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LRRR 300 Corner Array
Dynamic Analysis

PAGE i OF

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This ATM presents the results of the structural dynamics analysis performed on the Lunar Ranging Retro Reflector 300 Corner Array.

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Paragraph

Prepared by:

M. Goran

G. B. Min

K. Wadleigh

Approved by:

D. Dewhirst

J. Brueger

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(3) Leonard Meirovitch, Analytical Methods in Vibrations, The MacMillan Company, New York, 1967

(4) AL 410900, "Exhibit B, Design and Performance Specification for the Retro-Reflector Array Panels," Bendix Aerospace Systems Division, Ann Arbor, Michigan, November 1, 1970

1.0 Summary

1.1 Motivation of Analysis

The vibrations Analysis of the Lunar Ranging Retro Reflector 300 Corner Array (LRRR 300) was performed to answer four basic questions.

These questions are:

1. What loads are imposed on the LRRR 300 structure during qualification and acceptance testing, during launch and boost and during lunar landing? Are the loads assumed by the stress group adequate to ensure structural reliability?

2. What is the worst vibration environment seen by any single corner reflector mount? Does this vibration environment impose loads on any corner reflector mount that will cause either the reflector or the mount to fail?

3. What vibration test spectrum should be used at the small array attachment fittings when the small array is separately tested?

4. If the experiment is flown with the upper array removed (the off loaded condition) will the structure and the corner reflector mounts survive?

To answer the four above questions, it was necessary to analyze three different mathematical models of the LRRR 300 structure. One math model predicted vibration response perpendicular to the plane of the two arrays. The second model predicted vibration response in the plane of the arrays. The final model was constructed to find vibration response of the experiment in the off loaded condition.

1.2 Summary of Conclusions Drawn from Analysis

The analysis that was performed to answer the questions of paragraph 1.1 and the reasoning behind the answers make up the body of this report. The four paragraphs that follow give very brief summaries of the answers arrived at.

1. With the two arrays connected, vibration responses were such that the dynamic loads and deflections seen in the array structure were small (see Table 4.3.1). The dynamic environment therefore produced low stress levels in the array structure. The stress levels seen in the array structure are discussed in the "Structural Analysis Report" (1). In addition, analysis showed that the reaction loads carried through the LM interface fittings were well under the loads that the fittings were designed to carry (see Table 4.3.2 and (1)).

2. The analysis also showed that, when the experiment is subjected to its worst vibration environment (launch and boost qualification test), the worst 3 sigma response at a corner reflector mount in the array plane was 40 g. The worst 3 sigma response at a corner reflector mount perpendicular to the array plane was 48 g. Single corner reflector mounts have been subjected to sinusoidal vibration levels in excess of 50 g peak for considerable periods of time (2).

3. Vibration levels on the lower array at the attachments of the upper array were calculated both in the array plane and perpendicular to it. Using these calculated vibration levels, test levels for the upper array are proposed which are envelopes of the calculated responses. These envelope test levels are shown in Figures 3.1.7 and 3.2.7 of this report.

4. Analysis of the lower array in the "off loaded condition" showed that acceleration responses on the lower array are reduced by removing the upper array. Thus removing the upper array reduces loads seen in the lower array structure and reduces the acceleration levels seen by the corner reflector mounts.

2.0 Description of Dynamic Models

2.1 Dynamic Model Perpendicular to the Array Plane

The dynamic model of the structure perpendicular to the array was a lumped mass type model. The large array was modeled with 23 lumped masses and the small array was modeled with 8 lumped masses. Each lumped mass point was given one degree of freedom, motion perpendicular to the array plane. With 1 constraint, this gave a total of 30 degrees of freedom for the model. The locations of the lumped mass points on the lower array are shown in Figure 2.1.1 and on the upper array in Figure 2.1.2.

The structure consists of the corner reflector support cavity structure with a network of cross beams below. The upper array is about half as large as the lower array and is attached over one side of the lower array at four points as shown in figure 2.1.3.

The stiffness of the arrays between lumped mass points was evaluated by developing an equivalent beam stiffness for the cavity support structure and the lower ribs. The structure of each array was then modeled as a grid of beams, and a stiffness matrix for each separate array was found using Bendix Program No. 0313, Rigid Grid Analysis by Direct Element Method. The interface fittings, the hinges, and the clevis fittings were assumed to be very stiff compared to the arrays in this direction. The fittings were modeled as rigid links. The stiffness matrices found by this method are listed in Figure 2.1.4.

The mass matrix for the whole system was then evaluated. Thirty one lumped mass points were chosen as shown on Figure 2.1.1 and 2.1.2. The mass lumped at each location is shown in Table 2.1.1.

Again refering to Figures 2.1.1 and 2.1.2, it is seen that mass locations 3 and 16 are repeated on both arrays. This because the two arrays are connected by hinges at these points and the upper and lower arrays have the same motion. The clevis locations on both arrays are also connected, but no mass is lumped at the clevis locations on the upper array.

The stiffness matrix for the upper array was calculated with the four attachments between the arrays fixed in a plane. Since a plane is determined by 3 points, the displacement of any one of the 4 attachment fittings is determined by the other 3. Therefore 30 independent coordinates describe the system. The dependent coordinate chosen here is the displacement of the mass lumped at the rear clevis fitting (shown dotted). The mass matrix for the independent system was calculated

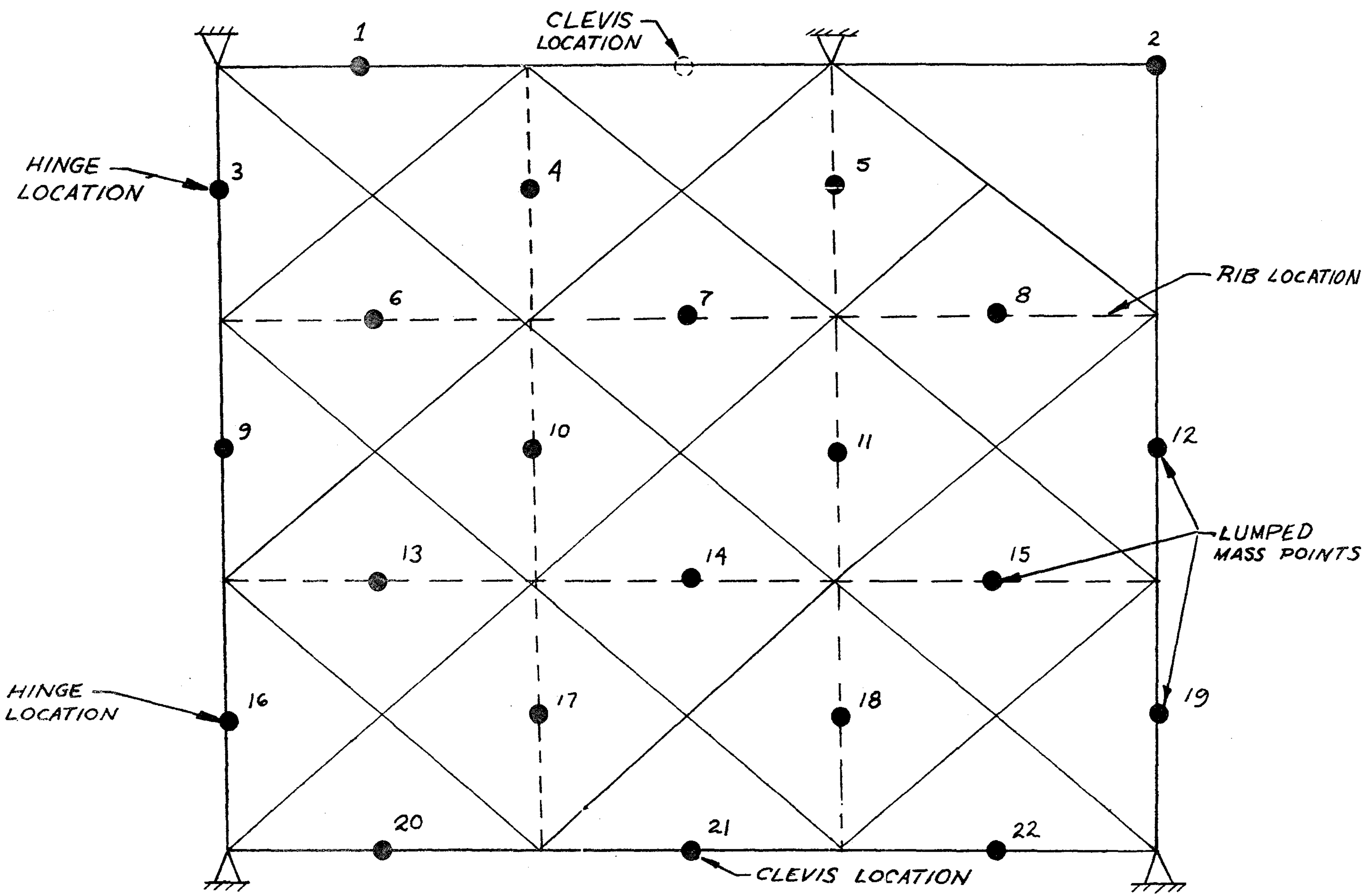

Figure 2.1.1
OUT OF PLANE DYNAMIC MODEL
LOWER ARRAY

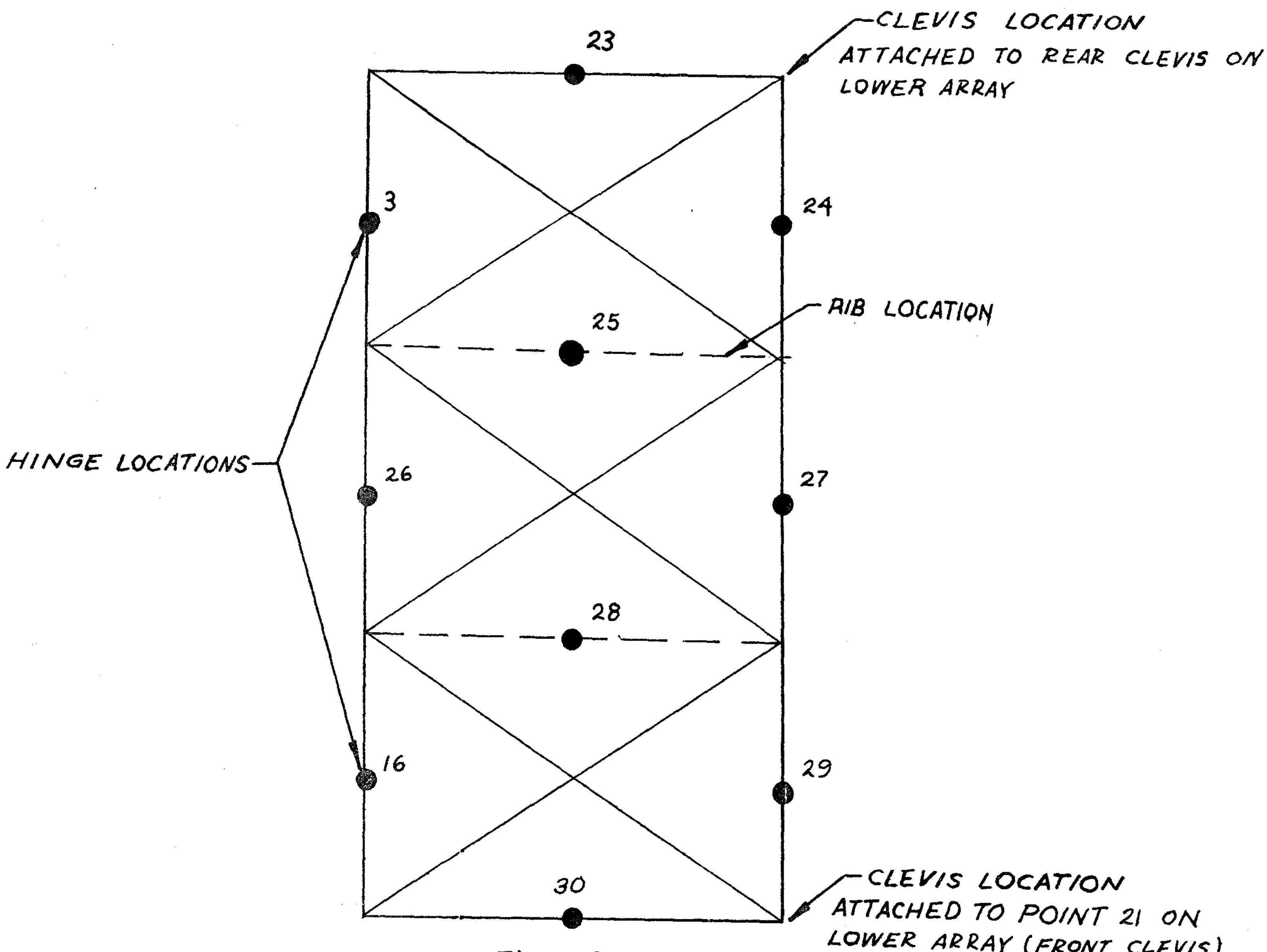

Figure 2.1.2

OUT OF PLANE DYNAMIC MODEL
UPPER ARRAY

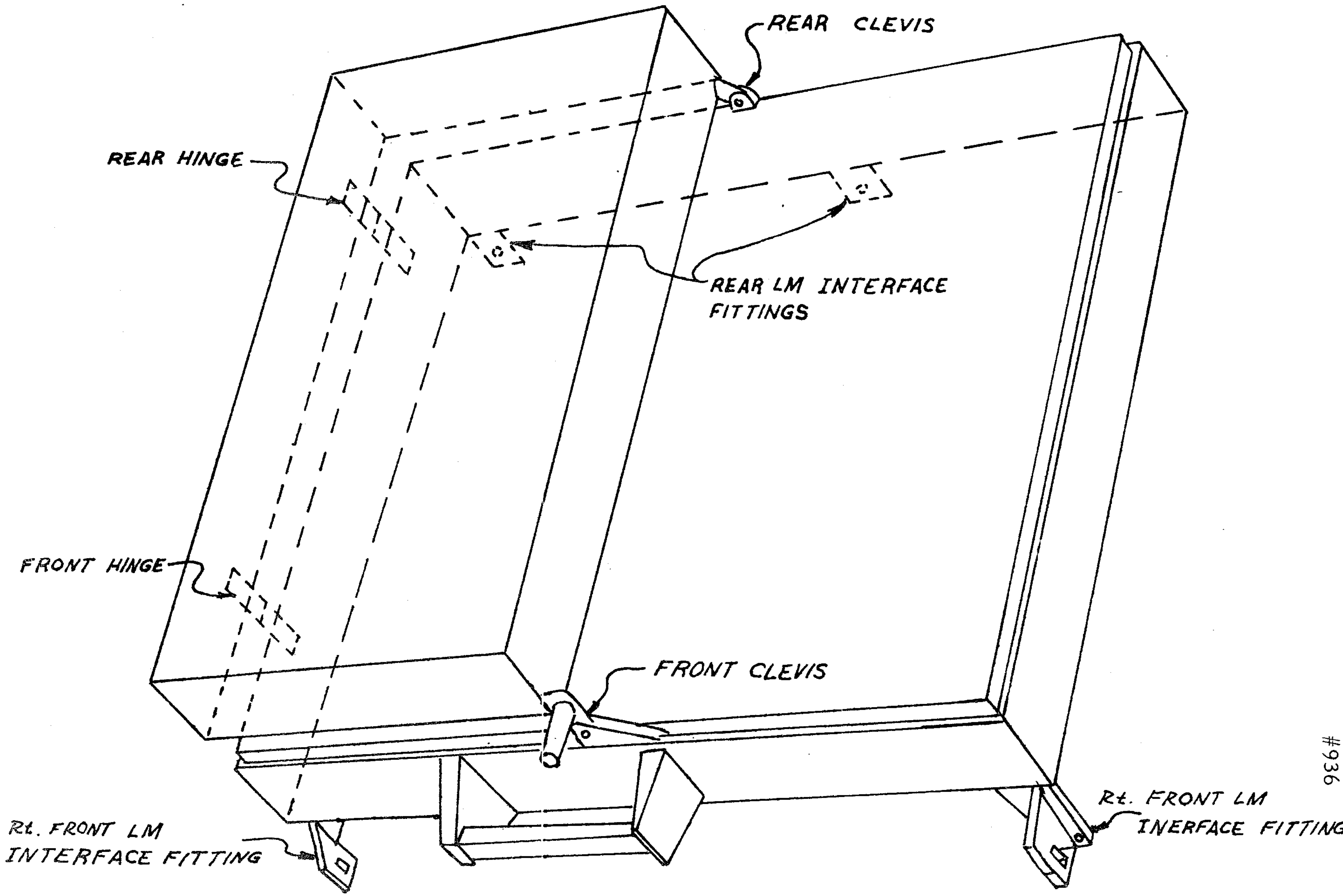

Figure 2.1.3
Fittings Between Array

SMALL K_L MATRIX

Figure 2.1.4

ROW 1

0.526007E 06	0.619771E 04	-0.258751E 05	-0.586831E 06	-0.332139E 05	0.137783E 06	0.151968E 06	0.792879E 03
-0.179200E 05	0.108560E 06	-0.249187E 05	0.832466E 03	-0.182693E 05	-0.183464E 05	0.368200E 04	0.839743E 04
-0.158291E 05	0.586827E 04	-0.137728E 04	0.420651E 04	0.241131E 04	-0.186492E 04		

ROW 2

0.619755E 04	0.451707E 05	0.207798E 04	0.416698E 05	0.422768E 05	-0.104759E 05	-0.110820E 05	-0.997391E 05
0.135560E 04	-0.854196E 04	0.325356E 05	0.178036E 04	0.166135E 04	-0.311063E 04	0.794736E 04	-0.755985E 03
0.205895E 04	-0.768521E 04	0.236573E 05	-0.715156E 03	0.966373E 03	0.163914E 04		

ROW 3

-0.258758E 05	0.207816E 04	0.796714E 06	0.124173E 06	-0.110109E 05	-0.402136E 06	0.114931E 05	-0.178587E 03
-0.308350E 06	0.126711E 06	-0.336583E 04	0.183522E 03	0.530359E 05	-0.198692E 05	0.156840E 04	0.135222E 06
-0.379419E 05	0.415821E 04	-0.722393E 03	0.760988E 04	0.586764E 04	-0.199680E 04		

ROW 4

-0.586831E 06	0.416697E 05	0.124171E 06	0.137515E 07	0.157231E 06	-0.656782E 06	-0.719730E 06	-0.419751E 04
0.926028E 05	0.623579E 05	0.127005E 06	-0.423432E 04	0.455632E 05	0.430710E 05	-0.168227E 05	-0.324326E 05
0.465149E 05	-0.213435E 05	0.600157E 04	-0.122832E 05	-0.591309E 04	0.655343E 04		

ROW 5

-0.332139E 05	0.422763E 05	-0.110095E 05	0.157230E 06	0.222509E 07	0.107649E 05	-0.935605E 06	-0.781997E 06
-0.360921E 04	0.172812E 06	-0.128168E 06	0.183392E 06	-0.247955E 05	0.732523E 05	0.622218E 05	0.742454E 04
-0.319240E 05	0.784836E 05	-0.729841E 05	0.100412E 05	-0.105942E 05	-0.202815E 05		

ROW 6

0.137783E 06	-0.104762E 05	-0.402131E 06	-0.656782E 06	0.107701E 05	0.142312E 07	0.283088E 06	0.181667E 04
-0.349771E 06	-0.833102E 06	-0.148442E 05	0.708673E 02	0.139974E 06	0.108985E 06	-0.209923E 04	0.523022E 05
0.583956E 05	-0.166617E 05	0.161691E 04	-0.170422E 05	-0.118346E 05	0.551046E 04		

ROW 7

0.151971E 06	-0.110823E 05	0.114928E 05	-0.719737E 06	-0.935610E 06	0.283094E 06	0.196371E 07	0.337456E 06
-0.952442E 04	-0.919661E 06	-0.867934E 06	-0.164802E 05	0.109928E 06	0.229610E 06	0.101326E 06	-0.211802E 05
0.623256E 05	0.539382E 05	-0.175639E 05	-0.152709E 05	-0.290374E 05	-0.129226E 05		

ROW 8

0.794101E 03	-0.997381E 05	-0.177781E 03	-0.419823E 04	-0.781992E 06	0.181503E 04	0.337455E 06	0.126060E 07
0.552431E 03	-0.249023E 05	-0.732704E 06	-0.514559E 06	-0.755012E 02	0.928844E 05	0.161395E 06	0.745074E 03
-0.125785E 05	0.337266E 05	0.124712E 06	0.386537E 04	-0.804031E 04	-0.117650E 05		

ROW 9

-0.179195E 05	0.135555E 04	-0.308351E 06	0.926002E 05	-0.361156E 04	-0.349767E 06	-0.952150E 04	0.551710E 03
0.624256E 06	0.248675E 06	-0.755613E 04	0.239801E 03	-0.352765E 06	-0.140040E 05	-0.536774E 03	-0.307700E 06
0.103841E 06	0.129098E 04	0.212649E 03	-0.157028E 05	-0.141233E 05	0.238393E 04		

ROW 10

0.108556E 06	-0.854108E 04	0.126708E 06	0.623666E 05	0.172810E 06	-0.833105E 06	-0.919660E 06	-0.249055E 05
0.248678E 06	0.219461E 07	0.351175E 06	-0.721839E 04	-0.816865E 06	-0.888690E 06	-0.118297E 05	0.123250E 06
-0.630851E 03	0.123725E 06	-0.411865E 04	0.981496E 05	0.655984E 05	-0.344919E 05		

ROW 11

-0.249232E 05	0.325351E 05	-0.336795E 04	0.127015E 06	-0.128169E 06	-0.148442E 05	-0.867942E 06	-0.732695E 06
-0.755619E 04	0.351173E 06	0.220552E 07	0.289476E 06	-0.122681E 05	-0.892186E 06	-0.829805E 06	-0.414727E 04
0.123871E 06	0.800029E 03	0.105419E 06	-0.344836E 05	0.655681E 05	0.981232E 05		

ROW 12

0.831960E 03	0.178041E 04	0.183608E 03	-0.423385E 04	0.183386E 06	0.721615E 02	-0.164810E 05	-0.514555E 06
0.238248E 03	-0.722023E 04	0.289482E 06	0.504962E 06	-0.158248E 03	-0.202744E 05	-0.331019E 06	0.400066E 02
0.248836E 04	0.903959E 05	-0.256296E 06	0.174709E 04	-0.120499E 05	-0.130786E 05		

Matrix (cont

Fig 2.1.4 (co

ROW							
ROW 13							
-0.182671E 05	0.166129E 04	0.530365E 05	0.455607E 05	-0.247895E 05	0.139971E 06	0.109922E 06	-0.751019E 02
-0.352763E 06	-0.816856E 06	-0.122698E 05	-0.160750E 03	0.144323E 07	0.312464E 06	0.937201E 04	-0.405790E 06
-0.744610E 06	-0.246566E 05	-0.232605E 04	0.121217E 06	0.109536E 06	-0.161408E 05		
ROW 14							
-0.183419E 05	-0.311053E 04	-0.198653E 05	0.430621E 05	0.732547E 05	0.108985E 06	0.229605E 06	0.928766E 05
-0.140068E 05	-0.888678E 06	-0.892174E 06	-0.202712E 05	0.312462E 06	0.196914E 07	0.314761E 06	0.581071E 04
-0.840621E 06	-0.840577E 06	0.849010E 04	0.116442E 06	0.238029E 06	0.116378E 06		
ROW 15							
0.368465E 04	0.794778E 04	0.156746E 04	-0.168299E 05	0.622217E 05	-0.209560E 04	0.101332E 06	0.161389E 06
-0.536060E 03	-0.118296E 05	-0.829803E 06	-0.331015E 06	0.936825E 04	0.314766E 06	0.144035E 07	-0.207598E 04
-0.251183E 05	-0.741450E 06	-0.415334E 06	-0.159572E 05	0.109048E 06	0.120540E 06		
ROW 16							
0.839656E 04	-0.755901E 03	0.135223E 06	-0.324306E 05	0.742239E 04	0.523003E 05	-0.211786E 05	0.745102E 03
-0.307702E 06	0.123248E 06	-0.414555E 04	0.400569E 02	-0.405790E 06	0.580760E 04	-0.207717E 04	0.797349E 06
0.140638E 06	-0.294745E 04	0.938292E 03	-0.229184E 05	-0.196414E 05	0.433221E 04		
ROW 17							
-0.158312E 05	0.205825E 04	-0.379443E 05	0.465163E 05	-0.319305E 05	0.584009E 05	0.623393E 05	-0.125710E 05
0.103840E 06	-0.648978E 03	0.123859E 06	0.248779E 04	-0.744603E 06	-0.840611E 06	-0.251137E 05	0.140633E 06
0.175013E 07	0.275589E 06	-0.347415E 04	-0.512893E 06	-0.487482E 06	0.392219E 05		
ROW 18							
0.586595E 04	-0.768547E 04	0.415846E 04	-0.213376E 05	0.784831E 05	-0.166670E 05	0.539435E 05	0.337313E 05
0.129335E 04	0.123717E 06	0.785721E 03	0.903948E 05	-0.246559E 05	-0.840572E 06	-0.741445E 06	-0.294849E 04
0.275598E 06	0.174912E 07	0.146535E 06	0.391974E 05	-0.487381E 06	-0.512730E 06		
ROW 19							
-0.137815E 04	0.236575E 05	-0.721833E 03	0.600419E 04	-0.729789E 05	0.161447E 04	-0.175655E 05	0.124711E 06
0.213244E 03	-0.411779E 04	0.105416E 06	-0.256298E 06	-0.232679E 04	0.848874E 04	-0.415332E 06	0.938044E 03
-0.347049E 04	0.146537E 06	0.774816E 06	0.461449E 04	-0.205537E 05	-0.240676E 05		
ROW 20							
0.420685E 04	-0.714823E 03	0.761127E 04	-0.122890E 05	0.100417E 05	-0.170448E 05	-0.152727E 05	0.386226E 04
-0.157025E 05	0.981548E 05	-0.344794E 05	0.174801E 04	0.121210E 06	0.116432E 06	-0.159590E 05	-0.229148E 05
-0.512878E 06	0.392025E 05	0.461554E 04	0.536519E 06	-0.715389E 05	0.371008E 05		
ROW 21							
0.241210E 04	0.966602E 03	0.586762E 04	-0.591458E 04	-0.105915E 05	-0.118334E 05	-0.290438E 05	-0.804225E 04
-0.141247E 05	0.656041E 05	0.655734E 05	-0.120503E 05	0.109540E 06	0.238029E 06	0.109045E 06	-0.196414E 05
-0.487496E 06	-0.487376E 06	-0.205519E 05	-0.715301E 05	0.517890E 06	-0.715486E 05		
ROW 22							
-0.186456E 04	0.163917E 04	-0.199742E 04	0.655247E 04	-0.202828E 05	0.551202E 04	-0.129233E 05	-0.117655E 05
0.238414E 04	-0.344905E 05	0.981273E 05	-0.130787E 05	-0.161436E 05	0.116374E 06	0.120538E 06	0.433233E 04
0.392270E 05	-0.512729E 06	-0.240663E 05	0.370982E 05	-0.715481E 05	0.536498E 06		

K_U MATRIX

Figure 2.1.4 (cont.)

