than the step excitation. The amplifications obtained by square step excitation were estimated to be improbable, and as such, the results were not plotted.

Figure 3 shows the transient response of ALSEP from 0-20 milliseconds, and Figure 4 shows the transient response from 20-200 milliseconds. The input pulse can be considered to be the sum of two functions. Figure 5 shows the superimposed response of the two ramp functions shown in Figures 1 and 2. In the dotted lines is shown the transient response, and in solid lines is shown the envelope of the computed values.

A steep rising transient amplification of 1.09 may be noticed at 4 milliseconds (refer to Figure 3). From 4 milliseconds to 30 milliseconds the envelope of transient acceleration falls steadily, and from 30-180 milliseconds the transient response stays almost constant. For the purpose of simplification in the computations, the decay part of the lunar landing pulse has not been computed. It is expected that the response g -vs- time history curve will drop steadily from 180 cps to 260 cps, registering a negative response at the termination of the pulse.

FIG. 3 TRANSIENT RESPONSE OF ALSEP  
FROM 0 - 20 milli-seconds

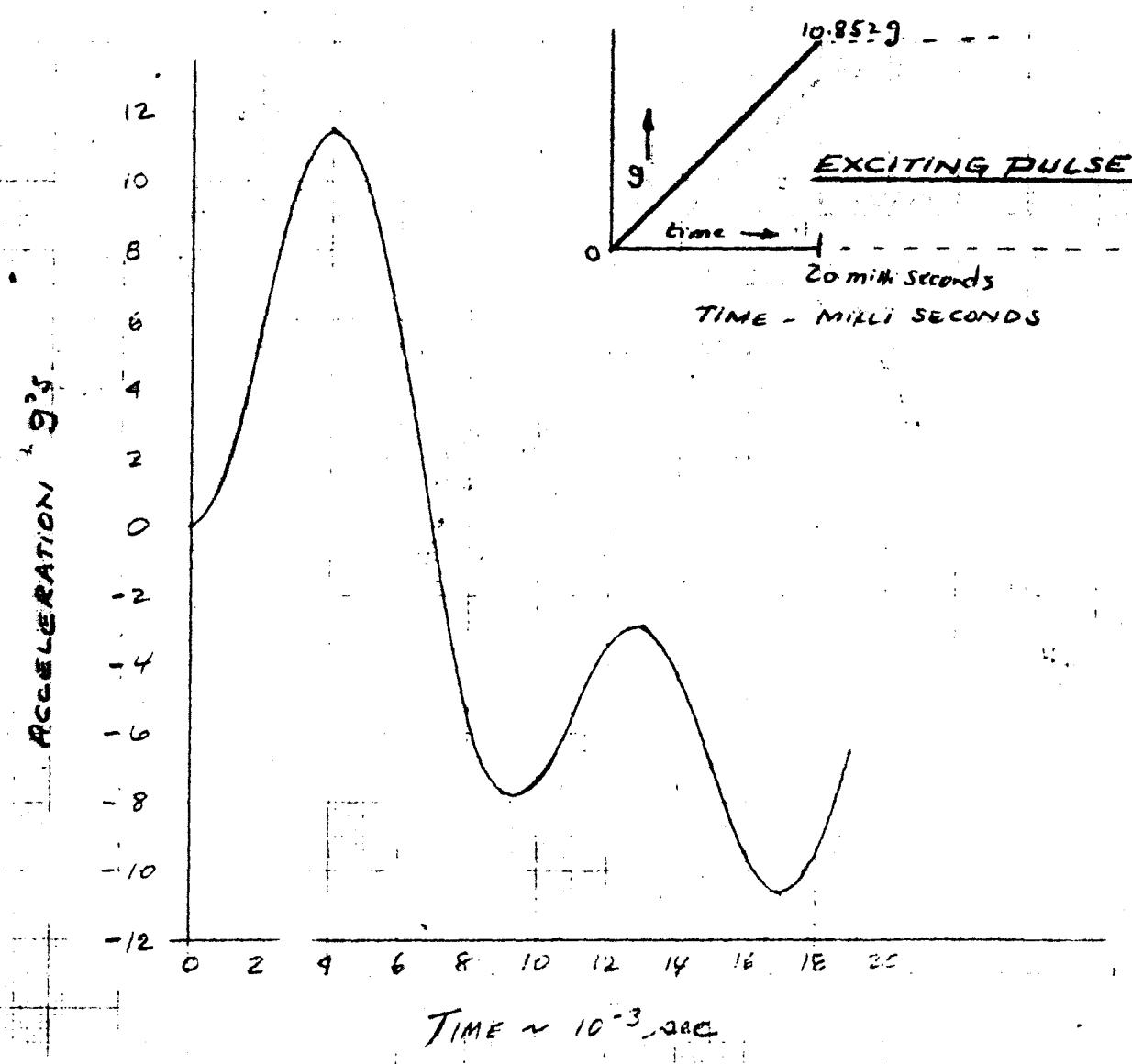


FIG. 4 TRANSIENT RESPONSE OF ALSEP  
FROM 20 - 200 milli-seconds

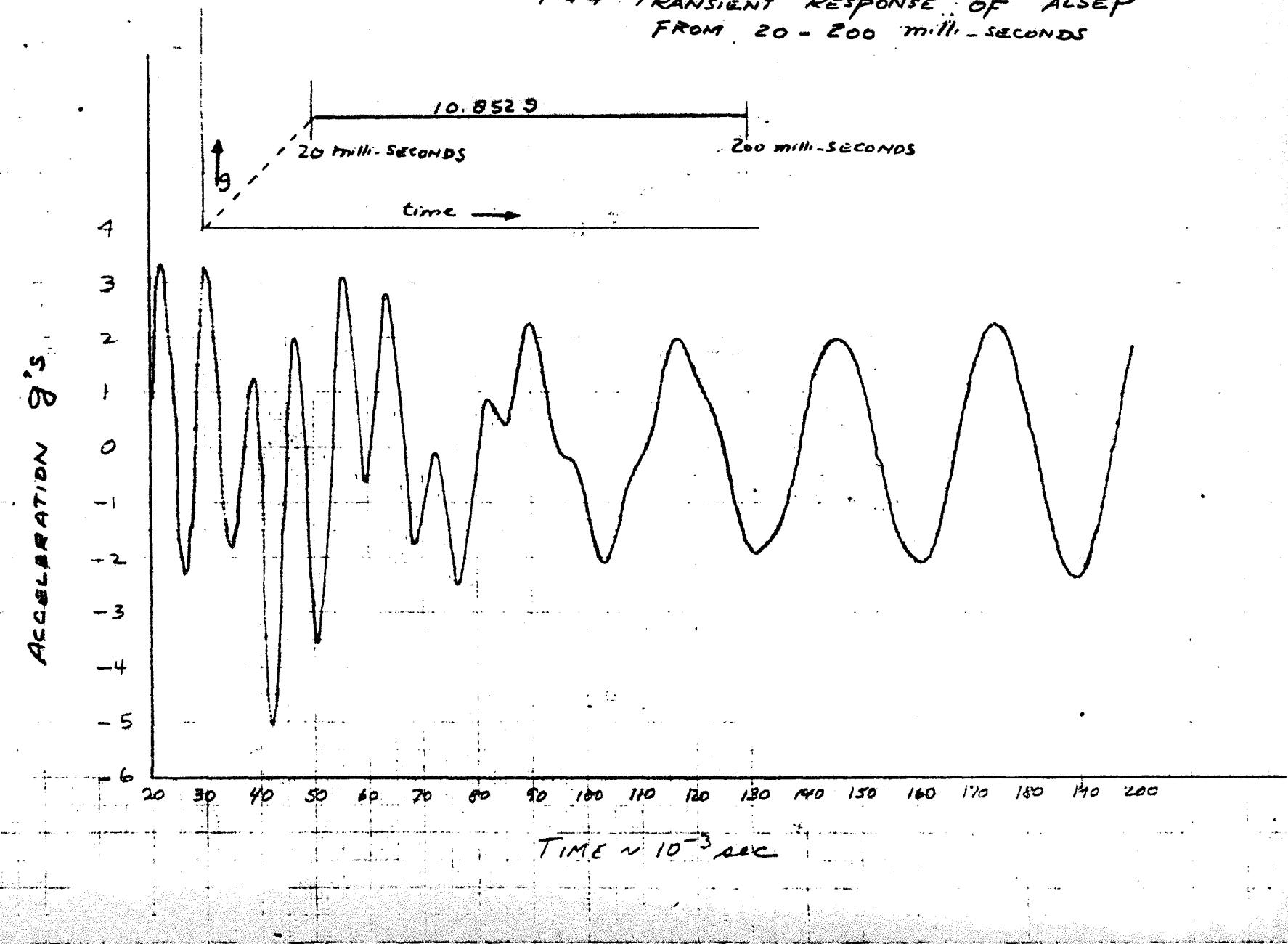
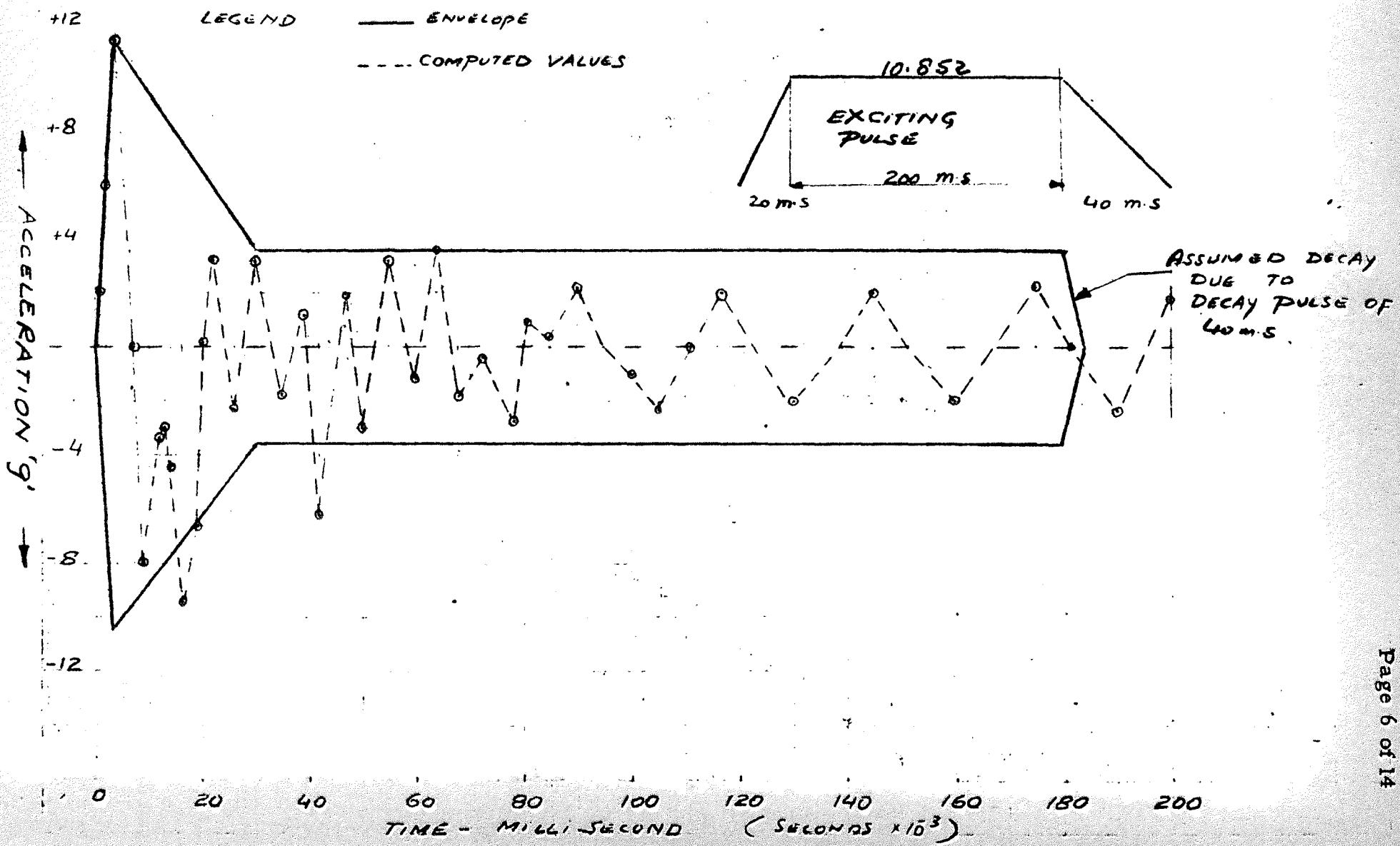