ROW								
1	0.219645E 06	-0.518918E C5	-0.143117E 05	0.366075E 05	0.214080E 04	0.296894E 05	0.710589E 03	-0.247245E 04
	-0.219645E 06	0.518918E 05	0.143117E 05	-0.366075E 05	-0.214080E 04	-0.296894E 05	-0.710589E 03	0.247245E 04
2	-0.518919E 05	0.798519E 06	-0.201561E 06	0.510754E 05	-0.280859E 06	0.333917E 05	0.133188E 06	0.215182E 04
	0.518919E 05	-0.798519E 06	0.201561E 06	-0.510754E 05	0.280859E 06	-0.333917E 05	-0.133188E 06	-0.215182E 04
3	-0.143121E C5	-0.201562E C6	0.701185E 06	-0.205677E 06	-0.182932E 06	-0.120458E 06	0.402414E 05	0.275100E 05
	0.143121E 05	0.201562E C6	-0.701185E 06	0.205677E 06	0.182932E 06	0.120458E 06	-0.402414E 05	-0.275100E 05
4	0.366075E C5	0.510757E C5	-0.205677E 06	0.271213E 06	0.129620E 06	-0.179171E 06	0.340586E 05	0.558781E 05
	-0.366075E 05	-0.510757E 05	0.205677E 06	-0.271213E 06	-0.129620E 06	0.179171E 06	-0.340586E 05	-0.558781E 05
5	0.214081E C4	-0.280859E C6	-0.182933E 06	0.129620E 06	0.411428E 06	-0.162511E 06	-0.269076E 06	0.292225E 04
	-0.214081E 04	0.280859E C6	0.182933E 06	-0.129620E 06	-0.411428E 06	0.162511E 06	0.269076E 06	-0.292225E 04
6	0.296896E C5	0.333914E C5	-0.120457E 06	-0.179172E 06	-0.162511E 06	0.711210E 06	-0.196401E 06	-0.134845E 05
	-0.296896E 05	-0.333914E C5	0.120457E 06	0.179172E 06	0.162511E 06	-0.711210E 06	0.196401E 06	0.134845E 05
7	0.710660E 03	0.133187E 06	0.402413E 05	0.340586E 05	-0.269076E 06	-0.196401E 06	0.617360E 06	-0.419941E 05
	-0.710660E 03	-0.133187E 06	-0.402413E 05	-0.340586E 05	0.269076E 06	0.196401E 06	-0.617360E 06	0.419941E 05
8	-0.247247E 04	0.215187E C4	0.275100E 05	0.558781E 05	0.292227E 04	-0.134843E 05	-0.419940E 05	0.218861E 06
	0.247247E C4	-0.215187E C4	-0.275100E 05	-0.558781E 05	-0.292227E 04	0.134843E 05	0.419940E 05	-0.218861E 06
9	-0.219645E 06	0.518918E C5	0.143117E 05	-0.366075E 05	-0.214080E 04	-0.296894E 05	-0.710589E 03	0.247245E 04
	0.219645E 06	-0.518918E 05	-0.143117E 05	0.366075E 05	0.214080E 04	0.296894E 05	0.710589E 03	-0.247245E 04
10	0.518919E 05	-0.798519E 06	0.201561E 06	-0.510754E 05	0.280859E 06	-0.333917E 05	-0.133188E 06	-0.215182E 04
	-0.518919E 05	0.798519E C6	-0.201561E 06	0.510754E 05	-0.280859E 06	0.333917E 05	0.133188E 06	0.215182E 04
11	0.143121E C5	0.201562E 06	-0.701185E 06	0.205677E 06	0.182932E 06	0.120458E 06	-0.402414E 05	-0.275100E 05
	-0.143121E 05	-0.201562E 06	0.701185E 06	-0.205677E 06	-0.182932E 06	-0.120458E 06	0.402414E 05	0.275100E 05
12	-0.366075E C5	-0.510757E C5	0.205677E 06	-0.271213E 06	-0.129620E 06	0.179171E 06	-0.340586E 05	-0.558781E 05
	0.366075E 05	0.510757E C5	-0.205677E 06	0.271213E 06	0.129620E 06	-0.179171E 06	0.340586E 05	0.558781E 05
13	-0.214081E C4	0.280859E C6	0.182933E 06	-0.129620E 06	-0.411428E 06	0.162511E 06	0.269076E 06	-0.292225E 04
	0.214081E 04	-0.280859E C6	-0.182933E 06	0.129620E 06	0.411428E 06	-0.162511E 06	-0.269076E 06	0.292225E 04
14	-0.296896E C5	-0.333914E C5	0.120457E 06	0.179172E 06	0.162511E 06	-0.711210E 06	0.196401E 06	0.134845E 05
	0.296896E 05	0.333914E 05	-0.120457E 06	-0.179172E 06	-0.162511E 06	0.711210E 06	-0.196401E 06	-0.134845E 05
15	-0.710660E 03	-0.133187E 06	-0.402413E 05	-0.340586E 05	0.269076E 06	0.196401E 06	-0.617360E 06	0.419941E 05
	0.710660E 03	0.133187E 06	0.402413E 05	0.340586E 05	-0.269076E 06	-0.196401E 06	0.617360E 06	-0.419941E 05

K_4 Matrix con.   Figure 2.1.1 (con.

ROW 16

0.247247E 04	-0.215187E C4	-0.275100E 05	-0.558781E 05	-0.292227E 04	0.134843E 05	0.419940E 05	-0.218861E 06
-0.247247E 04	0.215187E 04	0.275100E 05	0.558781E 05	0.292227E 04	-0.134843E 05	-0.419940E 05	0.218861E 06

Table 2.11
Lumped Masses

MASS LUMPED FOR LOWER ARRAY

…n	Weight (Lbs)	Attached Weight (Lbs)	Total Weight (Lbs)	Mass Lumped At Location $\frac{Lb/Sec.^2}{In}$	Remark (For Attached Weight
	1.37	.45 + .5	2.32	.00601	(1/2) Leg Hinges (1/8) Leveling Leg Assy
	1.344	.45 + .5 + .3	2.594	.00672	(1/2) Leg Hinges (1/8) Leveling Leg Assy +(1/2) Tie Pts.
	2.74	.1	2.84	.00735	Back Rest
	1.37	.2	1.57	.00406	(1/2) Array Hinges
	2.714	1.0	3.714	.00962	(1/4) Leveling Leg Assy
	2.714		2.714	.00703	
	2.74		2.74	.00709	
	2.688		2.688	.00696	
	2.74		2.74	.00709	
	1.37		1.37	.00354	
	2.714	1.0	3.714	.00962	(1/4) Leveling Leg Assy
	2.714		2.714	.00703	
	1.37		1.37	.00354	
	2.74		2.74	.00709	
	2.688		2.688	.00696	
	2.74		2.74	.00709	
	1.37	.2	1.57	.00406	(1/2) Array Hinges
	2.714	1.0	3.714	.00962	(1/4) Leveling Leg Assy
	2.714		2.714	.00703	
	1.37		1.37	.00354	
	1.37	.375	1.745	.00452	(1/2) Pins & Name Pl
	1.344	.5 + .3	2.144	.00555	Handle & Handling Socket + (1/2) Tie Pt.
	1.37	1.25	2.62	.00678	Suncompass Assy.

MASS LUMPED FOR UPPER ARRAY

Mass Joint Location	Synthesized Mass Numbering	Weight* On Each Location	Attached Weight (Lbs)	Total Weight On Each Location	Mass Lumped $\frac{(Lbs. Sec^2)}{In}$	Remark (For Attached Weight
2	23	1.5	.1	1.6	.00414	(1/4) Tie Points
4	3**	2.02	.2	2.22	.00575	(1/2) Hinges
6	24	2.02	.1	2.12	.00549	(1/4) Tie Points
8	25	4.55		4.55	.01178	
10	26	2.02		2.02	.00523	
12	27	2.02		2.02	.00523	
14	28	4.11		4.11	.01064	
16	16**	2.02	.2	2.02	.00523	(1/2) Hinges
18	29	2.02	.1	2.02	.00523	(1/4) Tie Points
20	30	1.94	.1	1.94	.00502	(1/4) Tie Points

*Weight on each location = $24.2 \times \frac{Ai}{At} = 24.2 \times \frac{Ai}{237.402}$

**Mass of stations 3.16 will be added to the stations 3.16 of the Lower Array

***Mass at this location is transformed to locations 3, 16, and 21 by making use of the fact that both the independent and dependent systems must have the same kinetic energy if all lumped masses have the same velocity in each system.

The only elements in the mass matrix affected by the constraint will involve the coordinates at the attachment points. These elements are calculated below. For the unconstrained system:

$$\text{Kinetic Energy} = \frac{1}{2}\left(M_{21}U_{21}^{2} + M_{16}U_{16}^{2} + M_{3}U_{3}^{2} + M_{d}U_{d}^{2}\right) \quad (1)$$

Where:

M_i = the mass lumped at the i th coordinate location,

U_i = the velocity of the mass lumped at the i th coordinate location

$(\ )_d$ = the displacement or velocity of the dependent coordinate

But since the 4 mass points are constrained to move in a plane:

$$U_d = U_{21} - 1.398U_{16} + 1.398U_3 \quad (2)$$

Where the constants are determined by geometry of the plane.

Substituting (2) into (1) gives:

$$\begin{aligned} K.\ E. = \frac{1}{2}\Big\{ & (M_{21} + M_d)\,U_{21}^{2} - 2.796\,M_d U_{21}\,U_{16} \\ & + (M_{16} + 1.9544\,M_d)\,U_{2}^{2} + 2.796\,M_d U_{21} U_3 \\ & - 3.4088\,M_d U_{16} U_3 + (M_3 + 1.9544\,M_d)\,U_3^{2}\Big\} \end{aligned} \quad (3)$$

Expressed in matrix form:

$$K.\ E. = \frac{1}{2}\begin{bmatrix}U_{21} & U_{16} & U_3\end{bmatrix}\begin{bmatrix}(M_{21} + M_d) & -1.398M_d & 1.398M_d \\ -1.398M_d & (M_{16} + 1.9544M_d) & -1.9544M_d \\ 1.398M_d & -1.9544M_d & (M_3 + 1.9544M_d)\end{bmatrix}\begin{Bmatrix}U_{21} \\ U_{16} \\ U_3\end{Bmatrix} \quad (4)$$

By definition of a mass matrix, the elements of the 3 x 3 matrix in (4) are the non-trivial elements of the mass matrix in the independent coordinates. These are placed in the independent, $[M_q]$ matrix as shown below

$$K.E. = \frac{1}{2}\left[U_{21},\ U_{16},\ U_3\right]\begin{bmatrix} M_{21,21} & M_{21,16} & M_{21,3} \\ M_{16,21} & M_{16,16} & M_{16,3} \\ M_{3,21} & M_{3,16} & M_{3,3} \end{bmatrix}\begin{Bmatrix} U_{21} \\ U_{16} \\ U_3 \end{Bmatrix} \qquad (5)$$

The remainder of the independent mass matrix is a diagonal matrix whose elements are the masses lumped at the appropriate points. The independent mass matrix is shown in Figure 2.1.5.

Next the two stiffness matrices calculated for the large and small arrays were used to calculate a stiffness matrix for the whole system. The stiffness for the upper array was calculated relative to a plane through its four attachments to the lower array. It is necessary to find the stiffness of the whole system relative to ground. To do this, the stiffness matrix for the upper array was transformed using the transformation matrix relating displacements relative to the plane and displacements relative to ground. This was accomplished as follows:

Let: q_i, (i = 1-22) = displacement of the i th lumped mass point on the lower array relative to ground

q_i, (i = 23-30) = displacement of the i th lumped mass point on the upper array relative to ground

u_{ri}, (i = 23-30) = displacement relative to ground of the i th point on the plane through any 3 of the 4 points attaching the two arrays, u_i has the same dimensions relative to the attachment points as the i th lumped mass

u_i, (i = 23-30) = displacement of the i th lumped mass point on the upper array relative to the above plane

u_i, (i = 1-22) = displacement of the i th lumped mass point on the lower array relative to ground

These coordinate definitions are shown in Figure 2.1.6.

MASS MATRIX

Figure 2.1.5

1	0.60100D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.73500D-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.22940D-01	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	-0.13130D-01	0.0	0.0
	0.0	0.0	0.94000D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.96200D-02	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.70300D-02	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.70300D-02
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0
	0.69600D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.70900D-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.35400D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.96200D-02	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0

Fig 2.1.5 (cont.)

	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.703000D-02	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.354000D-02
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.709000D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.696000D-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.709000D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	-0.131300D-01	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.224200D-01	0.0	0.0
	0.0	0.0	-0.940000D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.962000D-02	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.703000D-02
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.354000D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.452000D-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 2.1.5 (cont)

21	0.0	0.0	0.94000D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	-0.94000D-02	0.0	0.0
	0.0	0.0	0.12270D-01	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.67800D-02	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.41400D-02	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.54900D-02
	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.11780D-01	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.52300D-02	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.52300D-02	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.10640D-01	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.52300D-02	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.50200D-02

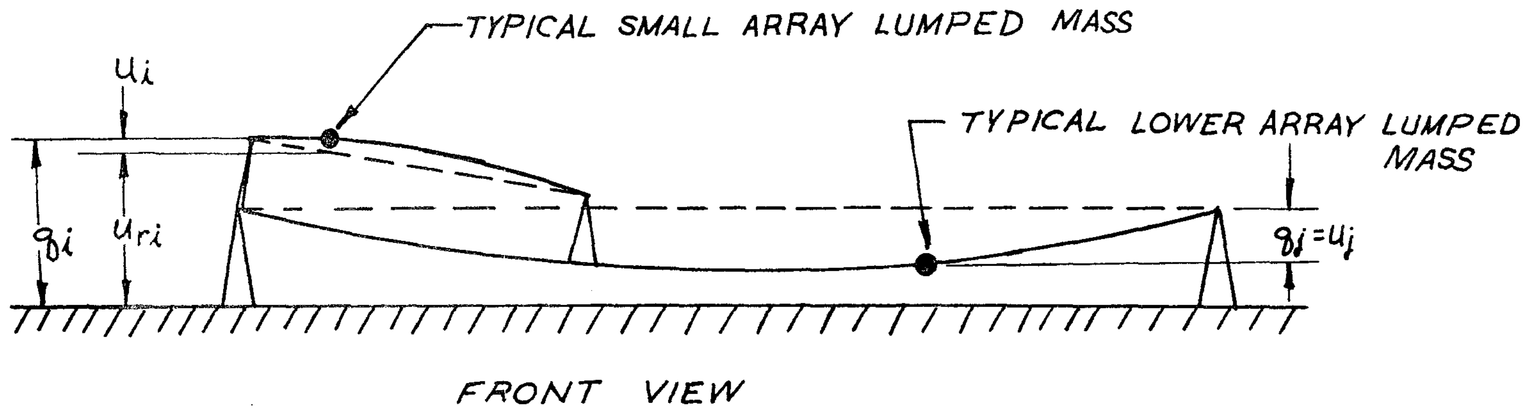

FIG. 2.1.6
DEFINITION OF COORDINATES

But since the u_{ri}'s are displacements of points on a rigid plane, they can be expressed in terms of the absolute displacements of the 3 independent coordinates determining the plane. In Figure 2.1.1 above, q_3, q_{16}, and q_{21} were chosen as the independent coordinates determining the plane through the attachment fittings. We can then write:

$$\begin{Bmatrix} u_{r\,23} \\ u_{r\,24} \\ \cdot \\ \cdot \\ \cdot \\ u_{r\,30} \end{Bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{Bmatrix} q_{21} \\ q_{16} \\ q_3 \end{Bmatrix} \qquad (6)$$

The $[C]$ matrix is determined by the geometry of the lumped mass locations on the plane.

$$\begin{Bmatrix} P_{23} \\ \cdot \\ \cdot \\ \cdot \\ P_{30} \end{Bmatrix} = \begin{bmatrix} K_u \end{bmatrix} \begin{Bmatrix} U_{23} \\ \cdot \\ \cdot \\ \cdot \\ U_{30} \end{Bmatrix} \qquad (7)$$

Where:

P_i, (i = 23-30) = the force at the i th coordinate

$[K_u]$ = the stiffness matrix calculated for the upper array relative to the plane through its attachment points.

By definition:

$$u_i = q_i - u_{ri} \qquad (i = 23\text{-}30) \qquad (8)$$

Substituting (8) into (7),

$$\{P\} = [K_u] \{q - u_r\} = [K_u]\{q\} - [K_u]\{u_r\} \qquad (9)$$

To balance these forces, there must be equal and opposite forces on the rigid plane.

$$\{P_r\} = [K_u]\{q\} + [K_u]\{u_r\} \qquad (10)$$

Expressing these two sets as one,

$$\begin{Bmatrix} P_{23} \\ \vdots \\ P_{30} \\ \cdots \\ P_{r\,23} \\ \vdots \\ P_{r\,23} \end{Bmatrix} = \begin{bmatrix} [K_u] & [-K_u] \\ & \\ [-K_u] & [K_u] \end{bmatrix} \begin{Bmatrix} q_i \\ \cdot \\ \cdot \\ \cdots \\ u_{ri} \\ \cdot \\ \cdot \end{Bmatrix} \qquad (11)$$

This equation allows calculation of dynamic loads on the structure.

This set of equations is also used to combine the two stiffness matrices to calculate a stiffness matrix for the whole system.

The coordinate transform can be written using (6)

$$\begin{Bmatrix} u_1 \\ \cdot \\ \cdot \\ u_{22} \\ \cdots \\ u_{23} \\ \cdot \\ \cdot \\ u_{30} \\ u_{r23} \\ \cdot \\ \cdot \\ u_{r30} \end{Bmatrix} = [\beta] \begin{Bmatrix} q_1 \\ \cdot \\ \cdot \\ \cdot \\ q_{30} \end{Bmatrix} \qquad (12)$$

The $[\beta]$ matrix is shown in Figure 2.1.7. It is seen that the $[\beta]$ matrix is simply a unit diagonal matrix with the $[C]$ matrix of equation (6) tacked on the bottom.

Figure 2.1.7

B MATRIX

ROW 1	0.1C00000E 01	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0		
ROW 2	0.0	0.1C000CC0E 01	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	0.0	0.0	0.0	0.0		
ROW 3	0.0	C.C	C.1C00GC0E 01	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.C	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 4	0.0	C.C	0.0	0.1000000E 01	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 5	0.0	0.C	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0		
ROW 6	0.0	C.0	C.0	0.0	0.0	0.1000000E 01	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 7	0.0	C.C	0.0	0.0	0.0	0.0	0.1000000E 01	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0		
ROW 8	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.1000000E 01
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0		
ROW 9	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.1C00000E 01	C.0	C.0	0.0	0.0	0.0	0.C	0.C
	0.C	0.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0		
ROW 10	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.1CCCCCCE 01	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0		
ROW 11	0.0	C.C	0.0	0.0	0.0	0.0	0.C	0.0
	0.0	0.0	0.1CC0CCC0E 01	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0		
ROW 12	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0

Fig. 2.1.7 (cont)

	0.0	C.0	C.0	0.1000000E 01	0.0	0.0	0.0	0.0
	0.0	0.0	C.C	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0		
ROW 13	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0		
ROW 14	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.1000000E 01	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0		
ROW 15	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0	0.1000000E 01	0.0
	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.1000000E 01
	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0		
ROW 17	0.C	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	C.1000000E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0		
ROW 18	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.1COCCCCE C1	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 19	0.0	0.0	C.C	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.1C00000E 01	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0		
ROW 20	0.0	C.C	C.0	0.0	0.0	0.0	0.C	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.1000000E 01	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0		
ROW 21	0.0	C.C	0.0	0.0	0.0	0.0	0.C	0.0
	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0		
ROW 22	0.0	C.C	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	0.0	0.0	0.0	0.1000000E 01	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0		
ROW 23	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.1000000E 01	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 24	0.0	0.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.10000C0E 01

Fig 2.1.7 (cont)

	0.0	0.0	0.0	0.0	0.0	0.0		
ROW 25	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.1000000E 01	0.0	0.C	0.0	0.0	0.0		
ROW 26	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.1CCCOCOE 01	0.0	0.0	0.0	0.0		
ROW 27	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.1CC0000E 01	0.0	0.0	0.0		
ROW 28	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	0.0	0.1000000E 01	0.0	0.0		
ROW 29	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.1000000E 01	0.0		
ROW 30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.1000000E 01		
ROW 31	0.0	C.C	C.12857CCE 01	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.7857000E 00
	0.0	0.0	C.0	0.0	0.5000000E 00	0.0	0.0	0.0
	0.0	0.C	C.0	0.0	0.0	0.0		
ROW 32	0.0	0.0	C.1224799E 01	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	-0.1224700E 01
	0.0	C.C	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.C	0.0	C.0	0.0	0.0	0.0		
ROW 33	0.0	0.0	C.81980CCE 0C	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3198000E 00
	0.0	0.C	C.0	0.0	0.5000000E 00	0.0	0.0	0.0
	0.0	C.C	0.0	0.0	0.0	0.0		
ROW 34	0.0	0.0	C.4744599E 0C	0.0	0.0	0.0	0.0	0.C
	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.52550C0E 00
	0.0	0.C	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.0	C.C	C.0	0.0	0.0	0.0		
ROW 35	0.0	0.C	C.69910CCE 0C	0.0	0.0	0.0	0.0	0.C
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.6991000E 00
	0.0	0.0	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.0	C.0	C.0	0.0	0.0	0.0		
ROW 36	0.0	0.0	C.3538CCCE 0C	0.0	0.0	0.0	0.C	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.C	0.1461999E 00
	0.0	0.C	0.0	0.0	0.5000000E 00	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0		

Fig 2.1.7 (cont)

ROW 37	0.0	0.0	C.2248CCCE OC	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	0.0	0.0	0.0	0.0	0.0	-0.2248000E 00
	0.0	0.C	C.0	0.0	0.1000000E 01	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		
ROW 38	0.0	0.0	-0.1120999E OC	0.0	0.0	0.0	0.0	0.0
	0.0	0.C	0.0	0.0	0.0	0.0	0.0	0.6120999E 00
	0.0	0.0	C.0	0.0	0.5000000E 00	0.0	0.0	0.0
	0.0	0.0	C.0	0.0	0.0	0.0		

Then the stiffness matrix for the whole system can be calculated:

$$K_s = [\beta]^T \begin{bmatrix} [K_L] & & 0 & \\ & [K_u] & & [-K_u] \\ 0 & [-K_u] & & [K_u] \end{bmatrix} [\beta] \qquad (13)$$

The $[K_s]$ matrix is shown in Figure 2.1.8.

With the system completely described, solution of the eigenvalue problem for the system was accomplished to find the mode shapes and frequencies of the model. These are listed in Figure 2.1.9. The mode shapes and frequencies were then used to calculate the system's frequency and time response through the method of modal analysis. Viscous modal damping of 10 percent of critical damping was assumed for all modes in calculating responses. Experience with previous retro-reflector arrays has shown this assumption to give good agreement between analysis and test results.

2.2 Dynamic Model In Plane of Array

The dynamic model predicting the structure's response to inputs in the plane of the array is shown in Figure 2.2.1. In this model the arrays are assumed to be rigid in comparison to the fittings connecting the experiment to the LM and the fittings connecting the two arrays. Each array is considered a rigid body with 3 degrees of freedom as shown in Figure 2.2.1. All flexibility is attributed to the 8 fittings shown as springs in the Figure. The rear LM interface fittings are seen to have been attributed no stiffness in the X coordinate direction. This is because they are free to slide over pins in this direction.

Figure 2.2.2 shows plan views of each array defining parameters and showing pertinent dimensions. Given this, a stiffness matrix for each array can be evaluated from the definitions of the various influence coefficients assuming both arrays fixed to ground. Figure 2.2.3 shows both stiffness matrices. To find a stiffness matrix for the whole system relative to ground the same procedure was followed as was used for the out-of-plane model.

Figure 2.1.8
[Ks] Matrix

```
ROW  1 0.5260000D 06 0.6198000D 04-0.2588000D 05-0.5868000D 06-0.3321000D 05 0.1378000D 06 0.1520000D 06 0.7928999D 03
      -0.1792000D 05 0.1086000D 06-0.2492000D 05 0.8325000D 03-0.1827000D 05-0.1835000D 05 0.3682000D 04 0.8397000D 04
      -0.1583000D 05 0.5868000D 04-0.1377000D 04 0.4207000D 04 0.2411000D 04-0.1865000D 04 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  2 0.6198000D 04 0.4517000D 05 0.2078000D 04 0.4167000D 05 0.4228000D 05-0.1048000D 05-0.1108000D 05-0.9974000D 05
       0.1356000D 04-0.8542000D 04 0.3254000D 05 0.1730000D 04 0.1661000D 04-0.3111000D 04 0.7947000D 04-0.7560000D 03
       0.2059000D 04-0.7685000D 04 0.2366000D 05-0.7152000D 03 0.9663999D 03 0.1639000D 04 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  3-0.2588000D 05 0.2078000D 04 0.1776005D 07 0.1242000D 06-0.1101000D 05-0.4021000D 06 0.1149000D 05-0.1786000D 03
      -0.3084000D 06 0.1267000D 06-0.3366000D 04 0.1835000D 03 0.5304000D 05-0.1987000D 05 0.1568000D 04-0.6024150D 06
      -0.3794000D 05 0.4158000D 04-0.7223999D 03 0.7610000D 04 0.7015359D 06-0.1997000D 04-0.2368629D 06-0.6153913D 06
      -0.4738502D 05-0.9827868D 05 0.2604436D 06 0.9368607D 04-0.9909156D 05-0.1182079D 05

ROW  4-0.5868000D 06 0.4167000D 05 0.1242000D 06 0.1375000D 07 0.1572000D 06-0.6566000D 06-0.7197000D 06-0.4198000D 04
       0.9260000D 05 0.6236000D 05 0.1270000D 06-0.4234000D 04 0.4556000D 05 0.4307000D 05-0.1682000D 05-0.3243000D 05
       0.4651000D 05-0.2134000D 05 0.6002000D 04-0.1229000D 05-0.5913000D 04 0.6553000D 04 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  5-0.3321000D 05 0.4228000D 05-0.1101000D 05 0.1572000D 06 0.2225000D 07 0.1076000D 05-0.9356000D 06-0.7820000D 06
      -0.3609000D 04 0.1728000D 06-0.1282000D 06 0.1834000D 06-0.2480000D 05 0.7325000D 05 0.6222000D 05 0.7425000D 04
      -0.3192000D 05 0.7848000D 05-0.7298000D 05 0.1004000D 05-0.1059000D 05-0.2028000D 05 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  6 0.1378000D 06-0.1048000D 05-0.4021000D 06-0.6558000D 06 0.1077000D 05 0.1423000D 07 0.2831000D 06 0.1817000D 04
      -0.3498000D 06-0.8331000D 06-0.1484000D 05 0.7087000D 02 0.1400000D 06 0.1090000D 06-0.2099000D 04 0.5230000D 05
       0.5840000D 05-0.1666000D 05 0.1617000D 04-0.1704000D 05-0.1183000D 05 0.5510000D 04 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  7 0.1520000D 06-0.1108000D 05 0.1149000D 05-0.7197000D 06-0.9356000D 06 0.2831000D 06 0.1964000D 07 0.3375000D 06
      -0.9524000D 04-0.9197000D 06-0.8679000D 06-0.1543000D 05 0.1099000D 06 0.2296000D 06 0.1013000D 06-0.2118000D 05
       0.6233000D 05 0.5394000D 05-0.1756000D 05-0.1527000D 05-0.2904000D 05-0.1292000D 05 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  8 0.7940999D 03-0.9974000D 05-0.1778000D 03-0.4198000D 04-0.7820000D 06 0.1815000D 04 0.3375000D 06 0.1261000D 07
       0.5523999D 03-0.2490000D 05-0.7327000D 06-0.5146000D 06-0.7550000D 02 0.9288000D 05 0.1614000D 06 0.7450999D 03
      -0.1258000D 05 0.3373000D 05 0.1247000D 06 0.3865000D 04-0.8040000D 04-0.1176000D 05 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW  9-0.1792000D 05 0.1356000D 04-0.3084000D 06 0.9260000D 05-0.3612000D 04-0.3498000D 06-0.9521000D 04 0.5517000D 03
       0.6243000D 06 0.2487000D 06-0.7556000D 04 0.2398000D 03-0.3528000D 06-0.1400000D 05-0.5367998D 03-0.3077000D 06
       0.1038000D 06 0.1291000D 04 0.2126000D 03-0.1570000D 05-0.1412000D 05 0.2384000D 04 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW 10 0.1086000D 06-0.8541000D 04 0.1267000D 06 0.6237000D 05 0.1728000D 06-0.8331000D 06-0.9197000D 06-0.2491000D 05
       0.2487000D 06 0.2195000D 07 0.3512000D 06-0.7219000D 04-0.8169000D 06-0.8887000D 06-0.1183000D 05 0.1233000D 06
      -0.6308999D 03 0.1237000D 06-0.4119000D 04 0.9815000D 05 0.6560000D 05-0.3449000D 05 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW 11-0.2492000D 05 0.3254000D 05-0.3368000D 04 0.1270000D 06-0.1282000D 06-0.1484000D 05-0.8679000D 06-0.7327000D 06
      -0.7556000D 04 0.3512000D 06 0.2206000D 07 0.2895000D 06-0.1227000D 05-0.8922000D 06-0.8298000D 06-0.4147000D 04
       0.1239000D 06 0.8000000D 03 0.1054000D 06-0.3448000D 05 0.6557000D 05 0.9812000D 05 0.0              0.0
       0.0           0.0            0.0           0.0           0.0           0.0

ROW 12 0.8320000D 03 0.1780000D 04 0.1836000D 03-0.4234000D 04 0.1834000D 06 0.7215999D 02-0.1648000D 05-0.5146000D 06
```

Figure 2.1.8 (cont)

```
        0.2382000D 03-0.7220000D 04 0.2895000D 06 0.5070000D 06-0.1582000D 03-0.2027000D 05-0.3310000D 06 0.4000999D 02
        0.2488000D 04 0.9040000D 05-0.2563000D 06 0.1747000D 04-0.1205000D 05-0.1308000D 05 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 13-0.1827000D 05 0.1661000D 04 0.5304000D 05 0.4556000D 05-0.2479000D 05 0.1400000D 06 0.1099000D 06-0.7509999D 02
      -0.3528000D 06-0.8169000D 06-0.1227000D 05-0.1508000D 03 0.1443000D 07 0.3125000D 06 0.9372000D 04-0.4058000D 06
      -0.7446000D 06-0.2466000D 05-0.2326000D 04 0.1212000D 06 0.1095000D 06-0.1614000D 05 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 14-0.1834000D 05-0.3111000D 04-0.1987000D 05 0.4306000D 05 0.7325000D 05 0.1090000D 06 0.2296000D 06 0.9288000D 05
      -0.1401000D 05-0.8887000D 06-0.8922000D 06-0.2027000D 05 0.3125000D 06 0.1969000D 07 0.3148000D 06 0.5811000D 04
      -0.8406000D 06-0.8406000D 06 0.8490000D 04 0.1164000D 06 0.2380000D 06 0.1164000D 06 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 15 0.3685000D 04 0.7948000D 04 0.1567000D 04-0.1683000D 05 0.6222000D 05-0.2096000D 04 0.1013000D 06 0.1614000D 06
      -0.5360999D 03-0.1183000D 05-0.8298000D 06-0.3310000D 06 0.9368000D 04 0.3148000D 06 0.1440000D 07-0.2076000D 04
      -0.2512000D 05-0.7415000D 06-0.4153000D 06-0.1596000D 05 0.1090000D 06 0.1205000D 06 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 16 0.8397000D 04-0.7558999D 03-0.6024150D 06-0.3243000D 05 0.7422000D 04 0.5230000D 05-0.2118000D 05 0.7450999D 03
      -0.3077000D 06 0.1232000D 06-0.4146000D 04 0.4006000D 02-0.4058000D 06 0.5808000D 04-0.2077000D 04 0.1687948D 07
        0.1406000D 06-0.2947000D 04 0.9382998D 03-0.2292000D 05-0.4836374D 06 0.4332000D 04 0.8400411D 05 0.6732061D 06
      -0.4384501D 05-0.2672137D 05-0.2198473D 06-0.1336269D 06 0.1637380D 06-0.1592884D 06

ROW 17-0.1583000D 05 0.2058000D 04-0.3794000D 05 0.4652000D 05-0.3193000D 05 0.5840000D 05 0.6234000D 05-0.1257000D 05
        0.1038000D 06-0.6490000D 03 0.1239000D 06 0.2488000D 04-0.7446000D 06-0.8406000D 06-0.2511000D 05 0.1406000D 06
        0.1750000D 07 0.2756000D 06-0.3474000D 04-0.5129000D 06-0.4875000D 06 0.3922000D 05 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 18 0.5866000D 04-0.7685000D 04 0.4158000D 04-0.2134000D 05 0.7848000D 05-0.1667000D 05 0.5394000D 05 0.3373000D 05
        0.1293000D 04 0.1237000D 06 0.7857000D 03 0.9039000D 05-0.2466000D 05-0.8406000D 06-0.7414000D 06-0.2948000D 04
        0.2756000D 06 0.1749000D 07 0.1465000D 06 0.3920000D 05-0.4874000D 06-0.5127000D 06 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 19-0.1378000D 04 0.2366000D 05-0.7217998D 03 0.6004000D 04-0.7298000D 05 0.1614000D 04-0.1757000D 05 0.1247000D 06
        0.2132000D 03-0.4118000D 04 0.1054000D 06-0.2563000D 06-0.2327000D 04 0.8489000D 04-0.4153000D 06 0.9380000D 03
      -0.3470000D 04 0.1465000D 06 0.7748000D 06 0.4614000D 04-0.2055000D 05-0.2407000D 05 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 20 0.4207000D 04-0.7147998D 03 0.7611000D 04-0.1229000D 05 0.1004000D 05-0.1704000D 05-0.1527000D 05 0.3862000D 04
      -0.1570000D 05 0.9815000D 05-0.3448000D 05 0.1748000D 04 0.1212000D 06 0.1164000D 06-0.1596000D 05-0.2291000D 05
      -0.5129000D 06 0.3920000D 05 0.4616000D 04 0.5365000D 06-0.7154000D 05 0.3710000D 05 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 21 0.2412000D 04 0.9665999D 03 0.7015361D 06-0.5915000D 04-0.1059000D 05-0.1163000D 05-0.2904000D 05-0.6042000D 04
      -0.1412000D 05 0.6560000D 05 0.6557000D 05-0.1205000D 05 0.1095000D 06 0.2380000D 06 0.1090000D 06-0.4836374D 06
      -0.4875000D 06-0.4874000D 06-0.2055000D 05-0.7153000D 05 0.1580090D 07-0.7155000D 05-0.1038257D 06-0.5929060D 06
        0.2530100D 06-0.3397350D 06 0.1791685D 06 0.2012550D 06-0.4168403D 06-0.1341930D 06

ROW 22-0.1865000D 04 0.1639000D 04-0.1997000D 04 0.6552000D 04-0.2028000D 05 0.5512000D 04-0.1292000D 05-0.1177000D 05
        0.2384000D 04-0.3449000D 05 0.9813000D 05-0.1308000D 05-0.1614000D 05 0.1164000D 06 0.1205000D 06 0.4332000D 04
        0.3923000D 05-0.5127000D 06-0.2407000D 05 0.3710000D 05-0.7155000D 05 0.5365000D 06 0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0

ROW 23 0.0           0.0          -0.2368629D 06 0.0           0.0           0.0           0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.8400409D 05
        0.0           0.0           0.0           0.0          -0.1038256D 06 0.0           0.2196000D 06-0.5189000D 05
      -0.1431000D 05 0.3661000D 05 0.2141000D 04 0.2969000D 05 0.7105999D 03-0.2472000D 04

ROW 24 0.0           0.0          -0.6153913D 06 0.0           0.0           0.0           0.0           0.0
        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.6732061D 06
        0.0           0.0           0.0           0.0          -0.5929060D 06 0.0          -0.5189000D 05 0.7985000D 06
```

Figure 2.1.8 (Cont)

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       -0.20160000D 06 0.51080000D 05-0.28090000D 06 0.33390000D 05 0.13320000D 06 0.21520000D 04

 ROW 25 0.0           0.0           -0.4738502D 05 0.0           0.0           0.0           0.0           0.0
        0.0           0.0            0.0           0.0           0.0           0.0           0.0          -0.4384501D 05
        0.0           0.0            0.0           0.0           0.25301000D 06 0.0         -0.14310000D 05-0.20160000D 06
        0.70120000D 06-0.20570000D 06-0.18290000D 06-0.12050000D 06 0.40240000D 05 0.27510000D 05

 ROW 26 0.0           0.0           -0.9827868D 05 0.0           0.0           0.0           0.0           0.0
        0.0           0.0            0.0           0.0           0.0           0.0           0.0          -0.2672137D 05
        0.0           0.0            0.0           0.0          -0.33973500D 06 0.0          0.36610000D 05 0.51080000D 05
       -0.20570000D 06 0.27120000D 06 0.12960000D 06-0.17920000D 06 0.34060000D 05 0.55880000D 05

 ROW 27 0.0           0.0            0.2604436D 06 0.0           0.0           0.0           0.0           0.0
        0.0           0.0            0.0           0.0           0.0           0.0           0.0          -0.2198473D 06
        0.0           0.0            0.0           0.0           0.17916850D 06 0.0          0.21410000D 04-0.28090000D 06
       -0.18290000D 06 0.12960000D 06 0.41140000D 06-0.16250000D 06-0.26910000D 06 0.29220000D 04

 ROW 28 0.0           0.0            0.9368607D 04 0.0           0.0           0.0           0.0           0.0
        0.0           0.0            0.0           0.0           0.0           0.0           0.0          -0.1336269D 06
        0.0           0.0            0.0           0.0           0.20125500D 06 0.0          0.29690000D 05 0.33390000D 05
       -0.12050000D 06-0.17920000D 06-0.16250000D 06 0.71120000D 06-0.19640000D 06-0.13480000D 05

 ROW 29 0.0           0.0           -0.9909169D 05 0.0           0.0           0.0           0.0           0.0
        0.0           0.0            0.0           0.0           0.0           0.0           0.0           0.1637381D 06
        0.0           0.0            0.0           0.0          -0.41684030D 06 0.0          0.71070000D 03 0.13320000D 06
        0.40240000D 05 0.34060000D 05-0.26910000D 06-0.19640000D 06 0.61740000D 06-0.41990000D 05