FIG 5: TRANSIENT RESPONSE OF ALSEP  
 (SUPERPOSED CURVES IN FIG - & FIG. )





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Recommendation

1. The time history curve shown in Figure 5 may be incorporated as design shock criteria in appropriate ICS's.
2. For experiments sensitive to long duration pulse, relaxation type structural supports may be employed to dampen the transients in a shorter time.

Analysis

TRANSIENT RESPONSE TO A STEP FUNCTION  
CONSIDERED TO BE SUM OF TWO RAMP FUNCTIONS

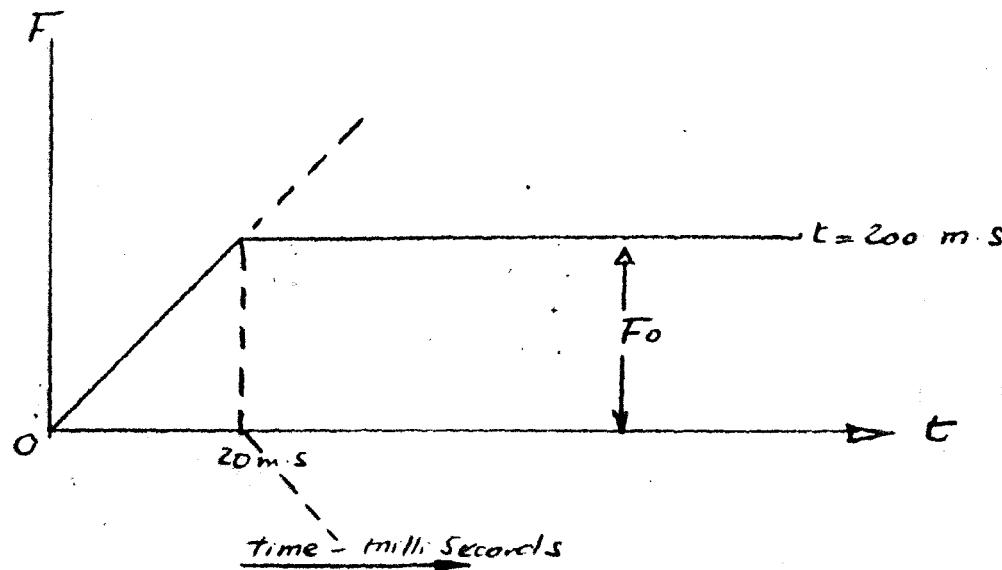


FIG 6.

THE INPUT CAN BE CONSIDERED TO BE THE SUM OF TWO RAMP FUNCTIONS, THE SECOND OF WHICH IS NEGATIVE AND DELAYED BY THE TIME  $t$ , FOR THE FIRST RAMP FUNCTION THE TERMS OF DUHAMEL'S LAW ARE

$$(1.0) \quad f(t) = F_0 \left( \frac{t}{t_1} \right)$$

$$(2.0) \quad g(t) = \frac{1}{m\omega_n} \sin \omega_n t = \frac{\omega_n}{K} \sin \omega_n t$$

AND THE RESPONSE BECOMES

$$(3.0) \quad x(t) = \frac{\omega_n}{K} \int_0^t \frac{F_0}{t_1} \sin \omega_n (t - \tau) d\tau \\ = \frac{F_0}{K} \left( \frac{t}{t_1} - \frac{\sin \omega_n t}{\omega_n t_1} \right) \quad t < t_1$$

FOR THE SECOND RAMP FUNCTION STARTING AT  $t_1$ ,  
THE SOLUTION CAN BE WRITTEN DOWN BY INSPECTION  
OF THE ABOVE EQN AS:

$$(4 \cdot 7) \quad x(t) = -\frac{F_0}{K} \left[ \frac{(t-t_1)}{t_1} - \frac{\sin \omega_n (t-t_1)}{\omega_n t_1} \right]$$

BY SUPERIMPOSING THESE TWO EQ'S THE RESPONSE FOR  
 $t > t_1$  BECOMES

$$(5 \cdot 0) \quad x(t) = \frac{F_0}{K} \left[ 1 - \frac{\sin \omega_n t}{\omega_n t_1} + \frac{1}{\omega_n t_1} \sin \omega_n (t-t_1) \right] \quad t > t_1$$