 ROW 30 0.0           0.0           -0.1182079D 05 0.0           0.0           0.0           0.0           0.0
        0.0           0.0            0.0           0.0           0.0           0.0           0.0          -0.1592884D 06
        0.0           0.0            0.0           0.0          -0.13419300D 06 0.0         -0.24720000D 04 0.21520000D 04
        0.27510000D 05 0.55880000D 05 0.29220000D 04-0.13480000D 05-0.41990000D 05 0.21890000D 06
```

Figure 2.1.9

MODE SHAPES & FREQUENCIES

	MODE 1	MODE 2	MODE 3	MODE 4
EIGENVALUE=	0.19651D-05	0.55108D-06	0.28643D-06	0.19232D-06
FREQUENCY=	0.11353D 03	0.21439D 03	0.29738D 03	0.36292D 03
EIGENVECTOR				
	-0.30581D-01	-0.17804D 00	-0.11513D 00	-0.14914D 00
	0.23246D 00	0.10000D 01	-0.29798D 00	0.10000D 01
	0.13529D 00	-0.18654D 00	-0.22949D 00	0.39701D 00
	0.76182D-01	-0.23406D 00	-0.36736D 00	-0.14462D 00
	0.11190D 00	0.80631D-01	-0.24461D 00	-0.19193D 00
	0.20352D 00	-0.26744D 00	-0.53199D 00	0.26420D 00
	0.20653D 00	-0.56150D-01	-0.48294D 00	-0.28270D 00
	0.24826D 00	0.39896D 00	-0.37626D 00	-0.28724D 00
	0.22776D 00	-0.34598D 00	-0.67376D 00	0.74952D 00
	0.31962D 00	-0.13978D 00	-0.58949D 00	-0.40864D-01
	0.33453D 00	0.15902D 00	-0.46738D 00	-0.44089D 00
	0.26736D 00	0.53369D 00	-0.32326D 00	-0.44336D 00
	0.33228D 00	-0.19440D 00	-0.60002D 00	0.29251D 00
	0.48068D 00	0.25863D-01	-0.44987D 00	-0.30072D 00
	0.35243D 00	0.26002D 00	-0.32838D 00	-0.50937D 00
	0.98560D-01	-0.16603D 00	-0.45395D 00	0.39596D 00
	0.56713D 00	-0.34741D-01	-0.30751D 00	-0.84251D-01
	0.57088D 00	0.10190D 00	-0.21357D 00	-0.38579D 00
	0.12266D 00	0.19046D 00	-0.14144D 00	-0.31060D 00
	0.39510D 00	0.85769D-02	-0.38062D-01	-0.65683D-01
	0.81226D 00	0.34505D-01	0.38173D-01	-0.21828D 00
	0.39508D 00	0.32868D-01	-0.97549D-04	-0.18763D 00
	0.51207D 00	-0.93823D-01	0.14589D 00	0.89583D-01
	0.92032D 00	-0.61167D-02	0.62539D 00	-0.86527D-01
	0.56505D 00	-0.10944D 00	0.30458D 00	0.31133D 00
	0.94719D 00	-0.15973D 00	-0.38358D 00	0.30884D 00
	0.10000D 01	-0.19403D-01	0.10000D 01	0.13134D 00
	0.54670D 00	-0.98357D-01	0.19214D 00	0.30276D 00
	0.90060D 00	0.10921D-01	0.47358D 00	-0.50921D-01
	0.45936D 00	-0.62522D-01	-0.19010D 00	0.75833D-01

Fig. 2.1.9 (con't

MODE 5	MODE 6	MODE 7	MODE 8
EIGENVALUE= 0.12965D-06	EIGENVALUE= 0.10408D-06	EIGENVALUE= 0.81465D-07	EIGENVALUE= 0.57610D-07
FREQUENCY= 0.44201D 03	FREQUENCY= 0.49333D 03	FREQUENCY= 0.55761D 03	FREQUENCY= 0.66309D 03
EIGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
0.27675D 00	0.14138D 00	-0.44726D-01	-0.25280D 00
-0.24522D-01	0.24049D 00	-0.40280D 00	-0.21050D 00
0.17291D 00	0.46092D 00	0.29299D 00	0.26078D 00
0.40416D 00	0.20747D 00	-0.19001D 00	-0.28797D 00
0.24383D 00	0.11447D-01	-0.29074D-01	-0.10487D 00
0.31730D 00	0.32868D 00	-0.42027D-01	0.66089D-01
0.38361D 00	0.84459D-01	-0.30120D 00	-0.30514D 00
0.40441D 00	-0.29375D-01	0.33751D 00	0.10022D 00
0.16195D 00	0.12343D 00	0.34566D 00	0.58417D 00
0.22327D 00	0.10351D 00	-0.38718D 00	-0.14599D 00
0.34296D 00	-0.22033D-02	-0.55031D-01	-0.14555D 00
0.49699D 00	-0.12570D 00	0.10000D 01	0.53355D 00
-0.56750D-02	-0.11329D 00	-0.15234D 00	0.22079D 00
0.21889D-01	0.29386D-02	-0.35397D 00	-0.16034D 00
0.28010D 00	-0.87508D-01	0.41892D 00	0.17605D 00
-0.14472D-01	-0.35701D 00	0.14454D 00	0.31627D 00
-0.33984D 00	-0.11150D 00	-0.32014D 00	0.43683D-01
-0.21189D 00	-0.86028D-01	-0.49263D-01	0.42024D-02
0.25216D 00	-0.83283D-01	0.55039D 00	0.29382D 00
-0.40521D 00	-0.12525D 00	-0.15264D 00	0.83474D-01
-0.73918D 00	-0.17711D 00	-0.10735D 00	0.10319D 00
-0.35542D 00	-0.83580D-01	-0.16362D-01	0.55635D-01
-0.11818D-01	0.10000D 01	0.83573D-01	0.56623D 00
0.94122D-01	0.61270D 00	-0.44843D-01	0.68651D-01
0.41042D 00	0.27924D-01	0.31087D 00	-0.35190D 00
-0.92185D 00	-0.34353D 00	0.81090D 00	-0.85753D 00
0.10000D 01	-0.41595D 00	-0.40047D 00	0.21741D 00
0.31079D 00	-0.47813D 00	0.15928D 00	-0.20047D 00
0.53485D-01	-0.47099D 00	-0.29568D 00	0.21271D 00
-0.31056D 00	-0.47236D 00	-0.32158D 00	0.10000D 01

Fig 2.1.9 (cont)

	MODE 9	MODE 10	MODE 11	MODE 12
EIGENVALUE=	0.40305D-07	0.28655D-07	0.20255D-07	0.16168D-07
FREQUENCY=	0.79276D 03	0.94020D 03	0.11183D 04	0.12517D 04
EIGENVECTOR	0.10000D 01	-0.44886D 00	-0.26620D 00	0.62424D 00
	0.27486D 00	0.22405D-01	-0.54929D-01	0.45396D-01
	-0.65856D-01	0.82913D-01	0.84989D-01	-0.15511D 00
	0.67274D 00	-0.18847D 00	-0.93814D-01	-0.36222D-01
	-0.70864D-01	-0.98485D-01	0.37654D 00	-0.11686D 00
	0.12293D 00	0.11095D 00	-0.18566D 00	-0.43502D 00
	0.28220D-02	0.69464D-02	0.36522D 00	-0.20839D 00
	-0.95205D-01	-0.25668D 00	0.27997D 00	-0.12117D 00
	-0.15303D-01	0.13608D 00	-0.22099D 00	-0.13476D 00
	-0.25516D 00	0.12930D 00	-0.10882D 00	-0.26126D 00
	-0.18592D 00	0.74927D-01	0.42198D 00	0.10536D 00
	0.10457D 00	-0.38240D 00	-0.54904D 00	-0.21260D 00
	-0.29082D 00	-0.76432D-01	-0.35473D 00	0.69182D-01
	-0.33480D 00	0.18622D 00	0.23164D-01	0.23492D 00
	0.40450D-01	0.17526D 00	-0.13715D 00	0.16077D 00
	0.75004D-01	0.69159D-01	-0.11738D-01	0.22422D 00
	-0.35274D 00	-0.45482D 00	-0.32151D 00	0.33832D 00
	0.12004D 00	0.62641D 00	-0.10921D 00	0.28290D 00
	0.83940D-01	-0.11951D 00	-0.39439D 00	-0.65759D-01
	-0.18614D 00	-0.83549D 00	-0.29629D 00	0.52156D 00
	0.25080D 00	-0.13511D 00	-0.68903D-01	-0.10117D 00
	0.36888D 00	0.10000D 01	-0.36388D 00	0.27436D 00
	-0.50192D-01	0.22733D-01	0.29860D 00	0.36508D 00
	-0.50610D-01	-0.54671D-01	-0.30187D-01	0.16664D 00
	-0.18019D 00	0.67075D-01	0.13512D 00	0.10000D 01
	0.14334D 00	-0.12460D 00	0.25851D 00	-0.18439D 00
	-0.43809D-01	0.29286D-01	-0.25170D 00	0.41822D-01
	0.28157D-01	0.17331D-01	0.11466D 00	-0.71911D 00
	0.22908D 00	-0.10136D 00	-0.89182D-01	-0.88705D 00
	0.54995D 00	-0.11329D 00	0.10000D 01	0.47407D 00

Fig 2.1.9 (cont)

ODE 13	MODE 14	MODE 15	MODE 16
IGENVALUE= 0.15975D-07	EIGENVALUE= 0.13276D-07	EIGENVALUE= 0.12680D-07	EIGENVALUE= 0.10913D-07
REQUENCY= 0.12592D 04	FREQUENCY= 0.13813D 04	FREQUENCY= 0.14134D 04	FREQUENCY= 0.15235D 04
IGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
0.21618D 00	-0.29000D 00	0.41173D 00	0.34548D 00
-0.32651D-02	0.80116D-01	0.16336D 00	-0.84774D-01
-0.79311D-02	-0.34792D 00	0.43362D 00	0.15792D 00
-0.40029D-01	0.15025D 00	-0.14177D 00	-0.92391D-01
0.92122D-01	-0.37455D 00	-0.58730D 00	0.40489D-01
-0.21552D 00	0.12490D 00	-0.37957D-01	-0.11151D 00
0.55296D-02	0.19699D-01	-0.40673D 00	-0.22874D 00
0.87197D-01	-0.54362D 00	-0.89217D 00	0.42709D 00
0.43406D-01	-0.73892D 00	0.19324D 00	-0.46818D 00
-0.19076D 00	0.45240D 00	-0.30410D 00	0.12366D 00
0.16732D 00	-0.36185D 00	0.43991D 00	-0.23217D 00
-0.25185D 00	0.50139D 00	-0.67458D 00	0.42927D 00
-0.33338D-01	0.14469D-01	-0.35988D 00	0.88401D-01
0.56123D-01	0.35178D-01	0.64053D 00	0.17094D 00
0.58383D-02	0.20490D 00	0.10000D 01	-0.50694D 00
0.16866D 00	-0.21703D 00	-0.71666D-01	-0.39336D 00
0.44907D-01	-0.94290D-02	0.80400D-01	0.37845D 00
0.92628D-01	-0.85318D-01	0.62702D 00	-0.16968D 00
-0.17530D 00	0.58428D 00	0.25350D 00	-0.20505D 00
0.15642D 00	-0.31256D 00	0.27685D 00	0.37501D 00
0.88498D-02	-0.61425D-01	-0.37893D 00	-0.50753D 00
0.55737D-01	-0.27357D 00	-0.89032D 00	0.86947D 00
0.10000D 01	0.10000D 01	-0.56663D 00	-0.58100D-01
-0.26164D 00	-0.23640D 00	-0.83446D-01	-0.15628D 00
-0.18871D 00	0.12519D 00	-0.12424D 00	0.13832D 00
0.17719D-01	0.20921D 00	-0.37855D 00	-0.66533D 00
0.28572D-01	-0.18386D 00	-0.30371D 00	-0.72195D 00
0.20810D 00	0.16305D 00	0.59959D 00	0.10000D 01
0.15651D 00	-0.36358D-01	-0.14828D 00	-0.53291D 00
-0.46664D 00	0.36352D 00	0.17558D 00	0.42361D 00

Fig 2.1.9 (cont)

ODE 17	MODE 18	MODE 19	MODE 20
IGENVALUE= 0.94248D-08	EIGENVALUE= 0.77534D-08	EIGENVALUE= 0.70912D-08	EIGENVALUE= 0.57876D-08
REQUENCY= 0.16394D 04	FREQUENCY= 0.18075D 04	FREQUENCY= 0.18900D 04	FREQUENCY= 0.20920D 04
IGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
-0.55100D 00	-0.11011D-02	0.25170D 00	-0.39331D 00
0.42116D-01	-0.22670D-01	0.11802D 00	-0.30989D-01
0.19603D-01	0.53657D-01	0.85610D-01	-0.80496D 00
0.24289D 00	-0.71259D-03	-0.19959D 00	0.34288D 00
0.90211D-01	-0.68101D-01	0.52453D 00	-0.48270D-01
0.24965D 00	0.17710D 00	-0.99020D 00	0.31655D 00
0.30506D 00	-0.16245D 00	0.89885D 00	-0.23538D 00
-0.18967D 00	0.12227D 00	-0.64253D 00	0.31165D 00
0.24245D-01	0.45125D-01	0.16149D 00	0.69407D 00
-0.11152D 00	-0.30084D-02	-0.40569D-01	-0.25273D 00
0.29039D-01	-0.79110D-03	0.65727D-01	-0.53000D-01
-0.12552D 00	0.49415D-01	-0.44047D 00	-0.11549D 00
-0.33701D 00	-0.13110D 00	0.90881D 00	0.24163D 00
-0.29476D 00	0.12325D 00	-0.75711D 00	-0.28210D 00
0.16678D 00	-0.75130D-01	0.44897D 00	0.72356D-01
0.45773D-01	-0.82594D-01	-0.62302D-02	0.91698D-01
-0.74431D-01	0.48201D-03	0.80874D-01	-0.66306D-01
-0.31437D-01	0.57614D-01	-0.37689D 00	0.44740D 00
0.18258D 00	-0.16437D 00	0.10000D 01	-0.59596D 00
0.10000D 01	0.15250D-01	-0.40670D 00	-0.38968D 00
-0.14474D-01	-0.10438D-01	0.17637D 00	0.10000D 01
0.25838D 00	-0.43922D-01	0.20973D 00	-0.50077D 00
0.38233D-02	-0.10758D-01	-0.15702D 00	0.60326D-01
-0.78325D-01	-0.72486D 00	0.83387D-02	0.83312D 00
0.63552D-01	0.41966D 00	0.25446D 00	0.29229D 00
-0.12715D 00	-0.27297D 00	-0.46467D 00	-0.79788D 00
-0.53059D-01	-0.42770D 00	-0.59059D 00	-0.96141D 00
0.57776D-01	-0.21578D 00	0.12289D 00	0.45220D 00
-0.57622D-01	0.10000D 01	0.43866D 00	-0.15749D 00
-0.30688D-01	-0.73850D-01	-0.13100D 00	-0.27218D 00

Fig 2.1.9 (cont)

DE 21	MODE 22	MODE 23	MODE 24
IGENVALUE= 0.45785D-08	EIGENVALUE= 0.43724D-08	EIGENVALUE= 0.40889D-08	EIGENVALUE= 0.38624D-08
REQUENCY= 0.22556D 04	FREQUENCY= 0.24069D 04	FREQUENCY= 0.24890D 04	FREQUENCY= 0.25609D 04
IGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
0.29633D 00	-0.89959D 00	0.19821D-01	0.29810D-01
-0.14395D-01	0.20868D-01	0.27657D-01	-0.39924D-02
-0.62335D 00	0.14067D 00	-0.25663D 00	-0.95755D-01
-0.33120D 00	0.10000D 01	-0.62207D-01	-0.15640D-01
0.83962D-01	-0.96449D-01	0.20998D 00	-0.12252D 00
0.50065D 00	-0.24325D 00	0.92668D-01	-0.60744D 00
-0.32786D 00	-0.17261D 00	0.15647D 00	0.55728D 00
0.20546D 00	0.10164D 00	-0.20655D 00	-0.30854D 00
0.10000D 01	-0.91934D-01	0.10000D 01	0.86164D 00
0.40343D-01	-0.86398D 00	-0.22547D 00	0.14747D 00
0.23454D 00	0.49503D-01	0.55821D-01	-0.23421D 00
-0.69190D 00	-0.13673D 00	0.63990D-01	0.10000D 01
0.55728D 00	0.38232D 00	-0.52884D 00	0.52107D-01
-0.46871D 00	-0.13657D 00	0.50086D 00	0.13621D 00
0.49633D 00	0.18402D 00	-0.41591D 00	-0.31906D 00
-0.98886D 00	-0.24665D 00	-0.16324D 00	-0.16020D 00
-0.19033D 00	0.83028D 00	0.24730D 00	-0.39650D 00
-0.87373D-01	0.14571D-01	-0.22740D 00	0.49309D 00
-0.13170D 00	-0.25731D 00	0.94641D 00	-0.79572D 00
0.35988D 00	-0.96069D 00	-0.30088D 00	0.36168D 00
-0.41554D 00	-0.11130D 00	0.66220D-01	0.28888D 00
0.13327D 00	0.47818D-01	0.10271D 00	-0.26591D 00
0.23279D 00	0.10739D-01	0.50709D-01	0.10872D-01
-0.42233D 00	-0.93059D 00	0.61336D-02	-0.63003D 00
-0.45819D-01	0.12709D-01	0.15471D-01	0.61869D-01
0.36570D 00	0.54131D-01	-0.67381D-03	-0.99529D-01
0.31577D 00	0.60703D 00	-0.18370D-01	0.34535D 00
-0.36002D-01	-0.17694D-01	0.23108D-01	0.48296D-01
-0.14649D 00	-0.50612D 00	-0.42232D-01	-0.44046D 00
0.31151D 00	0.83061D-01	0.21641D-01	0.13796D-02

Fig 2.19 (cont)

ODE 25	MODE 26	MODE 27	MODE 28
IGENVALUE= 0.37384D-C8	EIGENVALUE= 0.30801D-08	EIGENVALUE= 0.28635D-08	EIGENVALUE= 0.23246D-08
REQUENCY= 0.26030D 04	FREQUENCY= 0.28677D 04	FREQUENCY= 0.29742D 04	FREQUENCY= 0.33010D 04
IGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
0.32361D 00	-0.20762D 00	0.79828D-01	-0.98732D-01
-0.79121D-02	-0.10200D-02	0.70909D-02	-0.26011D-01
-0.28139D-02	0.69473D-01	0.76997D-01	0.52828D-01
-0.42551D 00	0.33373D 00	-0.15698D 00	0.28124D 00
0.59825D 00	0.22976D 00	0.10000D 01	-0.18404D 00
0.58404D 00	-0.35632D 00	0.27126D 00	-0.91066D 00
-0.13791D 00	-0.30267D 00	0.18388D 00	-0.19976D 00
0.22481D 00	0.33801D 00	-0.26305D 00	0.65309D 00
-0.64632D 00	-0.14844D-01	-0.19655D-01	0.10000D 01
0.23326D 00	0.53425D-02	-0.17403D 00	0.87141D 00
-0.34503D 00	-0.36178D 00	-0.79546D 00	-0.82944D 00
-0.64508D 00	-0.48735D 00	0.31334D 00	-0.83847D 00
-0.26244D 00	0.40295D 00	-0.18265D 00	-0.91935D 00
-0.30458D 00	0.26656D 00	-0.14645D 00	-0.14454D C0
0.47570D-02	-0.37185D 00	0.21940D 00	0.80692D 00
0.35546D 00	-0.11186D 00	-0.15011D 00	0.53981D-01
0.42682D-01	-0.37687D 00	0.40326D 00	0.30351D 00
0.29012D 00	0.58599D 00	0.62870D 00	-0.31723D 00
0.78965D 00	0.10000D 01	-0.56131D 00	-0.21630D C0
-0.16924D 00	0.36876D 00	-0.16791D 00	-0.13919D 00
0.10000D 01	-0.27739D 00	-0.56416D 00	-0.20373D-02
-0.25151D 00	-0.21633D C0	-0.18718D 00	0.70430D-01
-0.38978D-01	0.10598D-02	0.16383D-01	-0.51779D-02
-0.96656D 00	0.78740D-01	0.23120D 00	0.37236D-02
0.14938D 00	-0.27585D-01	-0.53250D-01	-0.11986D-02
-0.37510D 00	0.74346D-01	0.14005D 00	-0.27958D-02
0.50508D 00	-0.30177D-01	-0.97192D-01	-0.15171D-03
0.11806D 00	-0.23260D-01	-0.43914D-01	-0.18296D-02
-0.82715D 00	0.10497D 00	0.22210D 00	0.33034D-02
-0.15572D 00	0.37991D-01	0.62908D-01	-0.47605D-02

Fig 2.1.9 (cont)

MODE 29

EIGENVALUE= 0.19952D-08
FREQUENCY= 0.35631D 04

EIGENVECTOR

-0.13472D 00
0.29190D-01
-0.70724D-01
0.26747D 00
0.10000D 01
-0.14864D 00
-0.80140D 00
-0.48837D 00
-0.43554D-01
0.48694D-01
-0.25322D-01
0.26053D 00
0.21286D 00
0.74745D 00
0.42405D 00
0.92738D-01
-0.42938D 00
-0.75788D 00
-0.47768D 00
0.17281D 00
0.35097D 00
0.17092D 00
-0.55597D-02
-0.63253D-01
0.19707D-01
-0.53688D-01
0.21551D-01

0.15853D-01
-0.70547D-01
-0.26472D-01

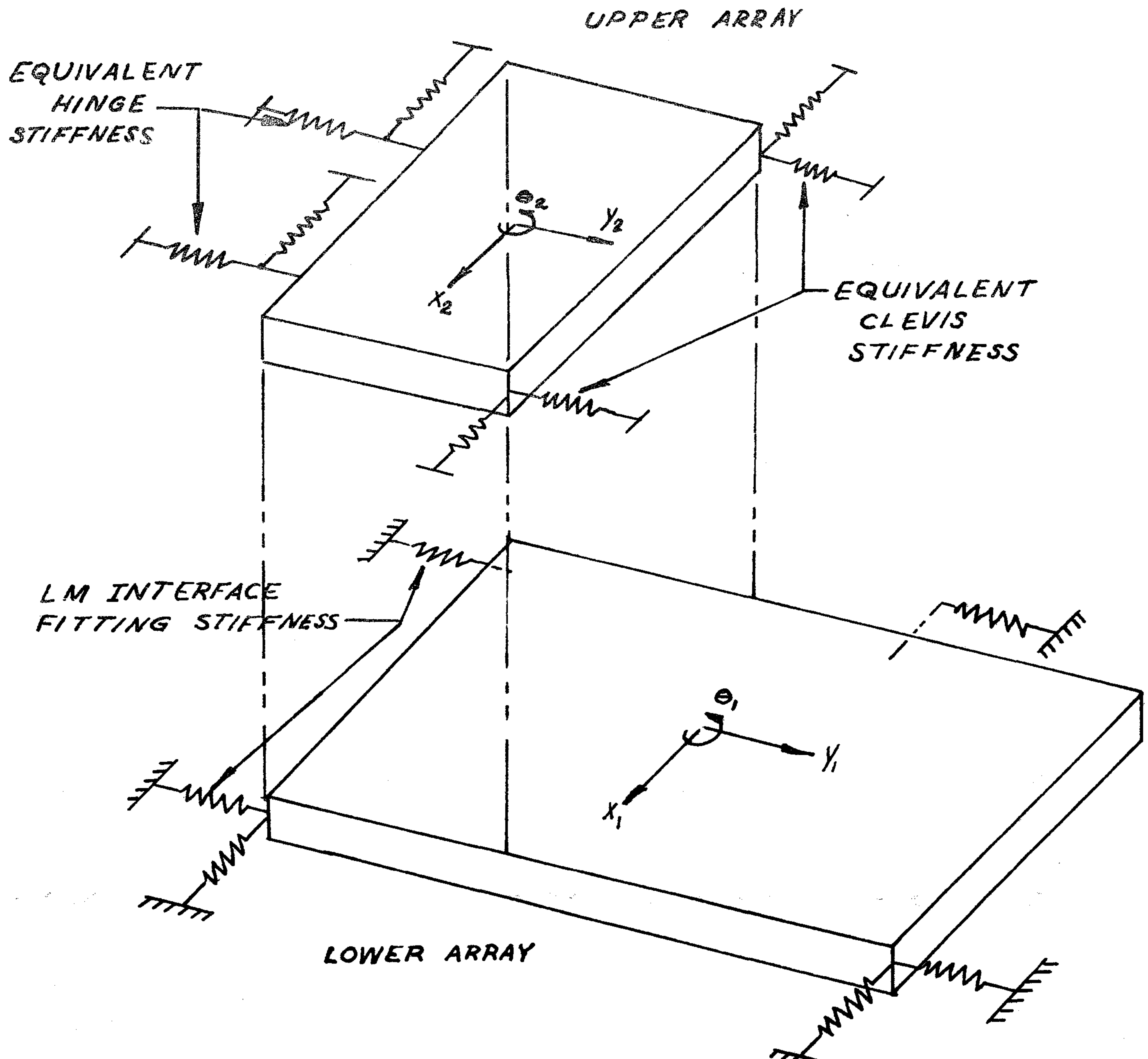
MODE 30

EIGENVALUE= 0.14337D-08
FREQUENCY= 0.42032D 04

EIGENVECTOR

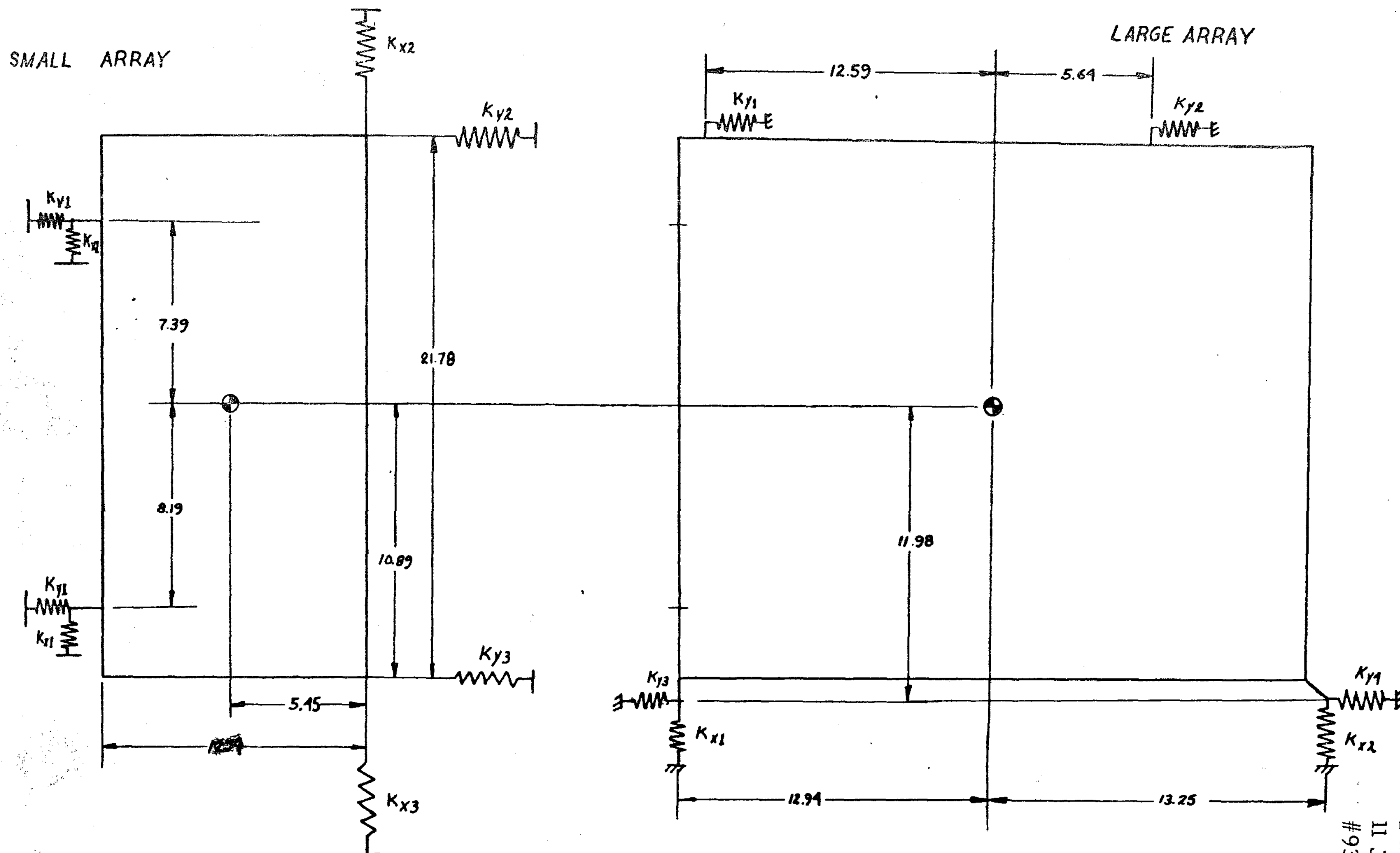
-0.71511D-01
0.22505D-01
0.59460D-01
0.20888D 00
0.45062D 00
-0.33214D 00
-0.87414D 00
-0.47696D 00
0.24007D 00
0.60611D 00
0.10000D 01
0.42398D 00
-0.35814D 00
-0.90804D 00
-0.52015D 00
-0.10903D-01
0.27214D 00
0.45621D 00
0.11676D 00
-0.81480D-01
-0.13948D 00
-0.72068D-01
-0.32406D-03
0.14487D-01
-0.54739D-02
0.13305D-01
-0.38730D-02

-0.45130D-02
0.17915D-01
0.59894D-02



IN PLANE DYNAMIC MODEL

Figure 2.2.1



TOP VIEWS OF ARRAYS

Figure 2.2.2

FIGURE 2.2.3
STIFFNESS MATRICES
IN PLANE MODEL

Lower Array

$$[K_L] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

$K_{X1} = 8.00\times10^5$ $\quad K_{Y2} = 8.00\times10^5$

$K_{X2} = 8.77\times10^5$ $\quad K_{Y3} = 9.08\times10^5$ (see Fig. 2.2.2)

$K_{Y1} = 8.85\times10^5$ $\quad K_{Y4} = 9.53\times10^5$

$$k_{11} = K_{X1} + K_{X2}$$

$$k_{12} = k_{21} = 0$$

$$k_{31} = k_{13} = -12.94^2 K_{X1} + 13.25^2 K_{X2}$$

$$k_{22} - K_{Y1} + K_{Y2} + K_{Y3} + K_{Y4}$$

$$k_{23} = k_{32} = 10.89^2(K_{Y1}+K_{Y2}) - 11.98^2(K_{Y3}+K_{Y4})$$

$$k_{33} = 11.98^2(K_{Y3}+K_{Y4}) + 10.89^2(K_{Y1}+K_{Y2}) + 12.94^2 K_{X1} + 13.25^2 K_{X2}$$

$$[K_L] = \begin{bmatrix} 1.68\times10^6 & 0 & -1.99\times10^7 \\ 0 & 3.55\times10^6 & -6.65\times10^7 \\ -1.99\times10^7 & -6.65\times10^7 & 7.53\times10^5 \end{bmatrix}$$

Upper Array

$$[K_U] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

$K_{X1} = 1.52\times10^4$ $\quad K_{Y1} = 1.76\times10^3$

$K_{X2} = 3.22\times10^2$ $\quad K_{Y2} = 6.17\times10^4$ (see Fig. 2.2.2)

$K_{X3} = 1.71\times10^4$ $\quad K_{Y3} = 3.61\times10^4$

$$k_{11} = 2K_{X1} + K_{X2} + K_{X3}$$

$$k_{21} = k_{12} = 0$$

$$k_{22} = 2K_{Y1} + K_{Y2} + K_{Y3}$$

$$k_{32} = k_{23} = 7.39^2 K_{Y1} + 10.89^2 K_{Y2} - 8.19^2 K_{Y1} - 10.9 K_{Y3}$$

$$k_{33} = 10.89^2(K_{Y2} + K_{Y3}) + K_{Y1}(7.39^2 + 8.19^2) + 5.452^2(2K_{X1} + K_{X2} + K_{X3})$$

$$k_{31} = k_{13} = 5.45^2(K_{X2} + K_{X3}) - 5.45^2(2 \times K_{X1})$$

$$[K_U] = \begin{bmatrix} 4.78\times10^4 & 0 & -3.88\times10^5 \\ 0 & 1.01\times10^5 & 3.02\times10^6 \\ -3.88\times10^5 & 3.02\times10^6 & 1.18\times10^7 \end{bmatrix}$$

The coordinate transformation employed is:

$$\begin{Bmatrix} X_L \\ Y_L \\ \theta_L \\ X_{ru} \\ Y_{ru} \\ \theta_{ru} \end{Bmatrix} = [\beta] \begin{Bmatrix} X_1 \\ Y_1 \\ \theta_1 \\ X_2 \\ Y_2 \\ \theta_2 \end{Bmatrix}$$

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & -7.49 & 1 & 0 & 0 \\ 0 & -1 & -7.49 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Where

X_L, Y_L, θ_L = Displacements of the lower array relative to ground

$X_{ru}, Y_{ru}, \theta_{ru}$ = Displacements of the upper array relative to the lower array

X_1, K_1, θ_1 = Generalized coordinates of the lower array relative to ground

X_2, Y_2, θ_2 = Generalized coordinates of the upper array relative to ground

The mass matrix is diagonal; made up of the mass or moment of inertia of the appropriate array.

$$[M] = \begin{bmatrix} M_L & & & & & O \\ & M_L & & & & \\ & & I_L & & & \\ & & & M_u & & \\ & & & & M_u & \\ O & & & & & I_u \end{bmatrix}$$

$$= \begin{bmatrix} .1555 & & & & & O \\ & .1555 & & & & \\ & & 15.0 & & & \\ & & & .0726 & & \\ & & & & .0726 & \\ O & & & & & 3.27 \end{bmatrix}$$

Where

M_u = Mass of the upper array in lb sec^2

M_L = Mass of the lower array in lb sec^2

I_L = Polar moment of inertia of the lower array, lb sec^{2}in.

I_u = Polar moment of inertia of the upper array, lb sec^{2}in.

The resulting eigenvalue problem could then be solved giving the mode shapes and natural frequencies of the system. These are shown in Figure 2.2.4. Modal analysis was used to calculate response of the system in the frequency domain. In calculating responses, 10% of critical viscous damping was used on all but mode 4 which was assigned 20% of critical damping. The value of 10% has been shown by experiment to be typical of such structures. Mode 4 was assigned 20% of critical damping since this mode will cause the rear LM interface fittings to slide on their pins and introduce more damping to the structure.

Figure 2.2.4

MODE 1

EIGENVALUE= 0.14891D-05
FREQUENCY= 0.13042D 03

EIGENVECTOR
0.14240D-01
-0.27280D-01
-0.15979D-02
0.10000D 01
0.10822D 00
-0.74693D-02

MODE 2

EIGENVALUE= 0.61479D-06
FREQUENCY= 0.20298D 03

EIGENVECTOR
0.29105D 00
0.18588D-01
-0.31334D-02
-0.27939D 00
0.10000D 01
-0.42419D 00

MODE 3

EIGENVALUE= 0.15536D-06
FREQUENCY= 0.40379D 03

EIGENVECTOR
0.23427D 00
0.50405D 00
0.57902D-01
-0.46694D-01
0.10000D 01
0.52526D-01

MODE 4

EIGENVALUE= 0.83034D-07
FREQUENCY= 0.55232D 03

EIGENVECTOR
0.10000D 01
-0.26126D 00
0.43344D-02
-0.55388D-02
-0.30928D 00
0.13915D-01

MODE 5

EIGENVALUE= 0.36088D-07
FREQUENCY= 0.83780D 03

EIGENVECTOR
0.32363D-01
-0.45351D 00
-0.55684D-01
-0.13514D 00
0.10000D 01
0.49501D-01

MODE 6

EIGENVALUE= 0.97458D-08
FREQUENCY= 0.16122D 04

EIGENVECTOR
0.27979D 00
0.10000D 01
-0.92726D-01
0.35371D-01
-0.12617D 00
0.63589D-02

2.3 Out-of-Plane Model of Off-Loaded Condition

To model the experiment with the upper array removed, a lumped mass model was again employed. Masses were lumped at 23 locations as shown in Figure 2.3.1. These are the same locations as used for the model of the lower array with the arrays connected. The stiffness matrix calculated for the lower array alone could then be employed directly. Since the attachments are no longer constrained to move in a plane by the upper array, the degree of freedom eliminated by the constraint can no longer be eliminated. The mass matrix becomes a diagonal matrix whose elements are simply the mass lumped at each node. Mass and stiffness matrices for this model are shown in Figure 2.3.2.

Solving the eigenvalue problem again gave mode shapes and frequencies which were used to calculate the frequency response of the system. The mode shapes and frequencies are shown in Figure 2.3.3. Again 10% of critical damping was used in calculating the system's response.

2.4 Vibration Environment

The qualification and acceptance vibration environments to be imposed on the LRRR 300 corner array in tests is that of the LM quadrant III. This then is the environment that was input to the base of the three models. The environment was taken from Exhibit B-1, Design and Performance Specification for the Laser Ranging Retro-Reflector Experiment (300 Corner Array), November 1, 1970. This environment is listed in Table 2.4.1 and the qualification levels shown in Figure 2.4.1.

The responses that were calculated here are responses to the qualification environment. Since the acceptance environment is less severe, the qualification environment then determines the dynamic loads on the structure.

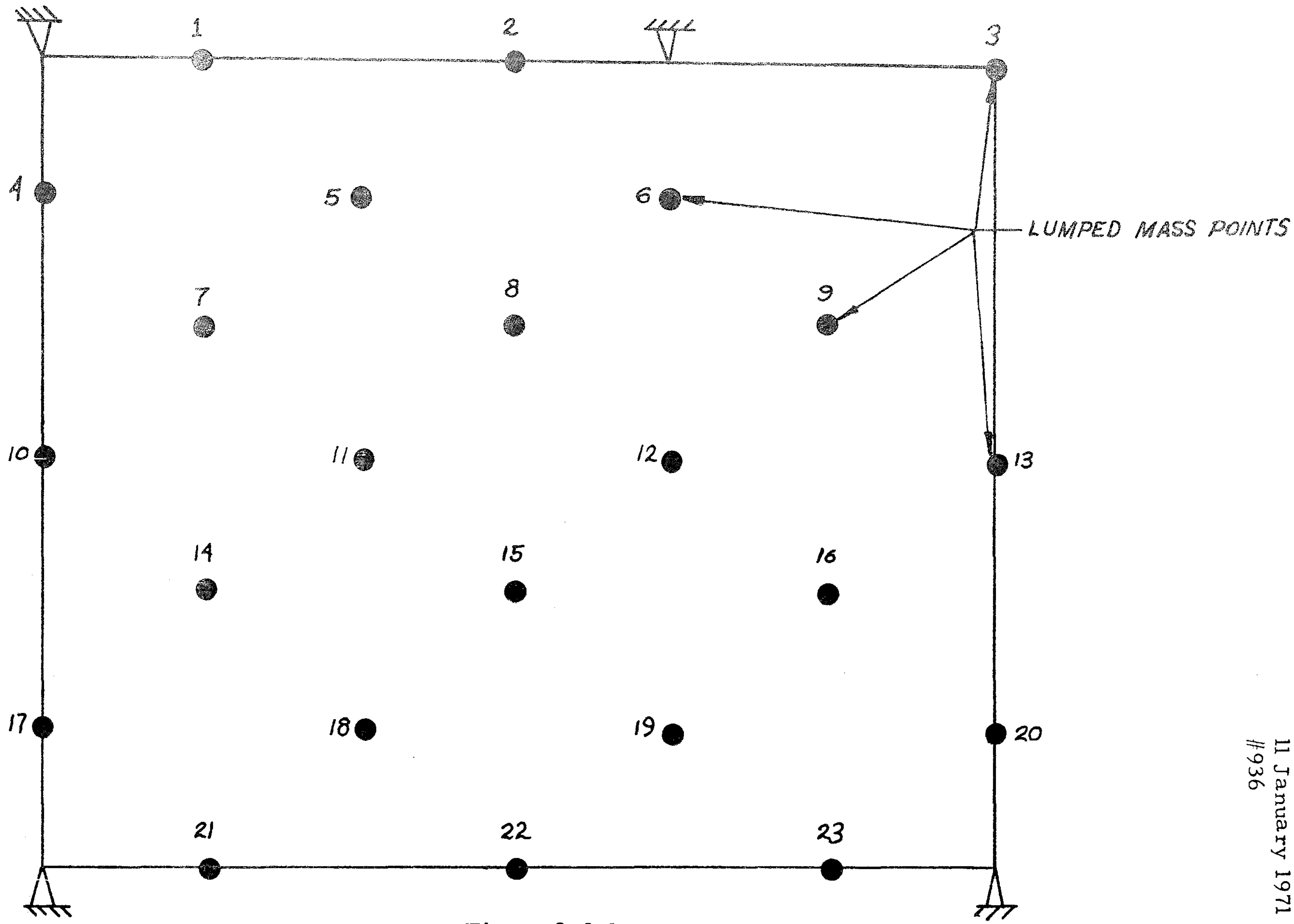


Figure 2.3.1

MODEL OF LR^3 IN OFF LOADED CONDITION

MASS MATRIX -OFF LOADED CONDITION Figure 2.3.2-

1	0.601000D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
2	0.0	0.672000-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
3	0.0	0.0	0.735000-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
4	0.0	0.0	0.0	0.40600D-02	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
5	0.0	0.0	0.0	0.0	0.96200D-02	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
6	0.0	0.0	0.0	0.0	0.0	0.70300D-02
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
7	0.0	0.0	0.0	0.0	0.0	0.0
	0.70900D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
8	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.69600D-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
9	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.70900D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
10	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.35400D-02	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
11	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.96200D-02	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
12	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.70300D-02
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	

	0.0	0.0	0.0	0.0	0.0	0.0
	0.35400D-02	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
14	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.70900D-02	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
15	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.69600D-02	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
16	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.70900D-02	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
17	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.40600D-02	0.0
	0.0	0.0	0.0	0.0	0.0	
18	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.96200D-02
	0.0	0.0	0.0	0.0	0.0	
19	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.70300D-02	0.0	0.0	0.0	0.0	
20	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.35400D-02	0.0	0.0	0.0	
21	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.45200D-02	0.0	0.0	
22	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.55500D-02	0.0	
23	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.67800D-02	

K MATRIX -OFF LOADED CO[illegible]TION

Figure 2.3.2 (cont)