### I<sup>ST</sup> RAMP FUNCTION

THE DIFFERENTIAL EQN OF MOTION OF THE LEM-  
ARSEP SYSTEM IS:

$$(6 \cdot 0) \quad m \ddot{x}_2(t) + c [ \dot{x}_2(t) - \dot{x}_1(t) ] + K_1 [ x_2(t) - x_1(t) ] =$$

$$(7 \cdot 0) \quad \text{WHERE } x_1(t) = \frac{F_0}{K} \left( \frac{t}{t_1} - \frac{\sin \omega_n t}{\omega_n t_1} \right)$$

FROM EQN (3)

$$(8 \cdot 0) \quad \dot{x}_1(t) = \frac{F_0}{K} \left( 1 - \frac{\cos \omega_n t}{t_1} \right)$$

LET  $\Delta = \frac{F_0}{K}$   
AND SUBSTITUTING (7.0) AND (8.0) INTO (6.0)  
WE HAVE:

$$(9 \cdot 0) \quad m \ddot{x}_2(t) + c [ \dot{x}_2(t) - \frac{\Delta}{t_1} (1 - \cos \omega_n t) ]$$

$$(10 \cdot 0) \quad + K_1 [ x_2(t) - \Delta \left( \frac{\omega_n t - \sin \omega_n t}{\omega_n t_1} \right) ] = 0$$

TAKING LAPLACIANS OF (9.0) & (10.0) WE HAVE

$$(11-0) - M \mathcal{L}[\ddot{x}_2(t)] + c \left[ \mathcal{L}[\dot{x}_2(t)] - \frac{\Delta}{t_1} \{ \mathcal{L}[1] - \mathcal{L}[\cos \omega_n t] \} \right] \\ + k_1 \left\{ \mathcal{L}[x_2(t)] - \frac{\Delta}{c \omega_n t_1} [ \omega_n \mathcal{L}(t) - \mathcal{L}(s \sin \omega_n t) ] \right\}$$

If the system is at rest when the impulse is applied, we have  $\dot{x}_2(0) = 0$

$$m s^2 \bar{x}_2(s) + c \left\{ s \bar{x}_2(s) - \frac{\Delta}{t_1} \left( \frac{1}{s} - \frac{s}{s^2 + \omega_n^2} \right) \right\} \\ + k_1 \left[ \bar{x}_2(s) - \frac{\Delta}{\omega_n t_1} \left\{ \omega_n \left( \frac{1}{s^2} \right) - \frac{\omega_n}{s^2 + \omega_n^2} \right\} \right] = 0$$

$$\text{OR } \bar{x}_2(s) \left\{ M s^2 + k_1 + c s \right\} = \frac{c \Delta}{t_1} \left( \frac{1}{s} - \frac{s}{s^2 + \omega_n^2} \right) \\ + \frac{k_1 \Delta}{t_1} \left( \frac{1}{s^2} - \frac{1}{s^2 + \omega_n^2} \right)$$

$$(12-0) \text{ OR } \bar{x}_2(s) = \frac{\Delta \left( \frac{s_c + k_1}{s^2} - \frac{k_1 + c s}{s^2 + \omega_n^2} \right)}{t_1 (M s^2 + c s + k_1)}$$

& (12-0) was solved using time frequency response program

2<sup>nd</sup> RAMP FUNCTION

SUBSTITUTING EQU (50) IN EQU (6) WE HAVE:

$$(13.0) \quad M \ddot{x}_2(t) + C \left[ \dot{x}_2(t) - \frac{F_0}{t, k} \{ \cos \omega_n(t-t_1) - \cos \omega_n t \} \right] \\ + K \left[ x_2(t) - \frac{F_0}{k \omega_n t_1} \{ \omega_n t_1 + \sin \omega_n(t-t_1) - \sin \omega_n t \} \right] = 0$$

TAKING LAPLACIAN TRANSFORMS.

$$Ms^2 \bar{x}_2(s) + C \left\{ s \bar{x}_2(s) - \frac{F_0}{t, k} \left( \frac{s e^{-t_1 s}}{s^2 + \omega_n^2} - \frac{s}{s^2 + \omega_n^2} \right) \right\} \\ + K \left[ \bar{x}_2(s) - \frac{F_0}{k \omega_n t_1} \left( \frac{\omega_n t_1}{s} + \frac{\omega_n e^{-t_1 s}}{s^2 + \omega_n^2} - \frac{\omega_n}{s^2 + \omega_n^2} \right) \right] = 0$$

Solving for  $\bar{x}_2(s)$

$$(14.0) \quad \bar{x}_2(s) \{ Ms^2 + Cs + K \} = \frac{F_0}{K t_1} \left\{ \frac{t_1}{s} + \frac{Cs(e^{-t_1 s} - 1) + e^{-t_1 s} - 1}{s^2 + \omega_n^2} \right\}$$

$$\bar{x}_2(s) = \frac{\frac{F_0}{K t_1} \left\{ \frac{t_1}{s} + \frac{(Cs+1)(e^{-t_1 s} - 1)}{s^2 + \omega_n^2} \right\}}{s(s^2 + \omega_n^2)(Ms^2 + Cs + K)}$$

$$(15.0) \quad \bar{x}_2(s) = \frac{\left( \frac{F_0}{K t_1} \right) [(s^2 + \omega_n^2)t_1 + s(Cs+1)(e^{-t_1 s} - 1)]}{s(s^2 + \omega_n^2)(Ms^2 + Cs + K)}$$



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PAGE 12 OF 14 PAGESAnalysis: STEP FUNCTION EXCITATION

Consider an undamped system, the response to the step-function excitation shown in Figure 2,  $f(t) = F_0$  is defined as:

$$(16.0) \quad g(t) = \frac{1}{M W_n} \sin W_n t$$

Using the \*convolutions integral, the response is

$$(17.0) \quad X(t) = \frac{F_0}{M W_n} \int_0^t \sin W_n (t - \xi) d\xi$$

$\xi$  is defined as time ( ) at a discrete pulse

$W_n$  is the natural frequency at RAD/SEC.

M is the mass in slugs

$F_0$  is a constant force in #

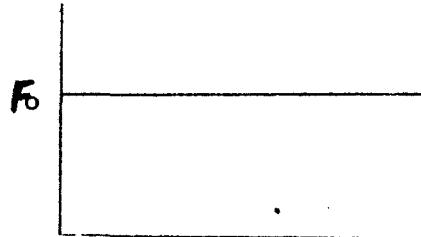


Figure 7

Step Function Excitation  $f(t) = F_0$ 

$$(18.0) \quad \therefore X(t) = \frac{F_0}{K} (1 - \cos W_n t)$$

Consider the differential equation for LEM-ALSEP transient mathematical model (Figure 1) If X is the relative motion, then the system is:

$$X + 2 \zeta W_n X' + W_n^2 X = \frac{F_0}{M}$$

Whose sol'n is the sum of the sol'ns to the homogeneous equations and that of the particular sol'n which for this case is.

$F_0/M W_n^2$  thus the equation

$$X(t) = X e^{-\zeta W_n t} \sin (\sqrt{1 - \zeta^2} W_n t - \phi) + \frac{F_0}{M W_n^2}$$

\*Refer to Chapter on "Transient and non-periodic vibration" by W. T. Thompson published by Prince Hall, Inc.



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is defined as the damping factor =  $\frac{C}{C_C}$

Fitted to the initial conditions of  $X(0) = X'(0) = 0$  Will result in the sol'ns which is given as

$$(19.0) \quad X = \frac{F_0}{K} \left[ 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \cos(\sqrt{1 - \zeta^2} \omega_n t - \psi) \right]$$

where  $\tan \psi = \frac{\zeta}{\sqrt{1 - \zeta^2}}$



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PARAMETRIC DATA FOR LEM-ALSEP TRANSIENT RESPONSE ANAL.

Natural Frequency       $W_n^* = 2 \times 35 = 219.9 \text{ rad/sec.}$

ALSEP Compt 1 Mass      -       $\frac{1265}{3864} = 0.3274 \text{ slugs}$

$F_o = \frac{126.5}{386.4} \times 10.852 \times 386.4 = 1372.9\#$

$K = \frac{126.5}{386.4} \times (219.5)^2$

= 171,169.974 #/in

$\Delta = \frac{F_o}{K} = \frac{1372.9}{171,169.974} = 0.00802 \text{ inches}$

\*Refer to test report ALSEP Compt. Model E-1

PLOT OF MAXR VS FREQUENCY

-01

016

014

012

010

008

006

004

002

-000

-002

.02

H  
A  
X  
R  
G

FOR EXCITING PULSE  
REFER TO THE ENVELOPE  
OF FIG 5.