```
ROW  1 0.5445000E 06-0.1206000E 06-0.5851000E 04-0.2205000E 05-0.5017000E 06-0.2842000E 05 0.1176000E 06 0.1301000E 06
       0.5930999E 03-0.1526000E 05 0.9282000E 05-0.2132000E 05 0.7447998E 03-0.1563000E 05-0.1568000E 05 0.3144000E 04
       0.7167000E 04-0.1353000E 05 0.5021000E 04-0.1187000E 04 0.3596000E 04 0.2061000E 04-0.1595000E 04

ROW  2-0.1206000E 06 0.7842000E 06 0.7838000E 05-0.2488000E 05-0.5536000E 06-0.3121000E 05 0.1313000E 06 0.1422000E 06
       0.1300000E 04-0.1730000E 05 0.1024000E 06-0.2342000E 05 0.5702998E 03-0.1719000E 05-0.1732000E 05 0.3497000E 04
       0.8006000E 04-0.1495000E 05 0.5515000E 04-0.1238000E 04 0.3970000E 04 0.2281000E 04-0.1755000E 04

ROW  3-0.5852000E 04 0.7838000E 05 0.5300000E 05-0.4087998E 03-0.1366000E 05 0.3916000E 05 0.2646000E 04 0.3131000E 04
      -0.9961000E 05-0.3732998E 03 0.1695000E 04 0.3019000E 05 0.1837000E 04-0.5665999E 02-0.4842000E 04 0.8297000E 04
       0.4417000E 02 0.5647998E 03-0.7134000E 04 0.2353000E 05-0.3182998E 03 0.1194000E 04 0.1464000E 04

ROW  4-0.2205000E 05-0.2488000E 05-0.4087000E 03 0.7975000E 06 0.1417000E 06-0.1002000E 05-0.4063000E 06 0.6982000E 04
      -0.2198000E 03-0.3078000E 06 0.1235000E 06-0.2623000E 04 0.1654000E 03 0.5358000E 05-0.1932000E 05 0.1458000E 04
       0.1350000E 06-0.3747000E 05 0.3983000E 04-0.6830999E 03 0.7484000E 04 0.5795000E 04-0.1941000E 04

ROW  5-0.5017000E 06-0.5536000E 06-0.1367000E 05 0.1417000E 06 0.1766000E 07 0.1793000E 06-0.7495000E 06-0.8201000E 06
      -0.5115000E 04 0.1048000E 06-0.9949000E 04 0.1435000E 06-0.4637000E 04 0.5770000E 05 0.5530000E 05-0.1929000E 05
      -0.3808000E 05 0.5707000E 05-0.2524000E 05 0.6875000E 04-0.1509000E 05-0.7523000E 04 0.7793000E 04

ROW  6-0.2842000E 05-0.3121000E 05 0.3916000E 05-0.1002000E 05 0.1793000E 06 0.2226000E 07 0.5540000E 04-0.9413000E 06
      -0.7820000E 06-0.2921000E 04 0.1687000E 06-0.1272000E 06 0.1834000E 06-0.2411000E 05 0.7394000E 05 0.6208000E 05
       0.7106000E 04-0.3133000E 05 0.7826000E 05-0.7293000E 05 0.9883000E 04-0.1068000E 05-0.2021000E 05

ROW  7 0.1176000E 06 0.1313000E 06 0.2646000E 04-0.4063000E 06-0.7495000E 06 0.5545000E 04 0.1445000E 07 0.3069000E 06
       0.2034000E 04-0.3527000E 06-0.8160000E 06-0.1876000E 05 0.1664000E 03 0.1371000E 06 0.1061000E 06-0.1514000E 04
       0.5364000E 05 0.5589000E 05-0.1574000E 05 0.1410000E 04-0.1638000E 05-0.1145000E 05 0.5217000E 04

ROW  8 0.1301000E 06 0.1422000E 06 0.3131000E 04 0.6981000E 04-0.8201000E 06-0.9413000E 06 0.3069000E 06 0.1989000E 07
       0.3377000E 06-0.1266000E 05-0.9011000E 06-0.8722000E 06-0.1638000E 05 0.1068000E 06 0.2265000E 06 0.1020000E 06
      -0.1973000E 05 0.5962000E 05 0.5494000E 05-0.1779000E 05-0.1455000E 05-0.2862000E 05-0.1324000E 05

ROW  9 0.5940000E 03 0.1301000E 04-0.9961000E 05-0.2191000E 03-0.5116000E 04-0.7820000E 06 0.2033000E 04 0.3377000E 06
       0.1261000E 07 0.5237000E 03-0.2473000E 05-0.7327000E 06-0.5146000E 06-0.1038000E 03 0.9286000E 05 0.1614000E 06
       0.7582998E 03-0.1260000E 05 0.3374000E 05 0.1247000E 06 0.3872000E 04-0.8036000E 04-0.1177000E 05

ROW 10-0.1526000E 05-0.1730000E 05-0.3732998E 03-0.3078000E 06 0.1048000E 06-0.2923000E 04-0.3527000E 06-0.1266000E 05
       0.5230999E 03 0.6246000E 06 0.2464000E 06-0.7040000E 04 0.2272000E 03-0.3524000E 06-0.1362000E 05-0.6138999E 03
      -0.3079000E 06 0.1042000E 06 0.1169000E 04 0.2399000E 03-0.1579000E 05-0.1417000E 05 0.2423000E 04

ROW 11 0.9281000E 05 0.1024000E 06 0.1695000E 04 0.1235000E 06-0.9936000E 04 0.1687000E 06-0.8160000E 06-0.9011000E 06
      -0.2474000E 05 0.2464000E 06 0.2208000E 07 0.3481000E 06-0.7145000E 04-0.8191000E 06-0.8910000E 06-0.1137000E 05
       0.1243000E 06-0.2584000E 04 0.1244000E 06-0.4280000E 04 0.9867000E 05 0.6590000E 05-0.3472000E 05

ROW 12-0.2132000E 05-0.2342000E 05 0.3019000E 05-0.2625000E 04 0.1436000E 06-0.1272000E 06-0.1877000E 05-0.8722000E 06
      -0.7327000E 06-0.7040000E 04 0.3481000E 06 0.2206000E 07 0.2895000E 06-0.1175000E 05-0.8917000E 06-0.8299000E 06
      -0.4386000E 04 0.1243000E 06 0.6353999E 03 0.1055000E 06-0.3460000E 05 0.6550000E 05 0.9818000E 05

ROW 13 0.7445000E 03 0.5697000E 03 0.1837000E 04 0.1655000E 03-0.4636000E 04 0.1834000E 06 0.1676000E 03-0.1638000E 05
      -0.5146000E 06 0.2257000E 03-0.7146000E 04 0.2895000E 06 0.5070000E 06-0.1709000E 03-0.2029000E 05-0.3310000E 06
       0.4587000E 02 0.2478000E 04 0.9040000E 05-0.2563000E 06 0.1750000E 04-0.1205000E 05-0.1306000E 05

ROW 14-0.1563000E 05-0.1719000E 05-0.5639000E 02 0.5358000E 05 0.5769000E 05-0.2411000E 05 0.1371000E 06 0.1068000E 06
      -0.1037000E 03-0.3524000E 06-0.8191000E 06-0.1176000E 05-0.1732000E 03 0.1444000E 07 0.3128000E 06 0.9295000E 04
      -0.4060000E 06-0.7443000E 06-0.2478000E 05-0.2299000E 04 0.1211000E 06 0.1095000E 06-0.1610000E 05
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Figure 2.3.2 (cont)

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ROW 15-0.1568000E 05-0.1731000E 05-0.4841000E 04-0.1932000E 05 0.5529000E 05 0.7394000E 05 0.1061000E 06 0.2265000E 06
        0.9285000E 05-0.1362000E 05-0.8909000E 06-0.8917000E 06-0.2028000E 05 0.3128000E 06 0.1970000E 07 0.3147000E 06
        0.5634000E 04-0.8403000E 06-0.8407000E 06 0.8517000E 04 0.1164000E 06 0.2380000E 06 0.1164000E 06

ROW 16 0.3146000E 04 0.2501000E 04 0.8298000E 04 0.1456000E 04-0.1930000E 05 0.6208000E 05-0.1510000E 04 0.1020000E 06
        0.1614000E 06-0.6132998E 03-0.1137000E 05-0.8299000E 06-0.3310000E 06 0.9292000E 04 0.3147000E 06 0.1440000E 07
       -0.2040000E 04-0.2519000E 05-0.7414000E 06-0.4153000E 06-0.1594000E 05 0.1091000E 06 0.1205000E 06

ROW 17 0.7166000E 04 0.8004000E 04 0.4414000E 02 0.1350000E 06-0.3808000E 05 0.7104000E 04 0.5364000E 05-0.1973000E 05
        0.7583999E 03-0.3079000E 06 0.1243000E 06-0.4385000E 04 0.4587999E 02-0.4060000E 06 0.5631000E 04-0.2041000E 04
        0.7974000E 06 0.1405000E 06-0.2891000E 04 0.9255999E 03-0.2288000E 05-0.1962000E 05 0.4314000E 04

ROW 18-0.1353000E 05-0.1495000E 05 0.5637000E 03-0.3747000E 05 0.5707000E 05-0.3134000E 05 0.5590000E 05 0.5963000E 05
       -0.1260000E 05 0.1042000E 06-0.2602000E 04 0.1243000E 06 0.2477000E 04-0.7443000E 06-0.8403000E 06-0.2518000E 05
        0.1405000E 06 0.1750000E 07 0.2755000E 06-0.3451000E 04-0.5130000E 06-0.4875000E 06 0.3926000E 05

ROW 19 0.5019000E 04 0.5511000E 04-0.7135000E 04 0.3984000E 04-0.2523000E 05 0.7826000E 05-0.1574000E 05 0.5494000E 05
        0.3374000E 05 0.1172000E 04 0.1244000E 06 0.6213999E 03 0.9040000E 05-0.2478000E 05-0.8407000E 06-0.7414000E 06
       -0.2892000E 04 0.2755000E 06 0.1749000E 07 0.1465000E 06 0.3923000E 05-0.4874000E 06-0.5127000E 06

ROW 20-0.1188000E 04-0.1240000E 04 0.2353000E 05-0.6825000E 03 0.6879000E 04-0.7293000E 05 0.1407000E 04-0.1779000E 05
        0.1247000E 06 0.2406000E 03-0.4280000E 04 0.1055000E 06-0.2563000E 06-0.2300000E 04 0.8516000E 04-0.4153000E 06
        0.9253999E 03-0.3447000E 04 0.1465000E 06 0.7748000E 06 0.4608000E 04-0.2056000E 05-0.2406000E 05

ROW 21 0.3596000E 04 0.3971000E 04-0.3180000E 03 0.7485000E 04-0.1509000E 05 0.9884000E 04-0.1638000E 05-0.1455000E 05
        0.3869000E 04-0.1579000E 05 0.9867000E 05-0.3460000E 05 0.1751000E 04 0.1211000E 06 0.1163000E 06-0.1594000E 05
       -0.2287000E 05-0.5130000E 06 0.3923000E 05 0.4609000E 04 0.5365000E 06-0.7153000E 05 0.3709000E 05

ROW 22 0.2061000E 04 0.2283000E 04 0.1195000E 04 0.5795000E 04-0.7526000E 04-0.1068000E 05-0.1145000E 05-0.2863000E 05
       -0.8038000E 04-0.1418000E 05 0.6590000E 05 0.6551000E 05-0.1205000E 05 0.1095000E 06 0.2380000E 06 0.1091000E 06
       -0.1962000E 05-0.4875000E 06-0.4874000E 06-0.2056000E 05-0.7152000E 05 0.5179000E 06-0.7155000E 05