-000    002    004    006    008    010    012    014    016    018

F-CPS

PLOT OF RESID VS FREQUENCY

-001

006 X-

X

FOR EXCITING PULSE  
REFER TO THE ENVELOPE  
OF FIG 5

004 I-

X

X

002 I-

R  
E  
S  
I  
D

-000 I-

X X X

X

-002 I X X

\*

X

-004 I -

X X

-006 XX

X

X

-008 -

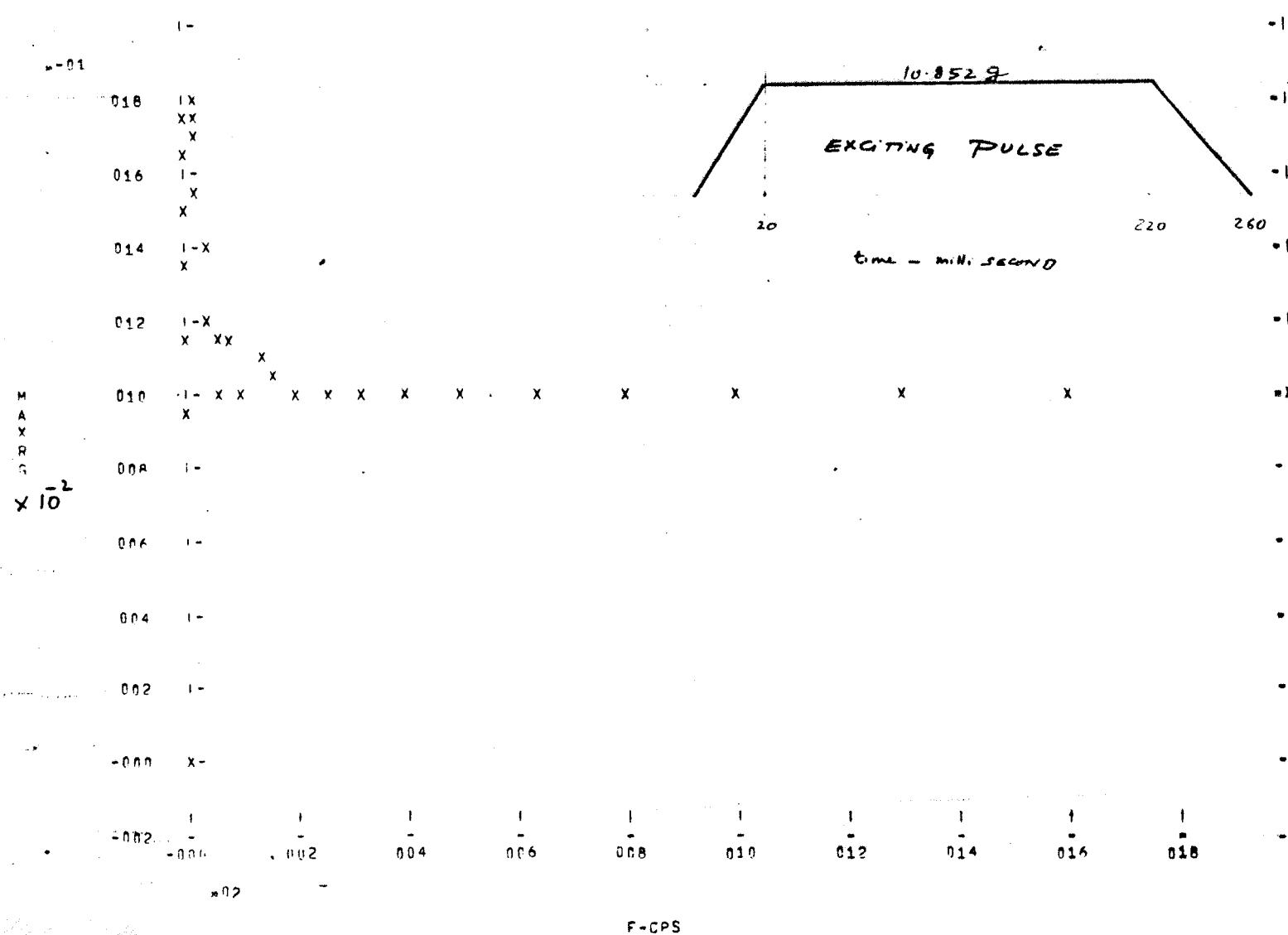
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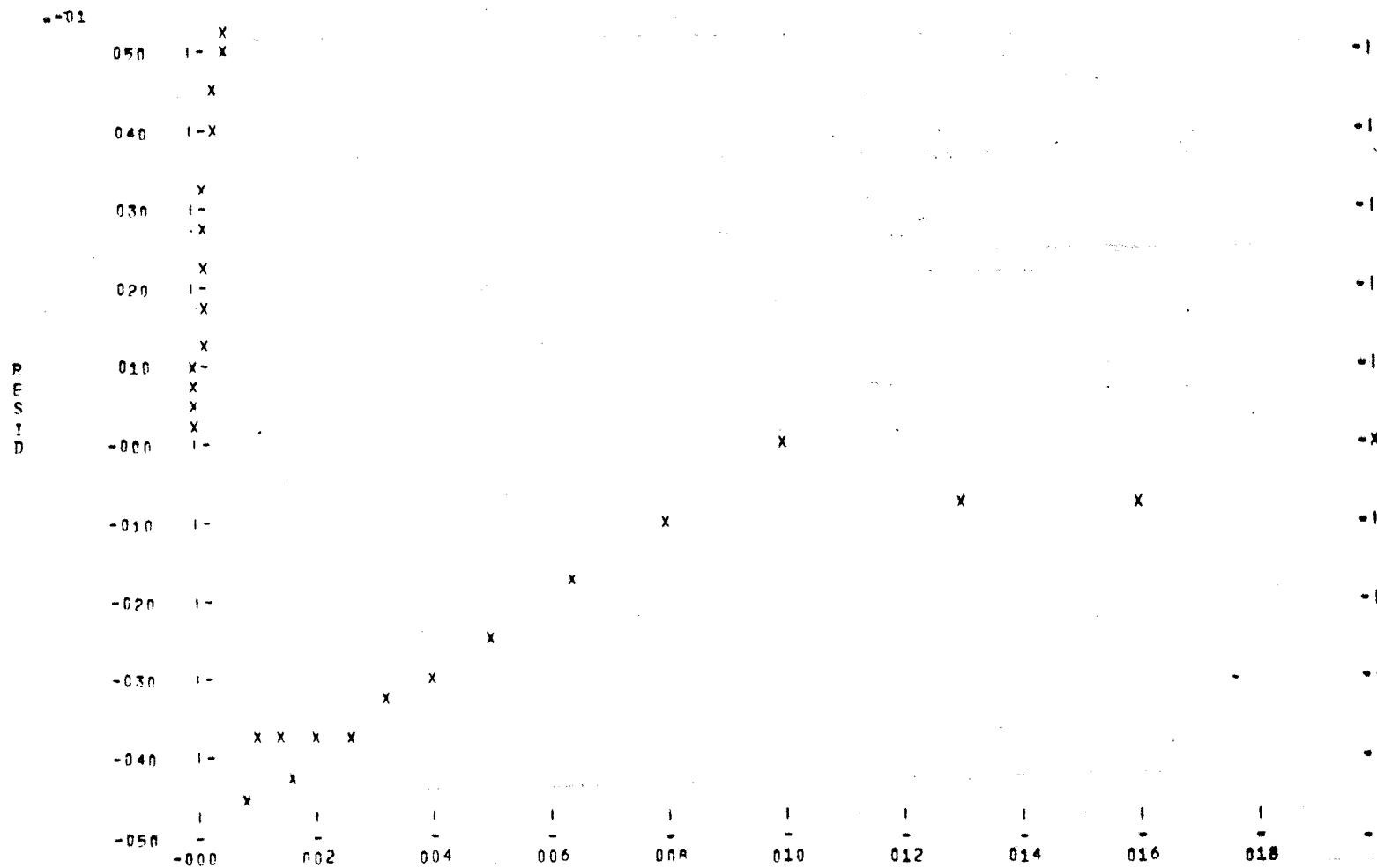
\*02

F-CPS

FIG A-1 PLOT OF MAXR VS FREQUENCY



PLOT OF RESID.VS FREQUENCY



F-CPS

FIG 2 PLOT OF MAXR VS FREQUENCY

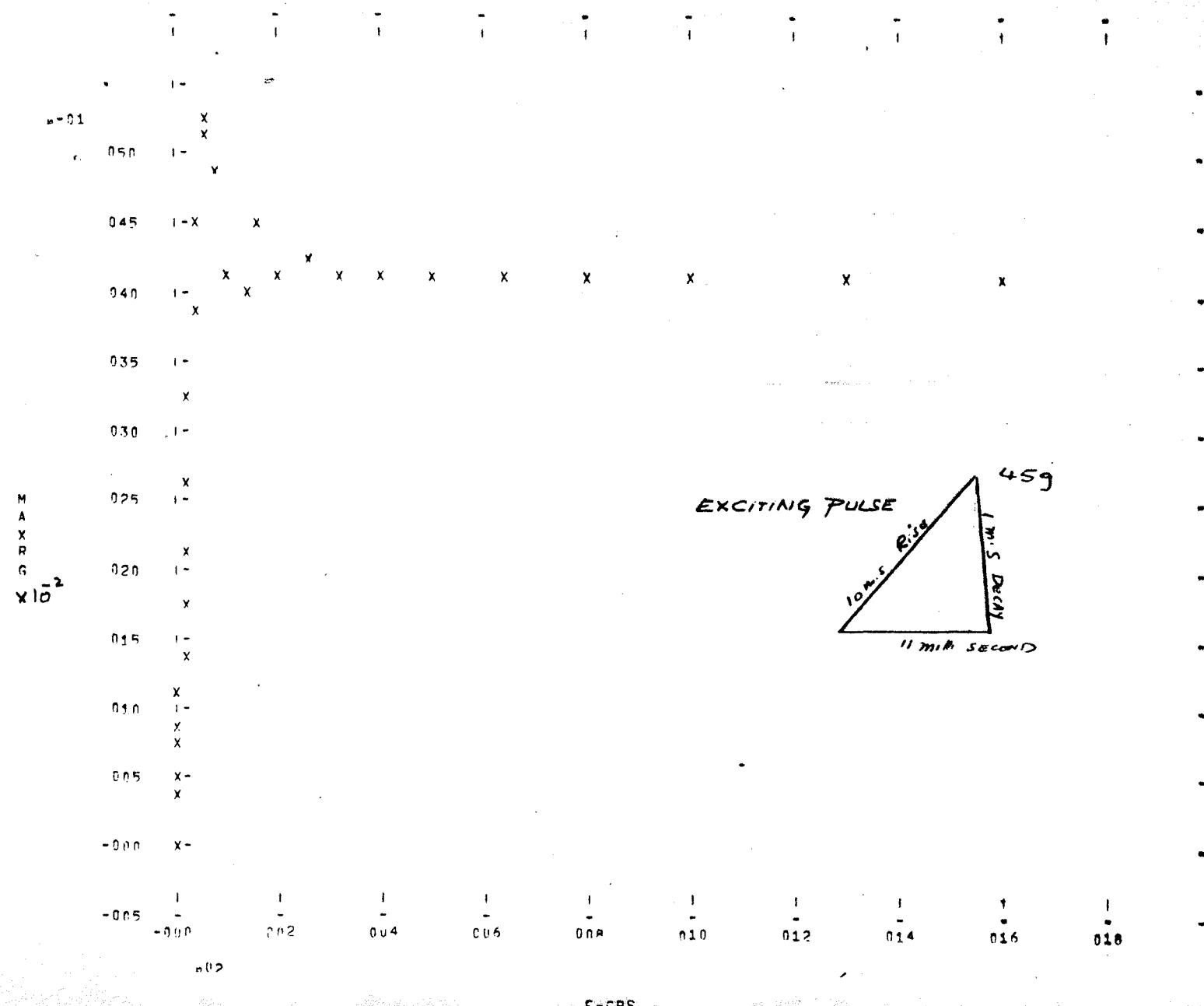
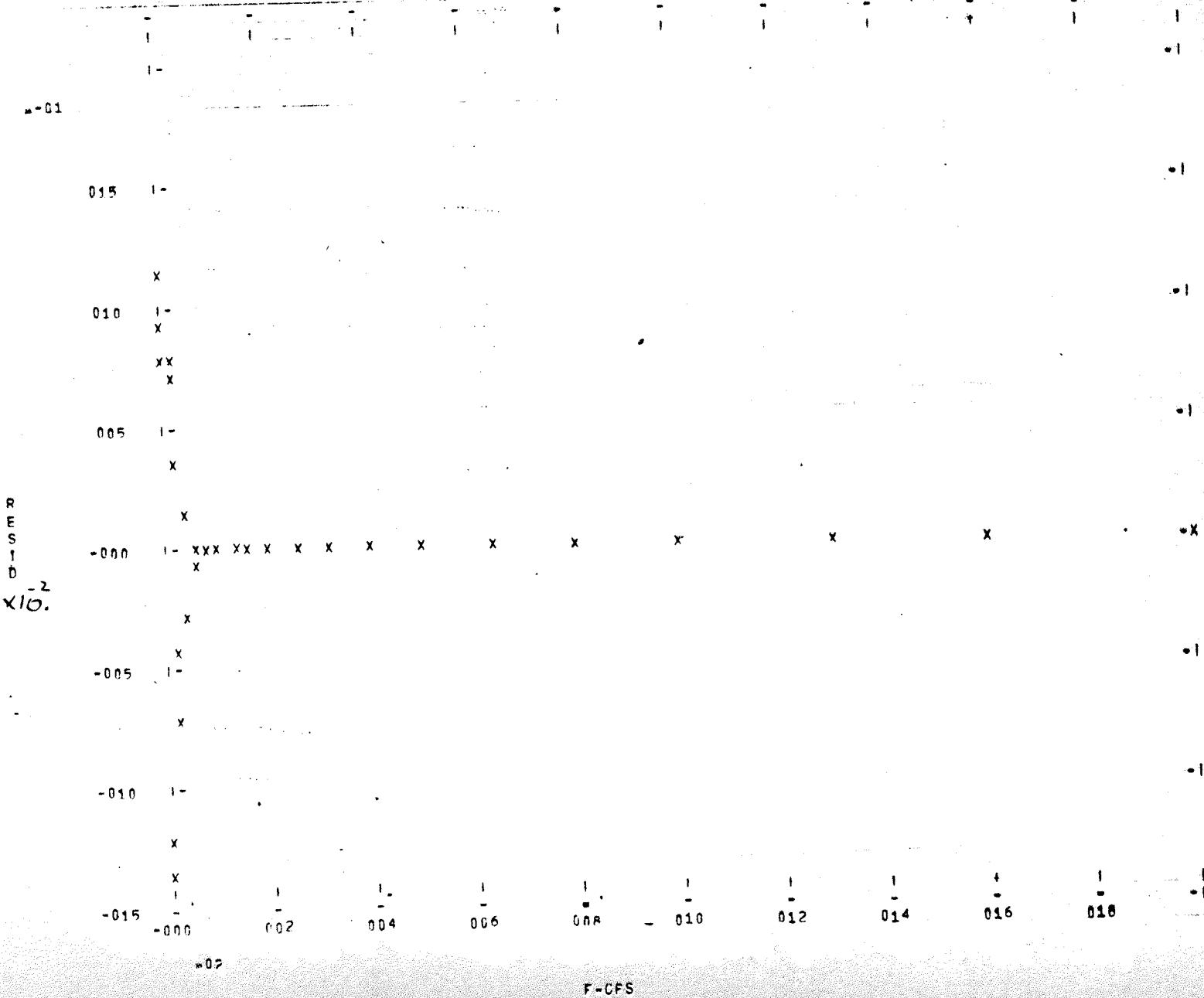