ROW 23-0.1595000E 04-0.1755000E 04 0.1464000E 04-0.1942000E 04 0.7791000E 04-0.2021000E 05 0.5218000E 04-0.1324000E 05
       -0.1177000E 05 0.2423000E 04-0.3472000E 05 0.9818000E 05-0.1308000E 05-0.1611000E 05 0.1164000E 06 0.1205000E 06
        0.4314000E 04 0.3926000E 05-0.5127000E 06-0.2406000E 05 0.3709000E 05-0.7155000E 05 0.5365000E 06
```

Figure 2.3.3

MODE SHAPES & FREQUENCIES

	MODE 1	MODE 2	MODE 3	MODE 4
EIGENVALUE=	0.83674D-06	0.51348D-06	0.16389D-06	0.13964D-06
FREQUENCY=	0.17399D 03	0.22213D 03	0.39314D 03	0.42590D 03
EIGENVECTOR	-0.90204D-01	-0.20004D 00	0.31109D-03	-0.40735D 00
	-0.13762D 00	-0.25066D 00	-0.91758D-01	-0.33294D 00
	0.71584D 00	0.10000D 01	0.94892D 00	0.17524D 00
	0.95291D-01	-0.20896D 00	0.50234D 00	-0.48512D-01
	0.66816D-01	-0.30848D 00	0.16656D 00	-0.56015D 00
	0.22629D 00	0.18263D-01	-0.52331D-01	-0.39533D 00
	0.21788D 00	-0.35448D 00	0.59223D 00	-0.30404D 00
	0.33713D 00	-0.17372D 00	0.52449D-01	-0.58221D 00
	0.55169D 00	0.29576D 00	-0.17952D 00	-0.69318D 00
	0.21328D 00	-0.40920D 00	0.10000D 01	-0.33638D-01
	0.45160D 00	-0.28715D 00	0.32410D 00	-0.28049D 00
	0.58776D 00	0.24122D-02	-0.20082D 00	-0.59662D 00
	0.62038D 00	0.43156D 00	-0.45753D 00	-0.90200D 00
	0.42798D 00	-0.31837D 00	0.54820D 00	0.63216D-01
	0.70539D 00	-0.16607D 00	-0.67280D-01	-0.79672D-01
	0.62235D 00	0.11517D 00	-0.42832D 00	-0.55596D 00
	0.11809D 00	-0.20055D 00	0.50021D 00	0.14805D-01
	0.74594D 00	-0.22242D 00	0.89943D-02	0.46829D 00
	0.79965D 00	-0.92635D-01	-0.36043D 00	0.20677D 00
	0.26468D 00	0.13665D 00	-0.31494D 00	-0.47065D 00
	0.49909D 00	-0.10611D 00	-0.12352D 00	0.55143D 00
	0.10000D 01	-0.18991D 00	-0.39133D 00	0.10000D 01
	0.50639D 00	-0.84837D-01	-0.27247D 00	0.45548D 00

Figure 2.3.3 (cont)

ODE 5	MODE 6	MODE 7	MODE 8
IGENVALUE= 0.61094D-07	EIGENVALUE= 0.44504D-07	EIGENVALUE= 0.28968D-07	EIGENVALUE= 0.14667D-07
REQUENCY= 0.64391D 03	FREQUENCY= 0.75443D 03	FREQUENCY= 0.93511D 03	FREQUENCY= 0.13142D 04
IGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
-0.36612D 00	0.10000D 01	0.42401D 00	-0.24221D 00
-0.23432D 00	0.70609D 00	0.30457D 00	-0.16309D 00
-0.41674D 00	0.36683D 00	0.11114D-02	0.10651D 00
0.47744D 00	0.22702D 00	-0.18937D 00	-0.22748D 00
-0.41894D 00	0.57569D 00	0.10396D 00	0.23996D 00
-0.13481D 00	-0.21555D 00	0.66965D-01	-0.74162D 00
0.15106D 00	0.19344D 00	-3.24147D 00	0.47813D 00
-0.46156D 00	-0.28885D 00	-0.82306D-01	-0.24351D 00
0.23713D 00	-0.11832D 00	0.24528D 00	-0.84562D 00
0.99220D 00	0.28125D 00	-0.16557D 00	-0.91814D 00
-0.25569D 00	-0.42126D 00	-0.16803D 00	0.70650D 00
-0.20705D 00	-0.36438D 00	-0.87336D-01	-0.71487D 00
0.10000D 01	0.36168D 00	0.38483D 00	0.10000D 01
0.31115D 00	-0.25976D 00	0.15988D 00	0.36759D-01
-0.32683D 00	-0.42480D 00	-0.13662D 00	0.12754D 00
0.33842D 00	0.11062D 00	-0.16658D 00	0.31573D 00
-0.52192D 00	0.57195D-01	0.21293D-01	-0.72466D 00
-0.59786D-01	-0.23562D 00	0.60388D 00	0.21326D 00
-0.64324D-01	0.22730D 00	-0.58048D 00	-0.81339D-01
0.55054D 00	0.21797D 00	0.11707D 00	0.95990D 00
0.19452D-01	0.13072D-01	0.10000D 01	-0.92317D-01
0.25998D-01	0.53210D 00	0.20662D 00	-0.21629D 00
0.67508D-02	0.55492D 00	-0.95796D 00	-0.16143D 00

Figure 2.3.3(cont)

DE 9	MODE 10	MODE 11	MODE 12
GENVALUE= 0.13178D-07	EIGENVALUE= 0.12370D-07	EIGENVALUE= 0.94401D-08	EIGENVALUE= 0.82991D-08
EQUENCY= 0.13864D 04	FREQUENCY= 0.14313D 04	FREQUENCY= 0.16381D 04	FREQUENCY= 0.17470D 04
GENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
-0.36470D 00	0.55885D 00	-0.85620D 00	-0.19706D 00
-0.19283D 00	0.23876D 00	-0.13134D 00	0.11280D 00
-0.16549D-01	0.17493D 00	0.14345D 00	0.40350D-01
0.10000D 01	0.44703D-01	-0.80343D 00	0.39864D-01
0.77293D-01	-0.27691D 00	0.45617D 00	0.50780D-01
0.12037D-01	-0.53252D 00	0.26777D 00	0.21403D-01
0.69896D 00	-0.40584D 00	-0.19321D 00	-0.11123D-01
0.31427D-02	-0.42650D 00	0.10000D 01	0.11392D 00
-0.14574D-01	-0.82736D 00	-0.74636D 00	-0.15542D 00
0.21645D 00	0.44843D 00	0.11349D 00	0.53844D-01
-0.18790D 00	-0.33194D 00	-0.96323D-01	-0.86560D-01
0.20888D 00	0.50627D 00	0.57109D-01	0.80084D-01
-0.28011D 00	-0.77150D 00	-0.39232D 00	-0.12647D 00
-0.84326D 00	0.96536D-01	0.83941D-01	-0.51824D-01
0.47337D-01	0.64064D 00	-0.90840D 00	-0.82624D-01
0.13608D 00	0.10000D 01	0.60816D 00	0.16027D 00
-0.77434D 00	0.42450D 00	0.92474D 00	0.18662D-01
-0.12462D 00	0.51658D-01	-0.46578D 00	0.53485D-01
0.91333D-01	0.64176D 00	-0.58334D-01	-0.85352D-01
-0.11779D 00	0.18277D 00	0.72324D 00	0.15503D 00
0.48775D 00	-0.12301D 00	0.22759D 00	0.10000D 01
0.34001D 00	-0.53148D 00	0.94212D 00	-0.72185D 00
-0.51520D 00	-0.81419D 00	-0.46316D 00	0.55078D 00

Figure 2.33 (cont)

MODE 13	MODE 14	MODE 15	MODE 16
EIGENVALUE= 0.80027D-08	EIGENVALUE= 0.55243D-08	EIGENVALUE= 0.42705D-08	EIGENVALUE= 0.30931D-08
FREQUENCY= 0.17791D 04	FREQUENCY= 0.21413D 04	FREQUENCY= 0.24355D 04	FREQUENCY= 0.25508D 04
EIGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
-0.78458D 00	0.17569D 00	-0.16626D-01	-0.51834D 00
0.10000D 01	0.29950D 00	-0.21247D-01	-0.63172D 00
0.75056D-01	0.86292D-01	0.29343D-01	-0.31797D-01
0.16149D-02	0.10000D 01	0.22765D 00	-0.20960D 00
0.28308D-01	-0.36131D 00	0.27546D-01	0.96892D 00
-0.71201D-01	0.47175D 00	-0.32421D-01	-0.13072D 00
-0.24007D-01	-0.40049D 00	-0.52442D 00	0.17630D 00
-0.15722D 00	0.44427D 00	0.58780D 00	-0.17643D 00
0.94113D-01	-0.35471D 00	-0.50017D 00	0.43999D-01
-0.65267D-02	0.11064D 00	0.71663D 00	-0.39567D-01
-0.25345D-01	-0.52849D-01	-0.58075D-01	-0.98889D 00
-0.83915D-02	0.48938D-01	-0.88201D-01	0.20446D 00
0.47449D-01	-0.31532D 00	0.99890D 00	-0.18333D 00
0.26606D-01	0.23283D 00	-0.87734D 00	-0.10276D 00
0.12847D 00	-0.23410D 00	0.10000D 01	0.11295D 00
-0.62432D-01	0.11532D 00	-0.88218D 00	0.69786D-01
-0.86262D-02	-0.86856D 00	0.79118D 00	0.16676D 00
0.47749D-01	0.36201D 00	-0.28562D-01	0.10000D 01

0.57294D-01	-0.42925D 00	-0.51153D-01	-0.29692D 00
-0.13340D 00	0.95350D 00	0.63170D 00	0.15954D 00
-0.20521D 00	-0.59608D 00	0.13997D-01	-0.99331D 00
0.68097D-02	0.83093D-01	0.21571D-01	-0.36049D 00
-0.76903D-01	0.22397D 00	0.13746D-01	0.22613D 00

Figure 2.3.3 (cont)

ODE 17	MODE 18	MODE 19	MODE 20
IGENVALUE= 0.36477D-08	EIGENVALUE= 0.31192D-08	EIGENVALUE= 0.30352D-08	EIGENVALUE= 0.27647D-08
REQUENCY= 0.26352D 04	FREQUENCY= 0.28497D 04	FREQUENCY= 0.28869D 04	FREQUENCY= 0.30269D 04
IGENVECTOR	EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
0.77714D-01	-0.23342D-02	-0.13269D 00	0.21265D 00
0.94995D-01	-0.26749D-02	-0.15058D 00	0.23774D 00
0.10972D-01	0.16663D-02	-0.11998D-01	0.29593D-01
-0.43661D 00	-0.84935D 00	0.25521D 00	-0.51046D 00
-0.16524D 00	-0.35024D-03	0.29764D 00	-0.53555D 00
0.19264D 00	0.27617D-01	0.71438D 00	0.10000D 01
0.38983D 00	-0.32292D-02	-0.19872D 00	0.64787D 00
-0.32633D 00	0.36361D-01	-0.90518D-01	0.20931D 00
0.19558D 00	-0.36439D-01	0.18349D 00	-0.46119D 00
-0.62514D-01	0.10000D 01	-0.11964D 00	-0.82018D-02
0.69309D-01	-0.43316D-01	-0.15925D 00	-0.26687D-01
0.55155D-01	-0.56466D-02	-0.79753D 00	-0.57070D 00
-0.62007D 00	0.61587D-01	-0.23452D 00	0.47685D 00
-0.22202D 00	0.19190D-01	0.29945D 00	-0.63324D 00
0.16115D 00	0.39230D-01	0.80452D-01	-0.19935D 00
-0.10039D 00	-0.53322D-01	-0.26171D 00	0.47429D 00
0.30040D 00	-0.86958D 00	-0.17214D 00	0.50851D 00
-0.13370D-01	-0.15180D-01	-0.68752D-01	0.57031D 00
-0.17335D 00	0.42415D-01	0.10000D 01	0.12768D 00
0.10000D 01	0.16663D-01	0.56965D 00	-0.97666D 00
-0.92712D-02	0.12044D-01	0.17461D 00	-0.33939D 00
0.80570D-01	-0.11754D-01	-0.37351D 00	-0.29025D 00
0.60604D-01	-0.14523D-01	-0.35809D 00	-0.23219D-01

Figure 2.3.3 (cont)

	MODE 21	MODE 22	MODE 23
EIGENVALUE=	0.23109D-08	0.19588D-08	0.14260D-08
FREQUENCY=	0.33108D 04	0.35961D 04	0.42147D 04

EIGENVECTOR	EIGENVECTOR	EIGENVECTOR
0.10070D 00	-0.14726D 00	-0.67070D-01
0.11105D 00	-0.15537D 00	-0.68997D-01
0.39997D-01	0.15564D-01	0.16648D-01
-0.27335D 00	0.12975D 00	0.84349D-01
-0.37092D 00	0.41279D 00	0.25619D 00
0.29010D 00	0.10000D 01	0.46435D 00
0.10000D 01	-0.26809D 00	-0.35161D 00
0.17368D 00	-0.86038D 00	-0.90356D 00
-0.75976D 00	-0.45636D 00	-0.47849D 00
-0.93238D 00	-0.88845D-02	0.22839D 00
-0.89465D 00	0.30751D-01	0.60468D 00
0.90102D 00	-0.64442D-01	0.10000D 01
0.94050D 00	0.21733D 00	0.42208D 00
0.97257D 00	0.28454D 00	-0.36503D 00
0.16759D 00	0.82327D 00	-0.89993D 00
-0.82711D 00	0.45943D 00	-0.51574D 00
-0.25350D 00	-0.13548D 00	0.92331D-01
-0.34347D 00	-0.47879D 00	0.27483D 00
0.28806D 00	-0.80550D 00	0.45059D 00
0.16600D 00	-0.47110D 00	0.11486D 00
0.15633D 00	0.19256D 00	-0.82346D-01
0.36260D-01	0.38960D 00	-0.15803D 00
-0.62806D-01	0.17752D 00	-0.69862D-01

Table 2.4.1

Quad III LM-10 Vibration Environment Qualification Levels

1. Launch and Boost Phases

a. Random (all axes - duration 1.0 min/axis)

20-40 hz	+12 db/oct
40-85	0.030 g^2/hz
85-110	+6 db/oct
110-400	0.050 g^2/hz
400-450	-6 db/oct
450-1100	0.040 g^2/hz
1100-2000	-12 db/oct

b. Sinusoidal Sweep (all axes)

5-12	.2 in. da
12-100	1.4 g. peak

Sweep: 5-100-5 hz
Rate: 3 oct/min

c. Sinusoidal Dwell (all axes-duration 10 sec)
6 hz 1.5 g-peak

2. Lunar Descent Phase

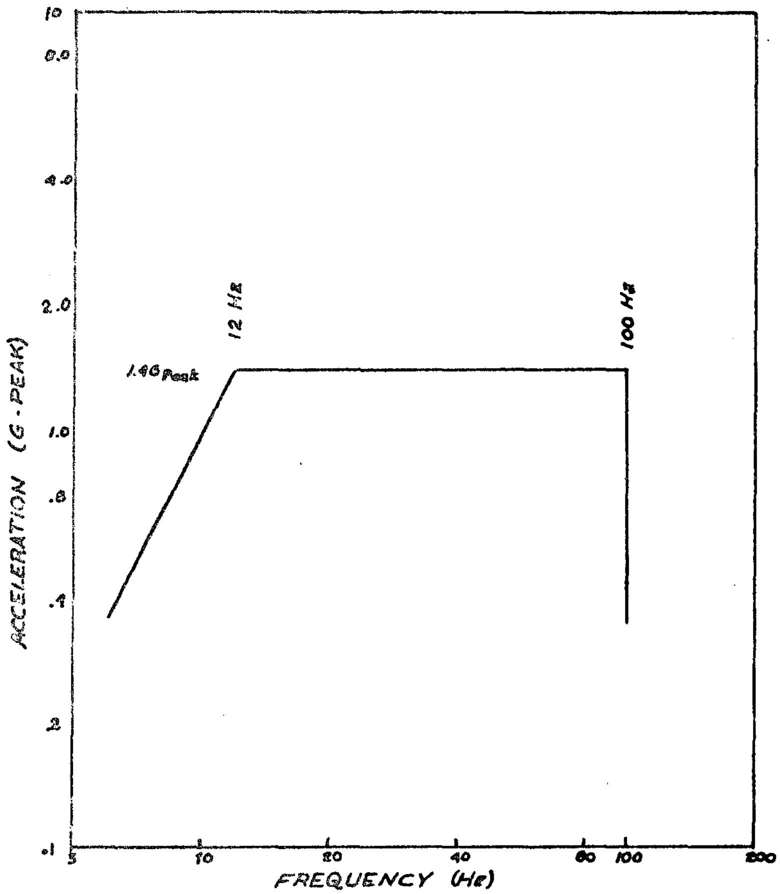
a. Random (all axes-duration 12.5 min)

20-2000	0.005 g^2/hz

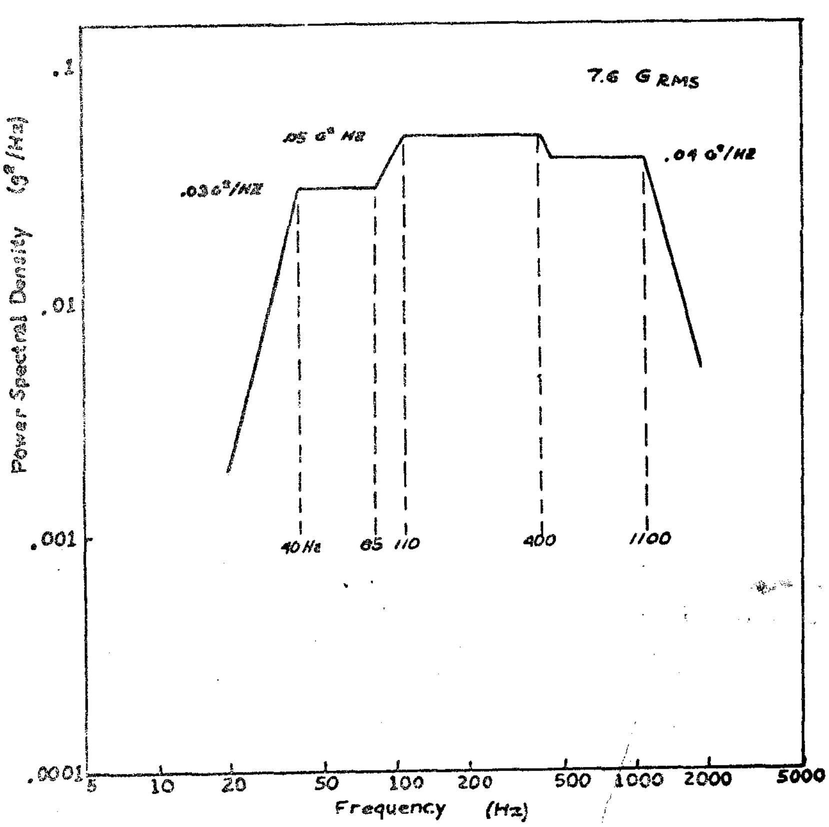
NOTE: Levels specified are to the GAC interface points in the LM.

Figure 2.4.1

QUAD III SINUSOIDAL ENVIRONMENT



QUAD III L&B RANDOM ENVIRONMENT



3.0 Computed Responses

3.1 Responses of Out of Plane Model of Both Arrays

The response at each lumped mass location of the out-of-plane model of both arrays was calculated to the sine environment and to the random environment. Transmissibility, sine response, and random response at five location on the experiment are shown in Figure 3.1.1 through Figure 3.1.5. The random responses shown are responses to the "Launch & Boost Environment". The "Lunar Descent" random environment gave responses that were in all cases less severe than responses to the L&B environment. Out-of-plane responses are summarized in Table 3.1.1.