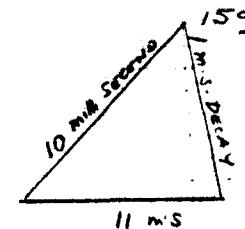
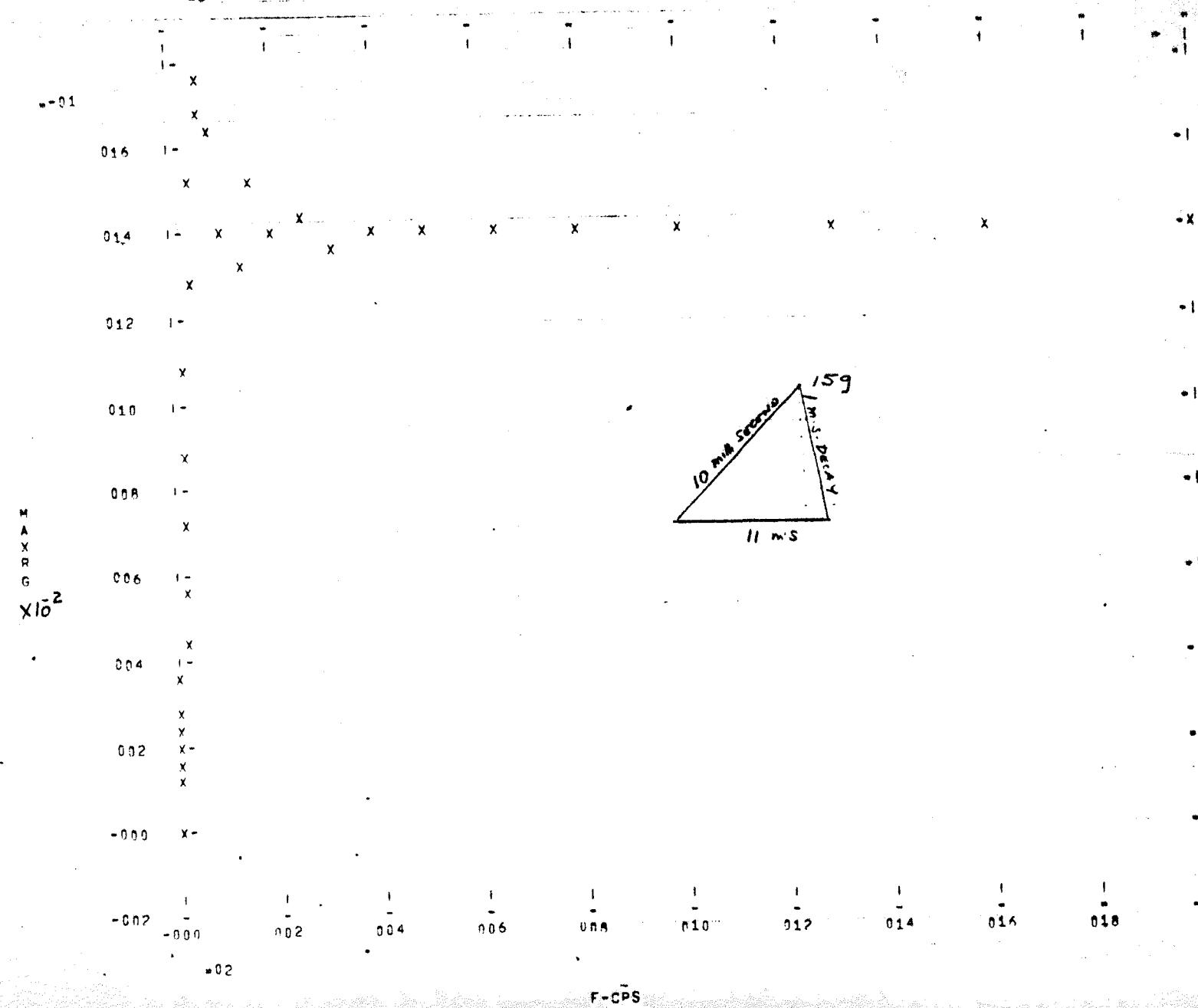


FIG A-1.1 PLOT OF RESID VS. FREQUENCY



F-CFS

PLOT OF MAXR VS FREQUENCY



F-CPS

PLOT OF RESID VS FREQUENCY