The most severe response was found to be at the center of the right edge of the small array, lumped mass location 27 of Figure 2.1.2. The response to the L&B random excitation at this location is shown in Figure 3.1.6. This response has a root mean square value of 15.87 g, giving a 3-sigma peak value of 47.6 g. Thus, the greatest acceleration perpendicular to the plane of the arrays likely to be seen by a corner reflector mount is 47.6 g. The greatest power spectral density of this response is in the frequency range between 100 and 400 Hz. If the corner mount design can survive sinusoidal vibration well in excess of 47 g peak perpendicular to the array face, then the corner mount design is of sufficient strength in this direction.

Response at the 3 independent attachment points between the arrays determine the vibration levels seen by the upper array. An envelope enclosing the responses at all the attachments could then be used for testing the upper array by itself. The three responses were superimposed and a suitable envelope, reduced by a factor of 1.69 to acceptance level, is shown in Figure 3.1.7.

3.2 Response of the In Plane Model

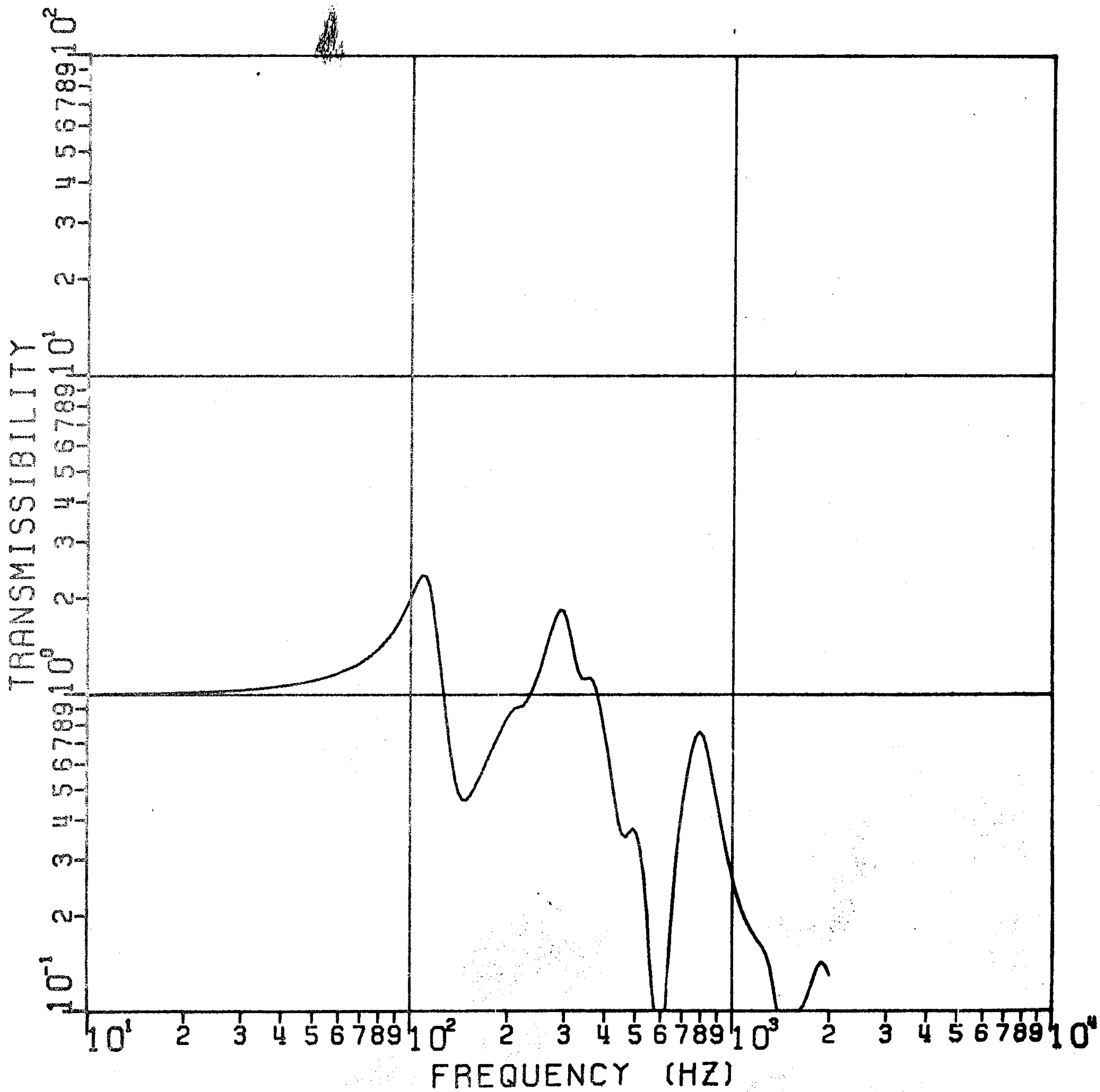
Responses in the six coordinate directions of the in plane model (coordinate directions shown in Figure 2.2.1) are shown in Figure 3.2.1 through 3.2.6. Response of the angular coordinates θ_1 and θ_2, are given in "pseudo g" units of $\frac{rad}{sec^2} \cdot \frac{1}{386}$. These responses are summarized in Table 3.2.1

The worst vibration level seen in-plane by a corner mount will occur along the edge of one of the two arrays. The response will be the sum of translational response of the center of gravity plus the rotational response about the c.g. times the distance to the edge from the c.g.. The maximum responses at corner locations on both arrays to excitation in the X and Y directions are shown in Table 3.2.2.

FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.1 A TRANSMISSIBILITY

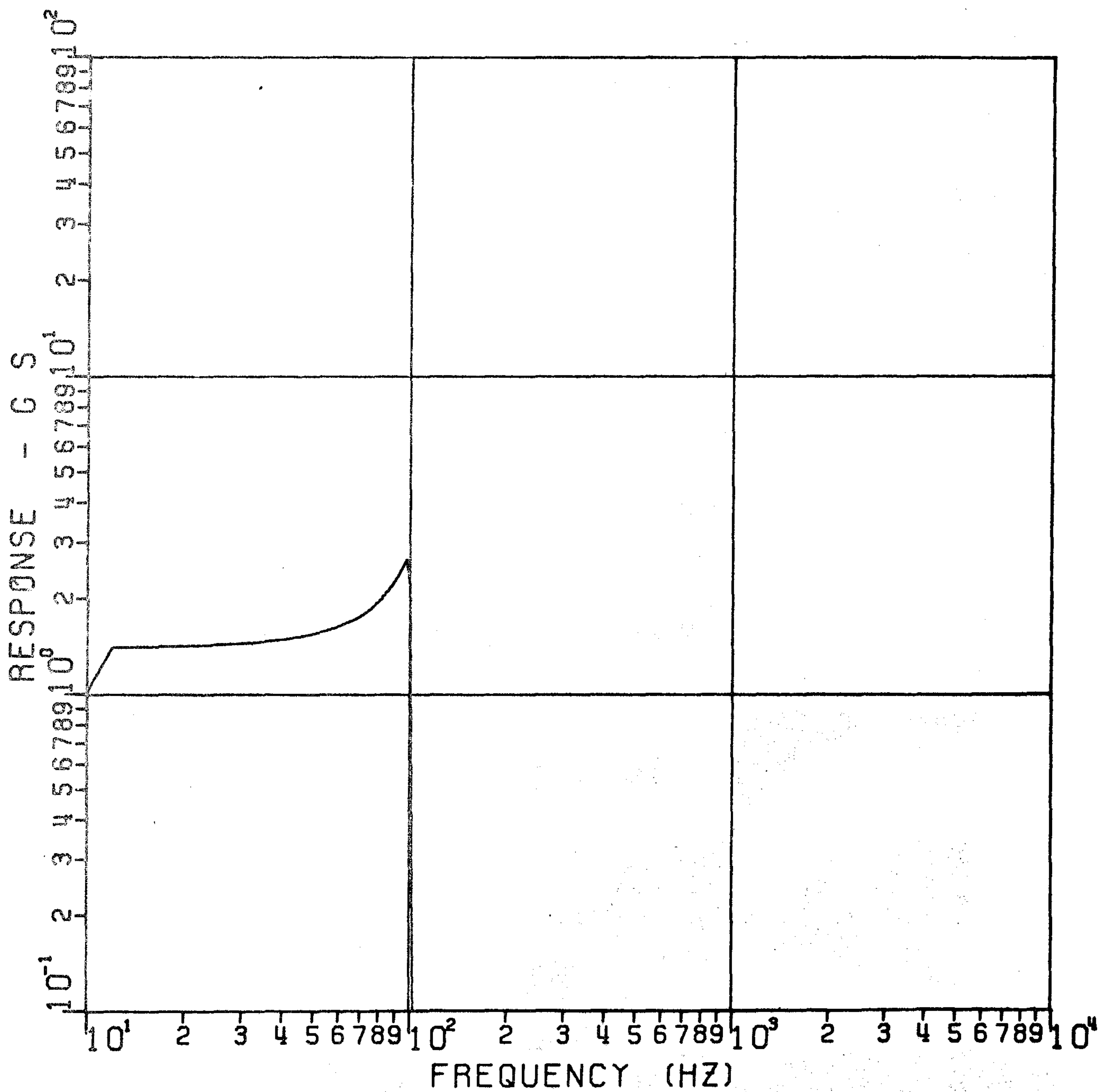
LOCATION 2



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.1 B SINE RESPONSE

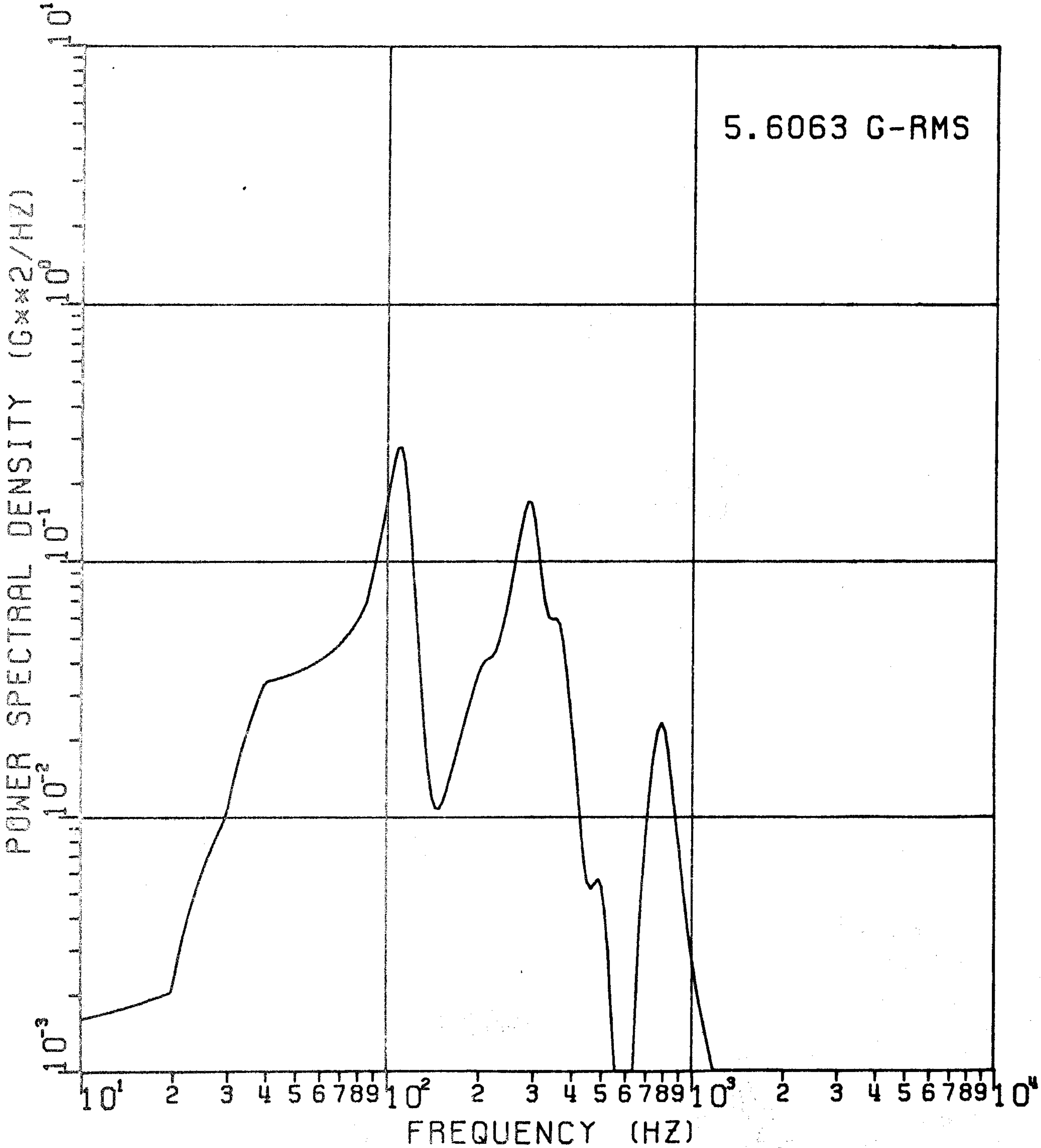
LOCATION 2



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.1 C RANDOM VIBRATION SPECTRUM

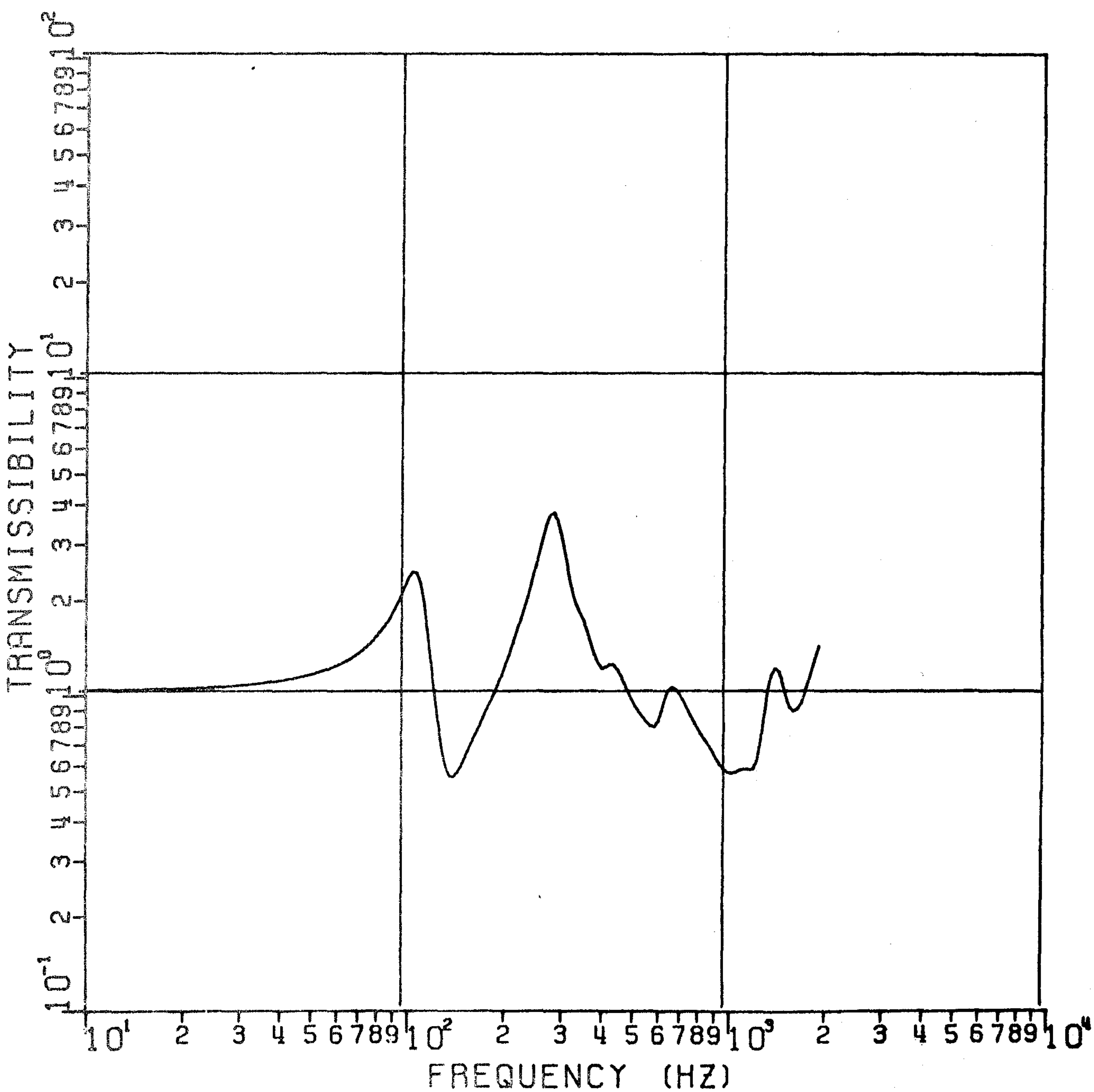
LOCATION 2



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.2 A TRANSMISSIBILITY

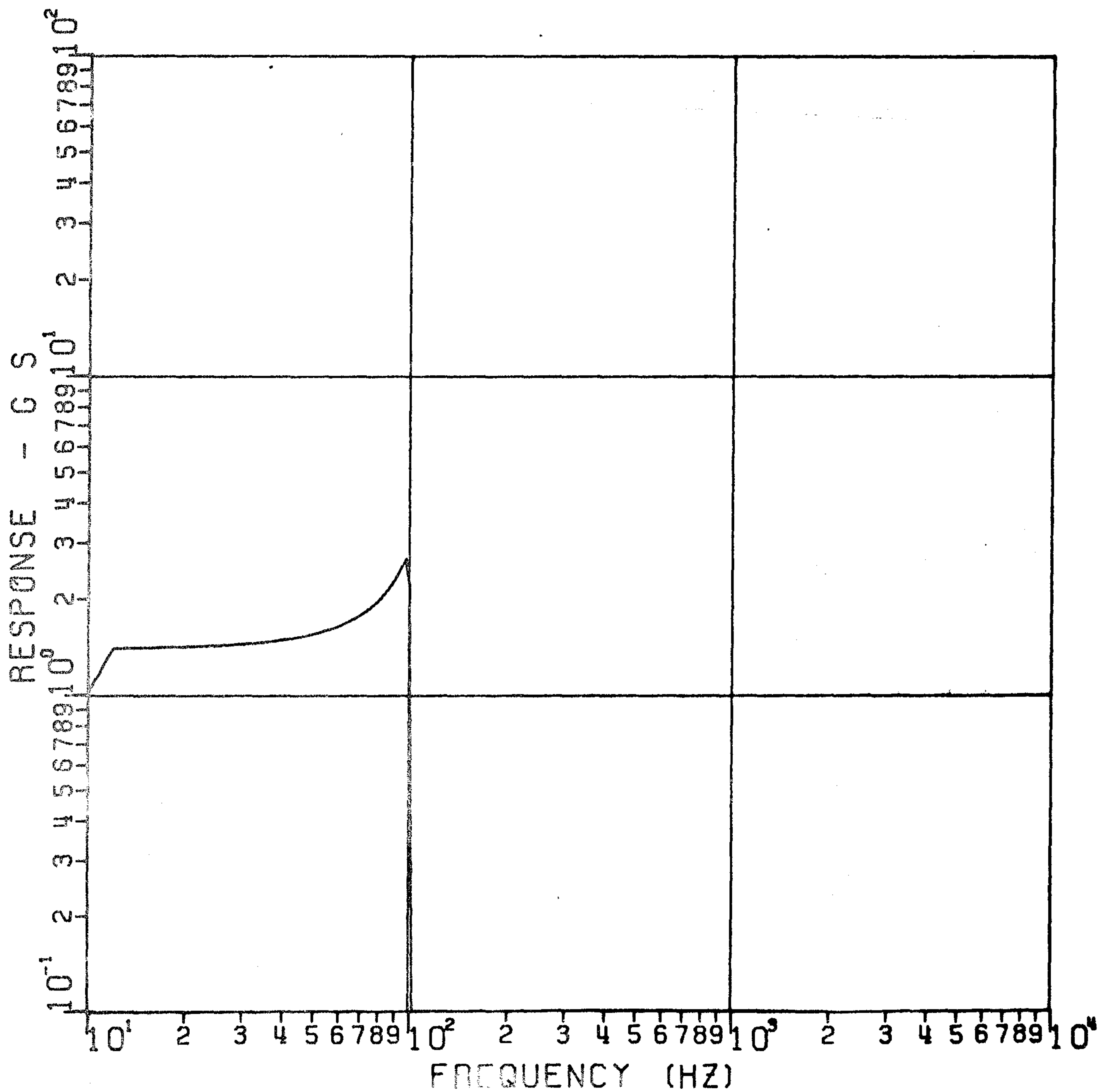
LOCATION 9



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.2 B SINE RESPONSE

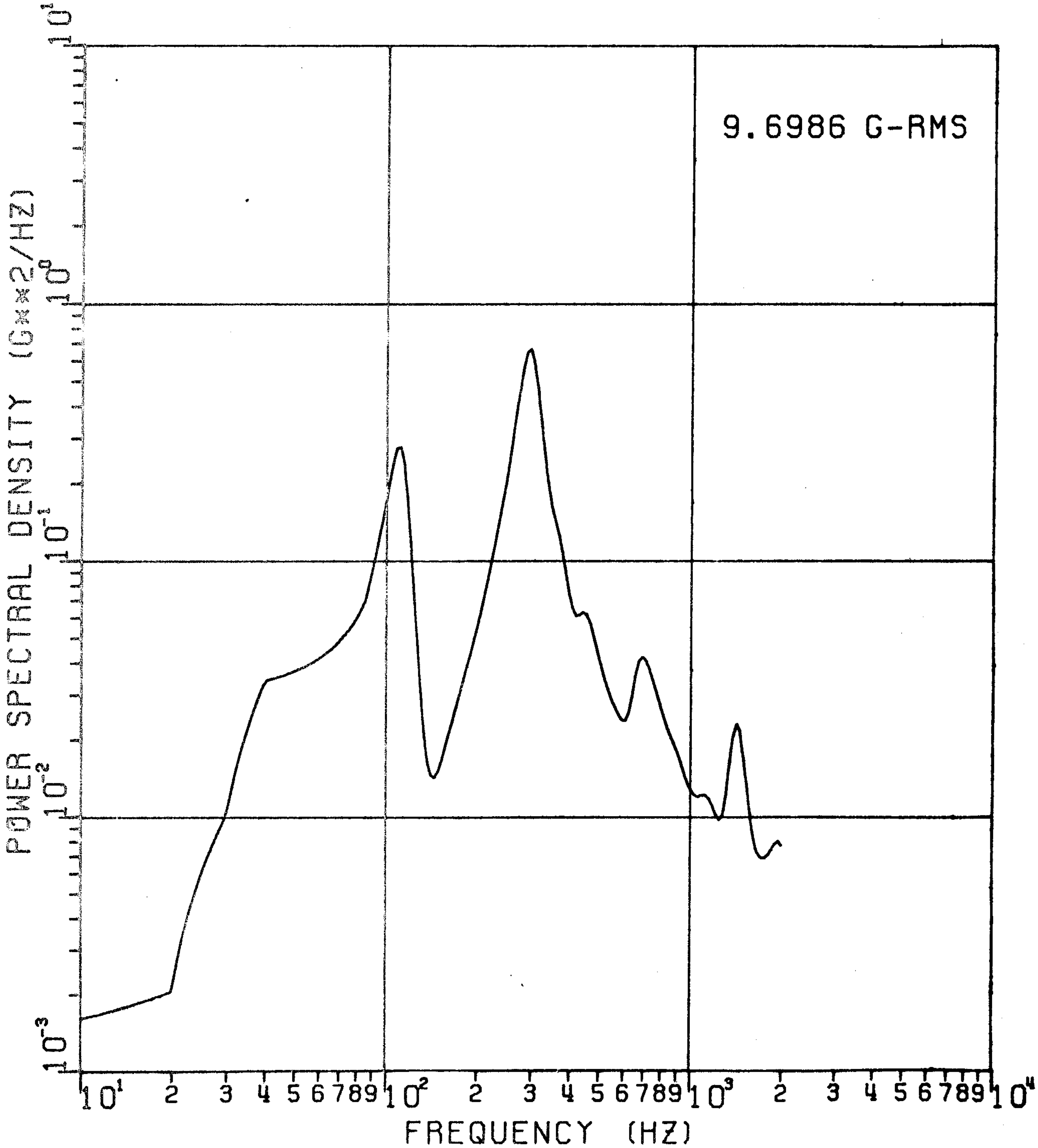
LOCATION 9



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.2 C RANDOM VIBRATION SPECTRUM

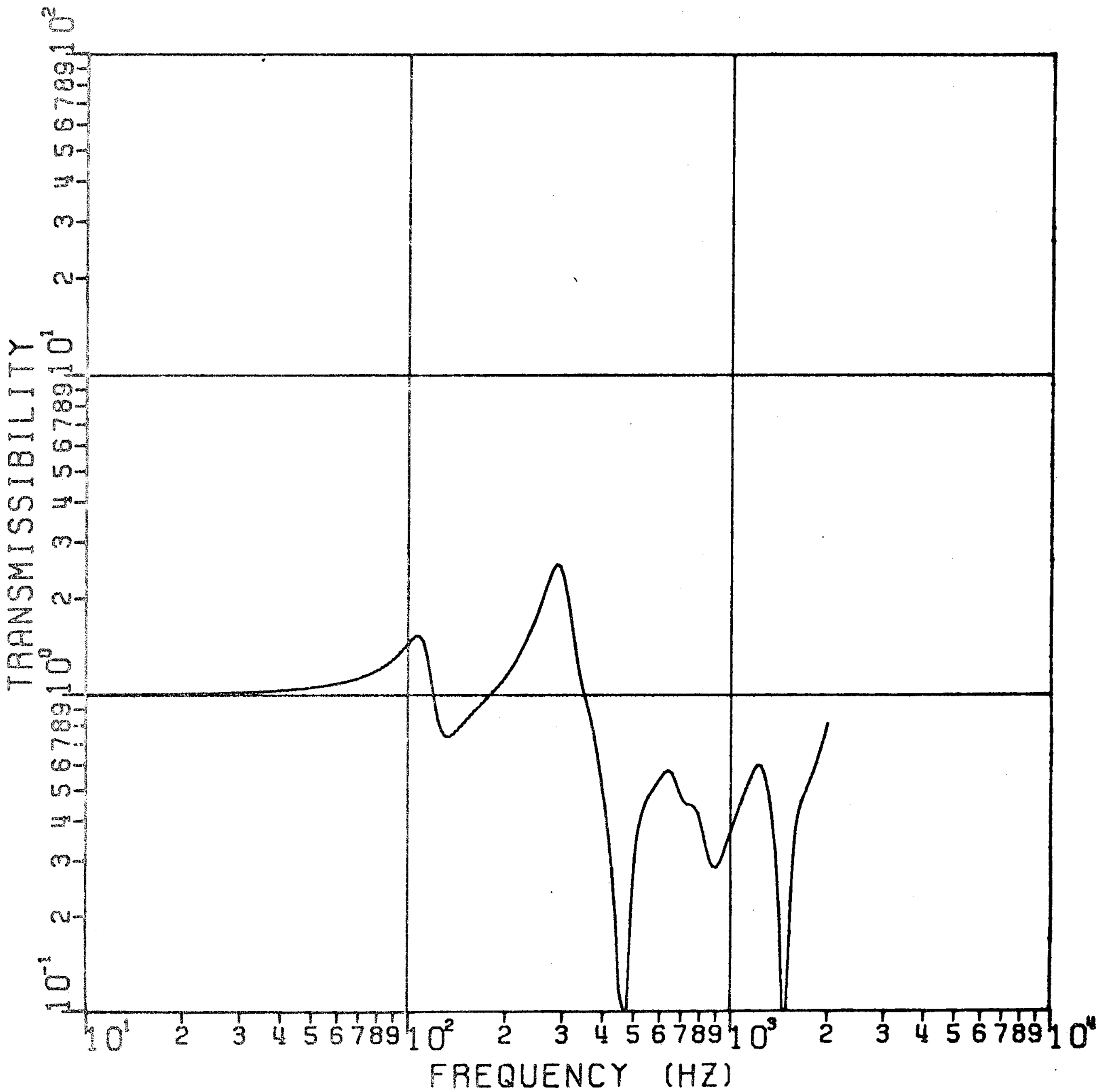
LOCATION 9



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 31.3A TRANSMISSIBILITY

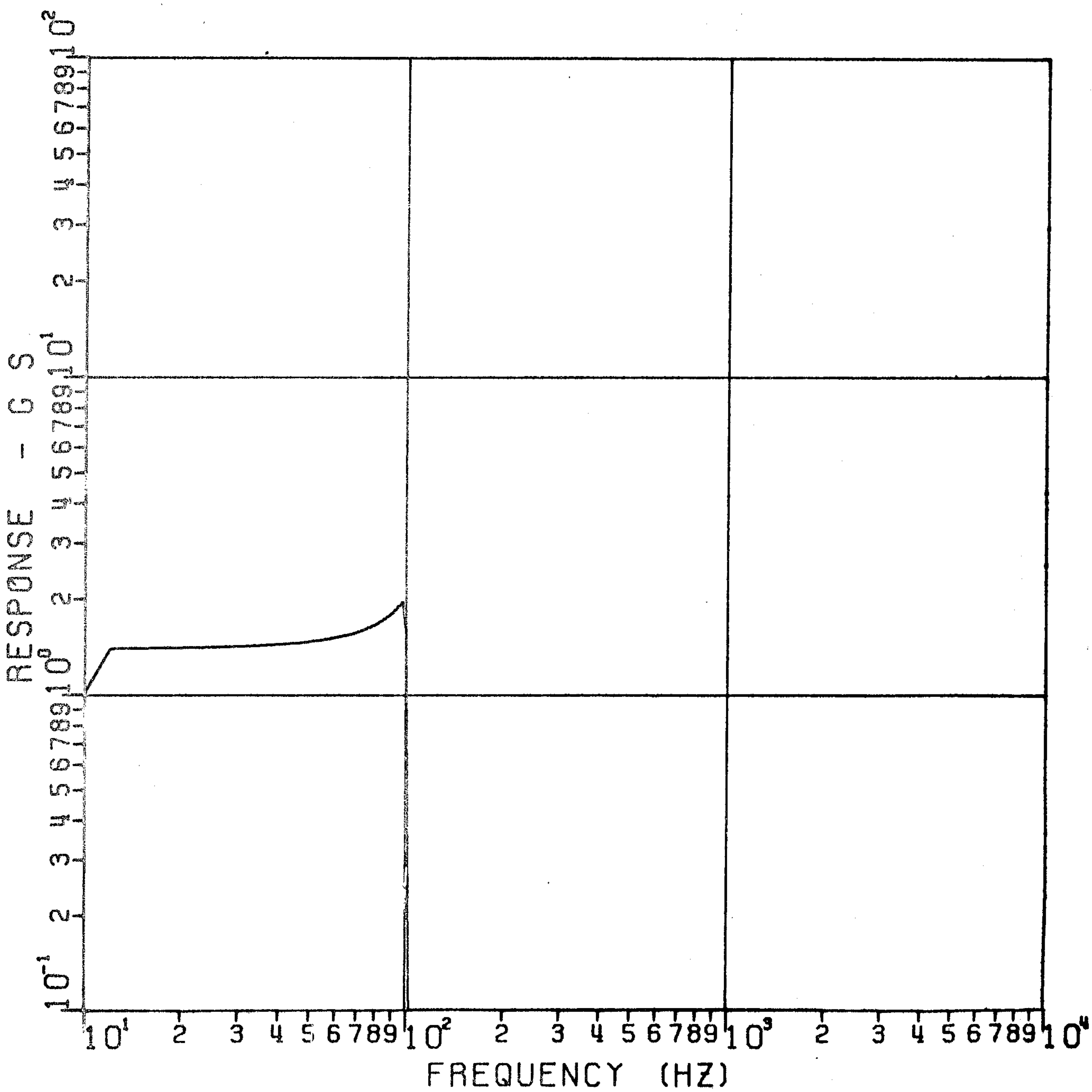
LOCATION 16



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.13B SINE RESPONSE

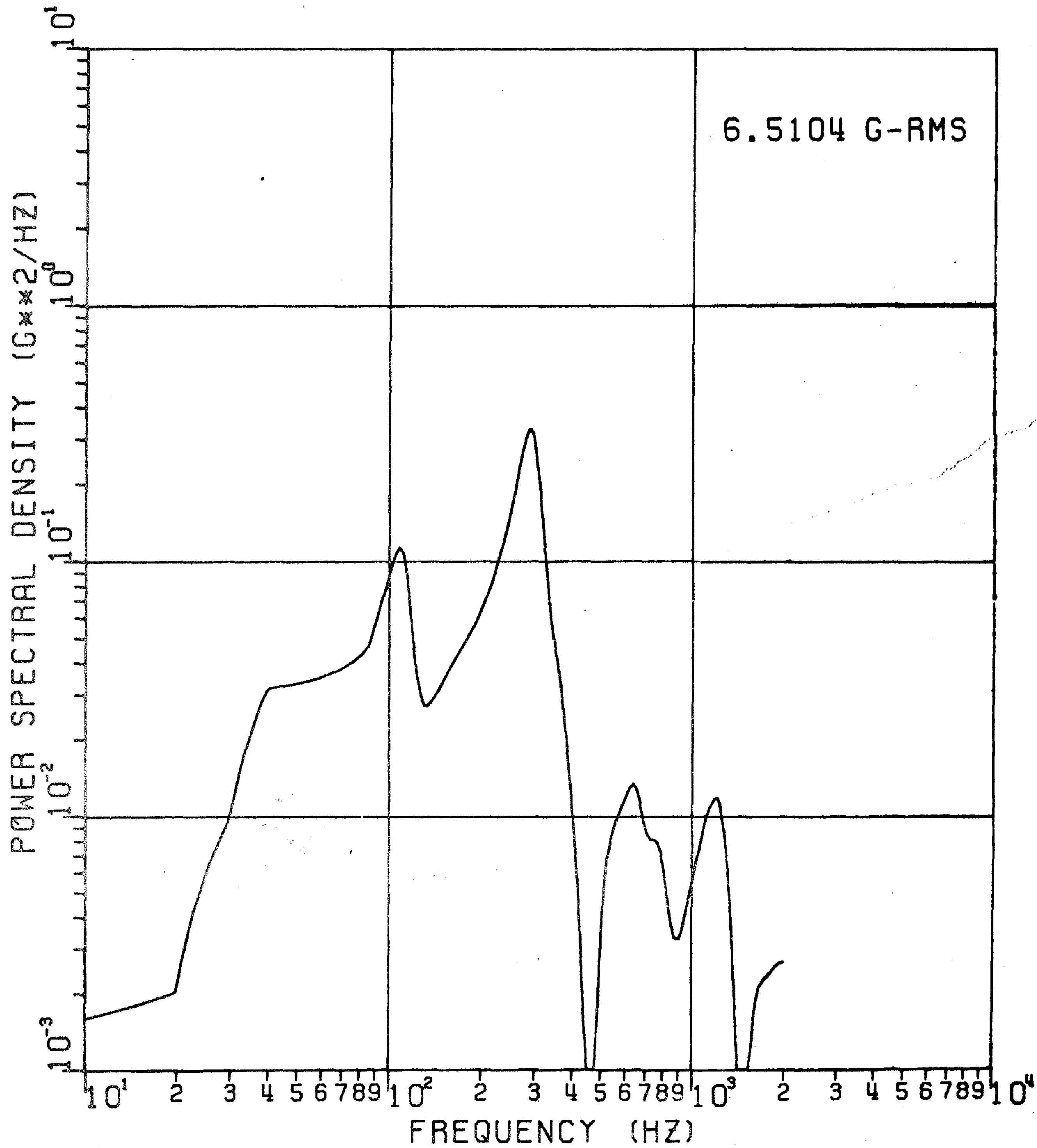
LOCATION 16



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.3C RANDOM VIBRATION SPECTRUM

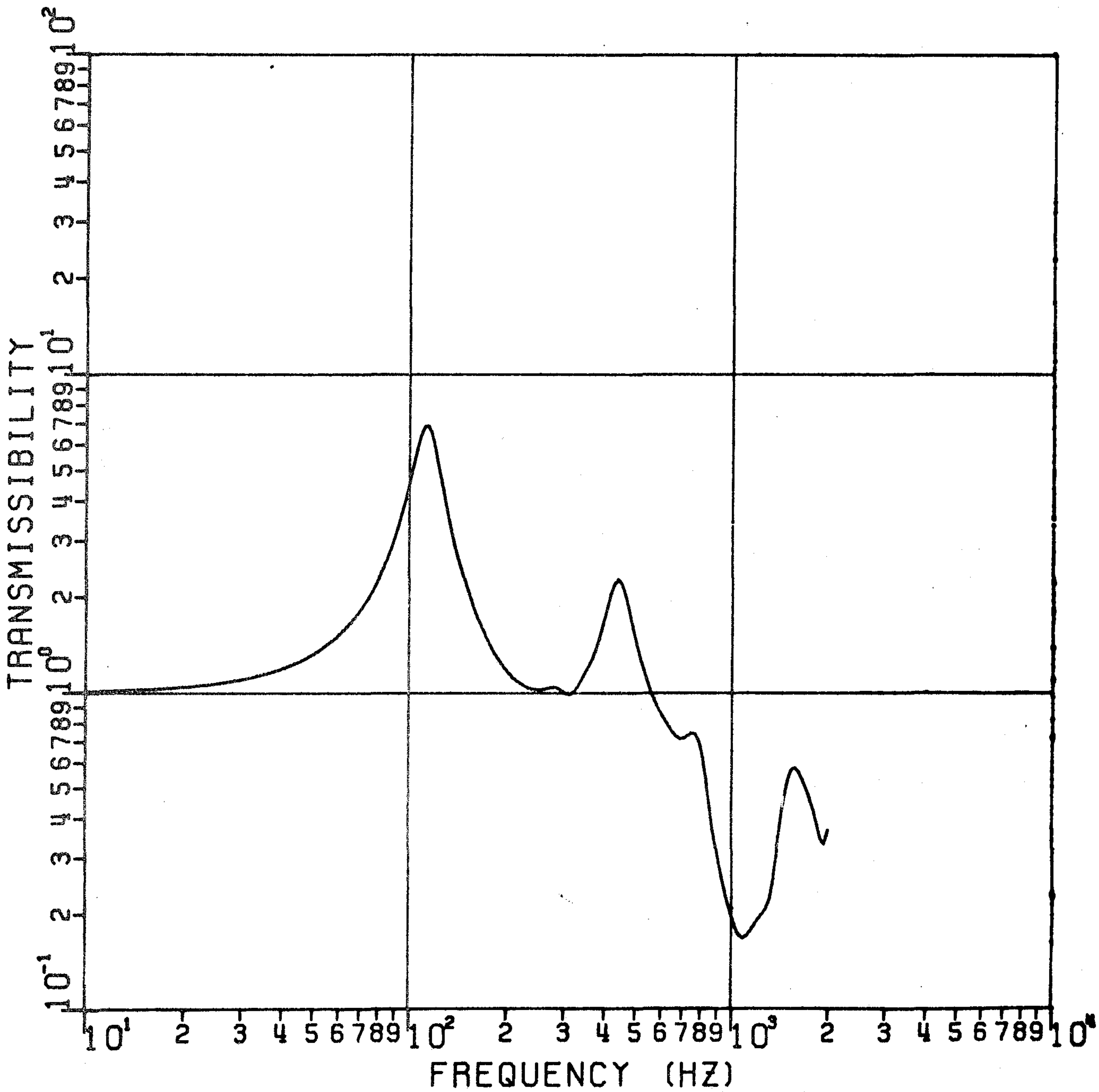
LOCATION 16



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.4 A TRANSMISSIBILITY

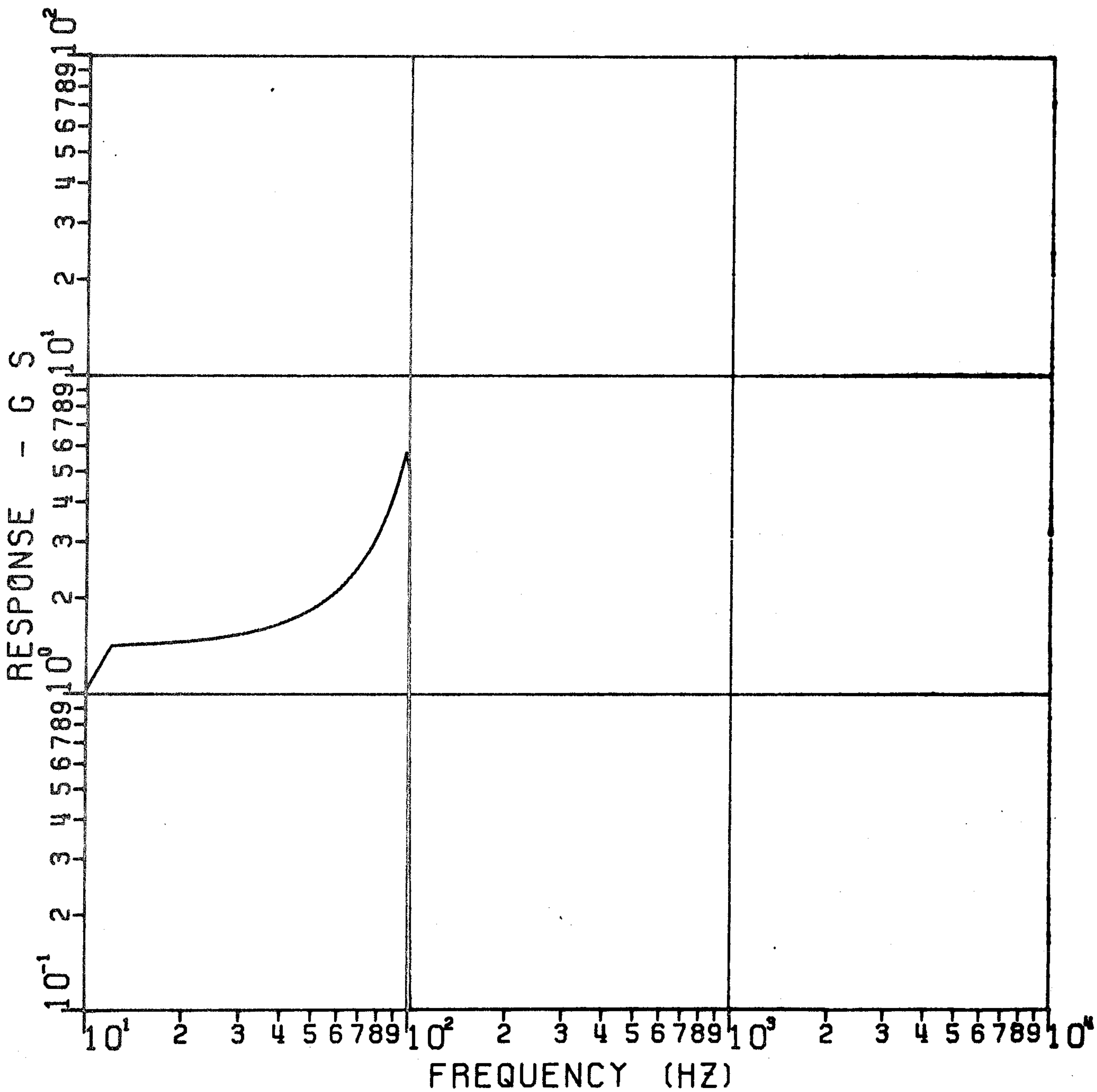
LOCATION 21



FREQUENCY RESPONSE OF LR3 300, 30 D.F. MODEL

FIGURE 3.1.4 B SINE RESPONSE

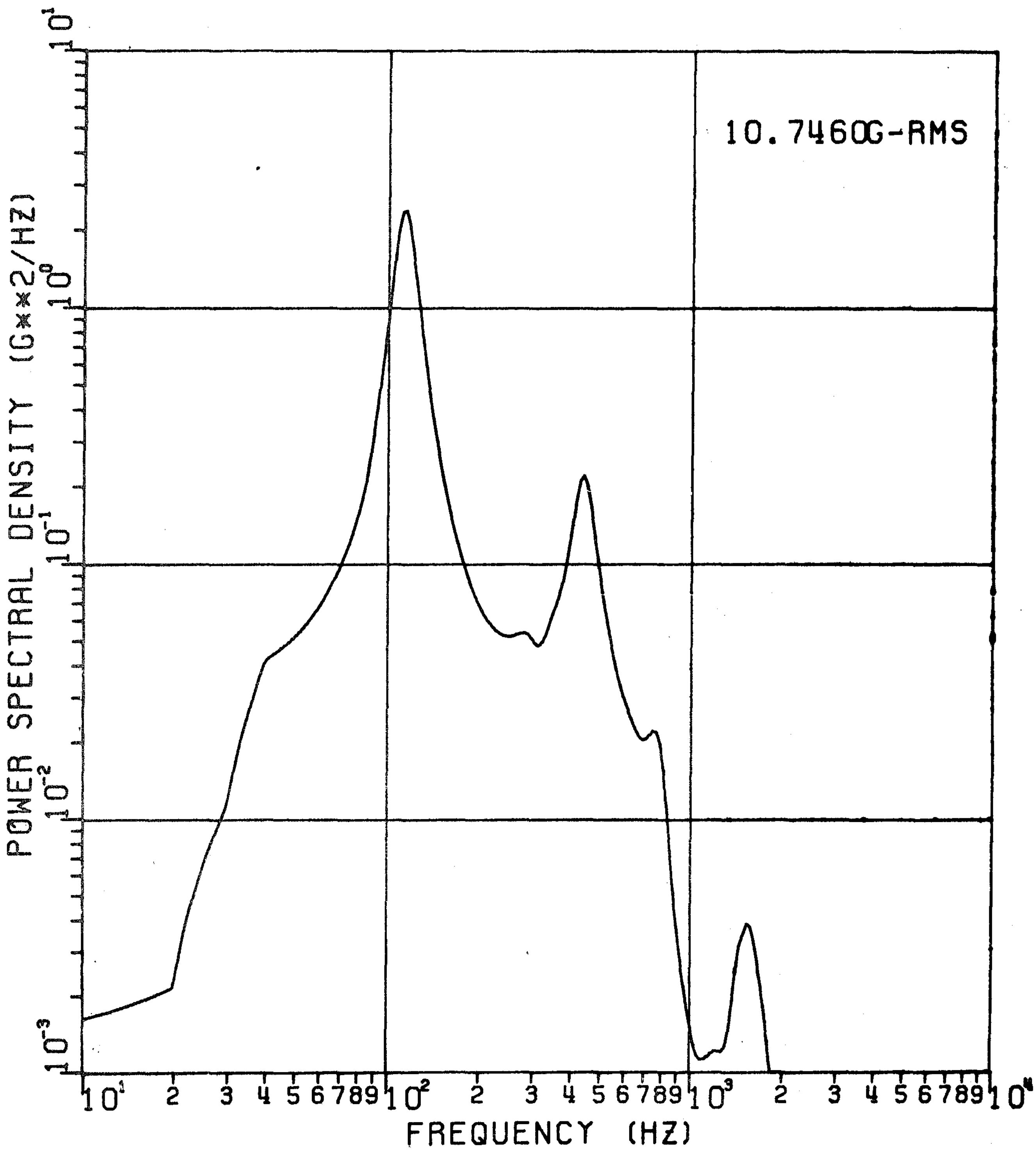
LOCATION 21



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.1.4C RANDOM VIBRATION SPECTRUM

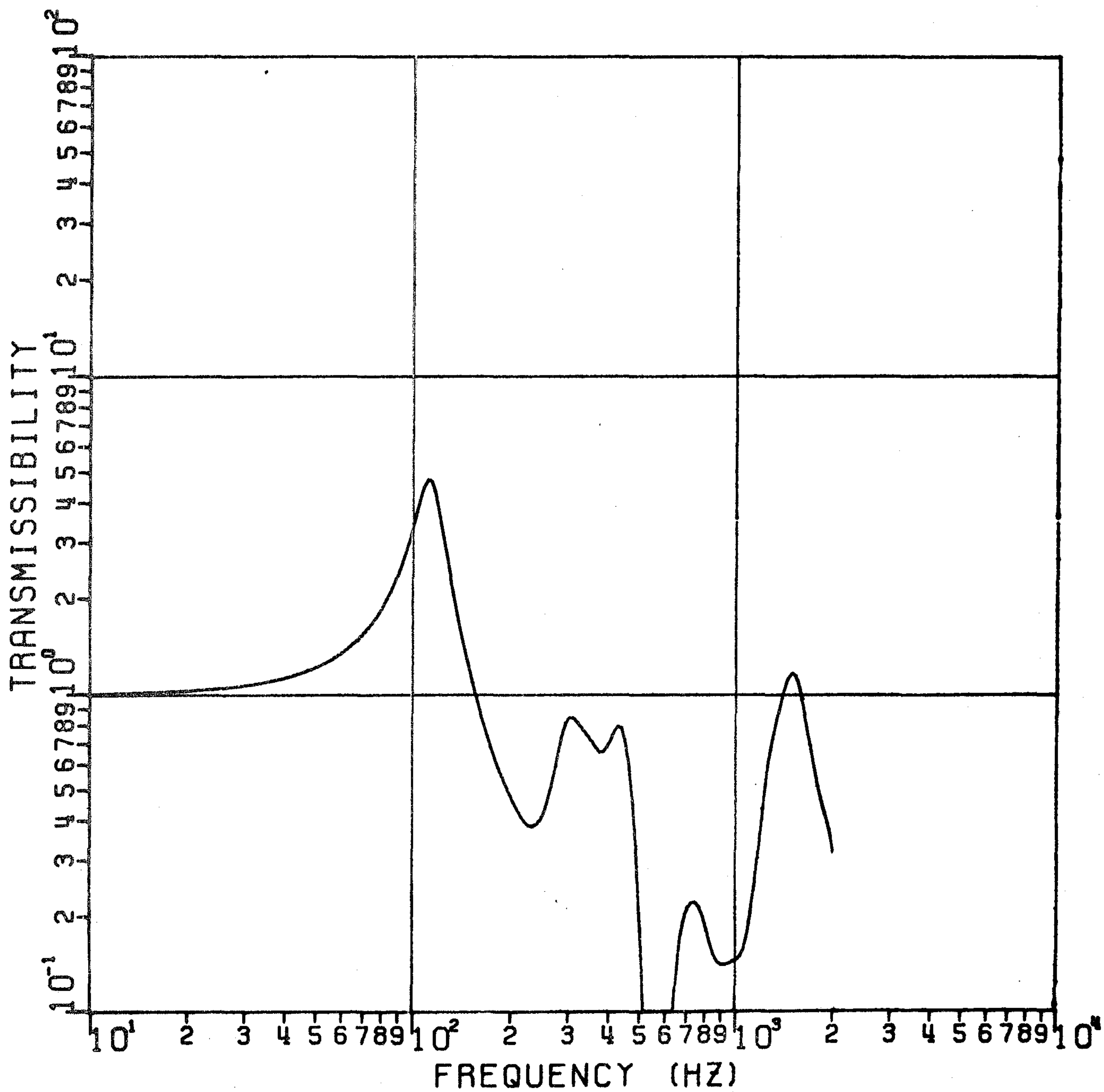
LOCATION 21



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.15A TRANSMISSIBILITY

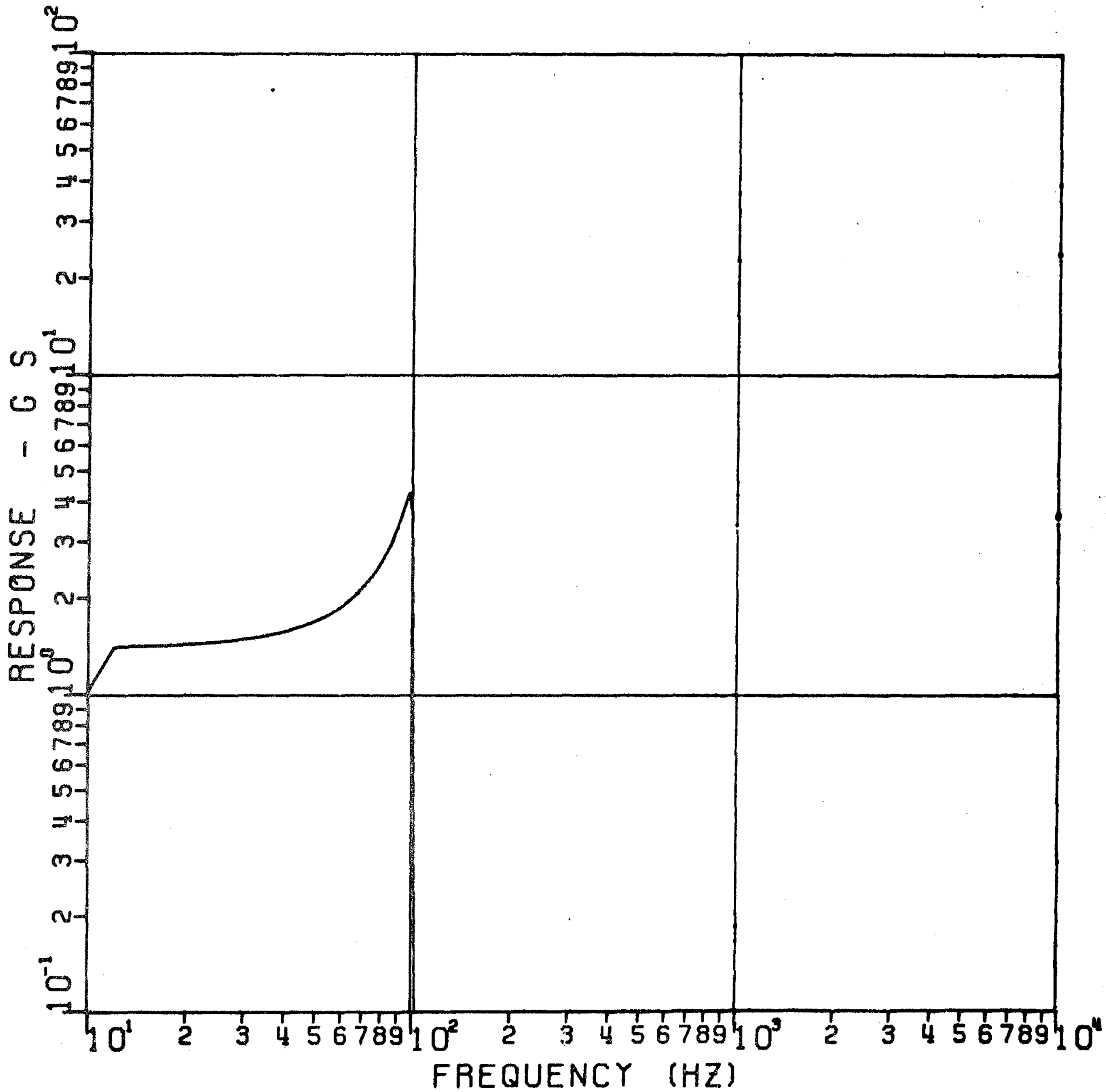
LOCATION 28



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.15B SINE RESPONSE

LOCATION 28



FREQUENCY RESPONSE OF LR3 300, 30 D.F.MODEL

FIGURE 3.15C RANDOM VIBRATION SPECTRUM

LOCATION 28

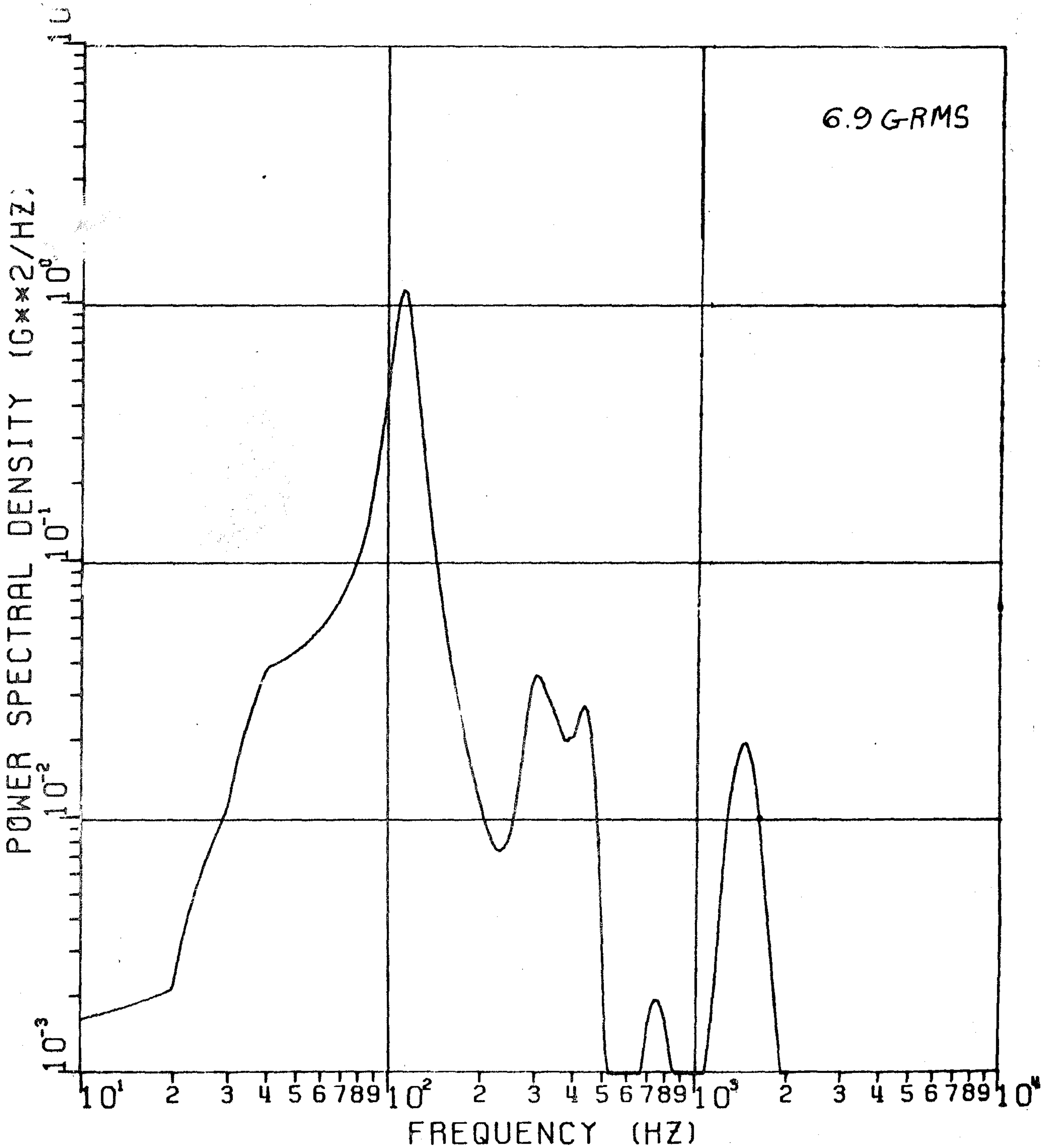


TABLE 3.1.1
SUMMARY OF OUT OF PLANE RESPONSES

Location (See Fig. 2.1.1 & 2.1.2)	Maximum Sine Resp. g's peak	Random Resp. g's RMS
1	1.4	10.9
2	2.6	5.6
3	2.0	5.7
4	1.85	8.2
5	2.0	6.1
6	2.5	8.3
7	2.5	6.8
8	2.8	6.4
9	2.7	9.7
10	3.2	8.4
11	3.2	7.0
12	2.9	7.2
13	3.2	7.9
14	4.0	7.7
15	3.3	5.8
16	2.0	6.5
17	4.4	8.0
18	4.5	6.8
19	2.0	6.3
20	3.5	9.7
21	5.7	10.8
22	3.5	8.4
23	4.1	7.1
24	6.3	11.9
25	4.4	7.8
26	6.5	14.0
27	6.7	15.9
28	4.3	6.9
29	6.2	11.4
30	3.8	8.4

Figure 3.1.6

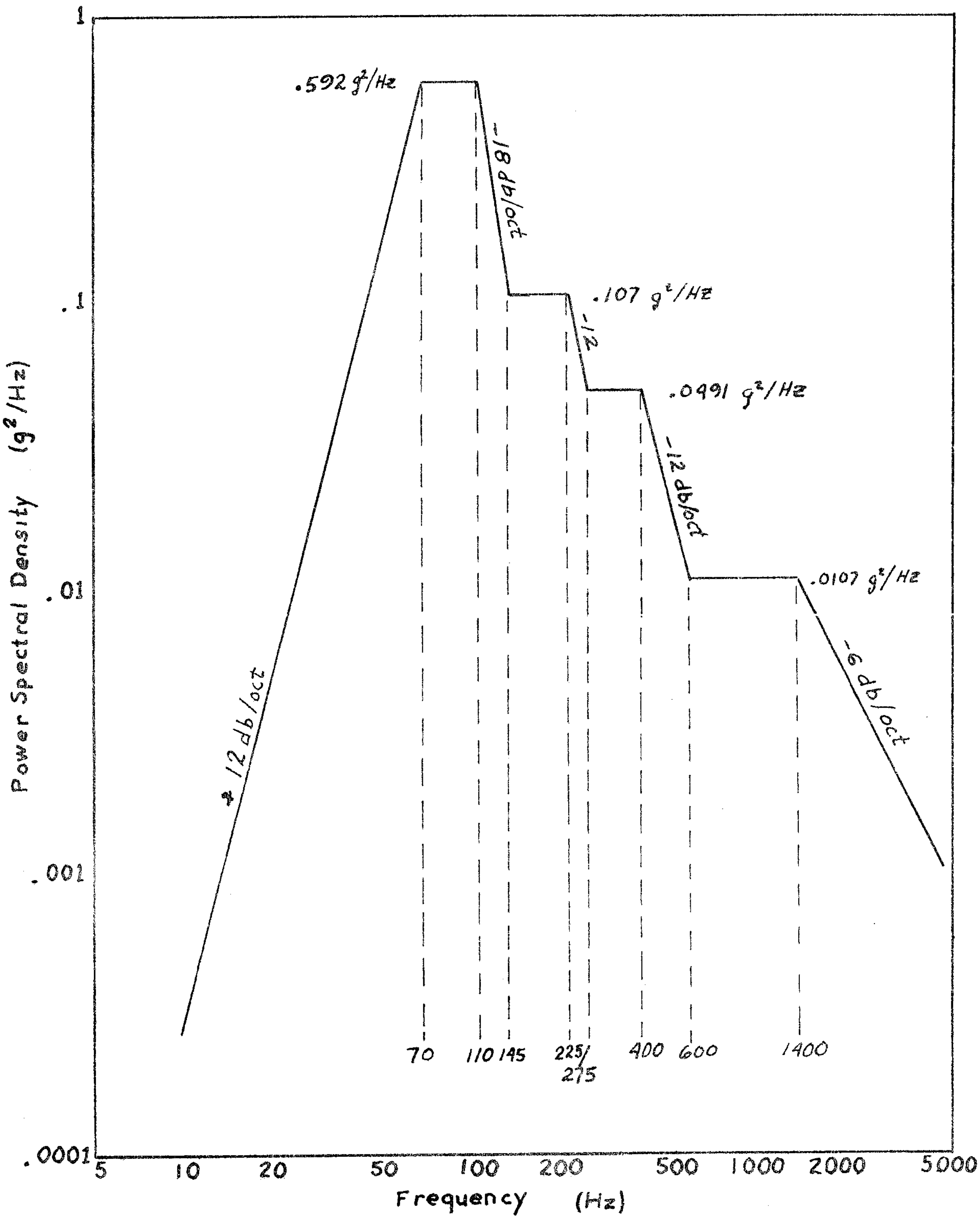
WORST RESPONSE AT A CORNER LOCATION

LOCATION 27
PERPENDICULAR TO ARRAY

UPPER ARRAY ACCEPTANCE RANDOM VIBRATION LEVELS

Figure 3.1.7

Z AXIS - PERPENDICULAR TO ARRAY

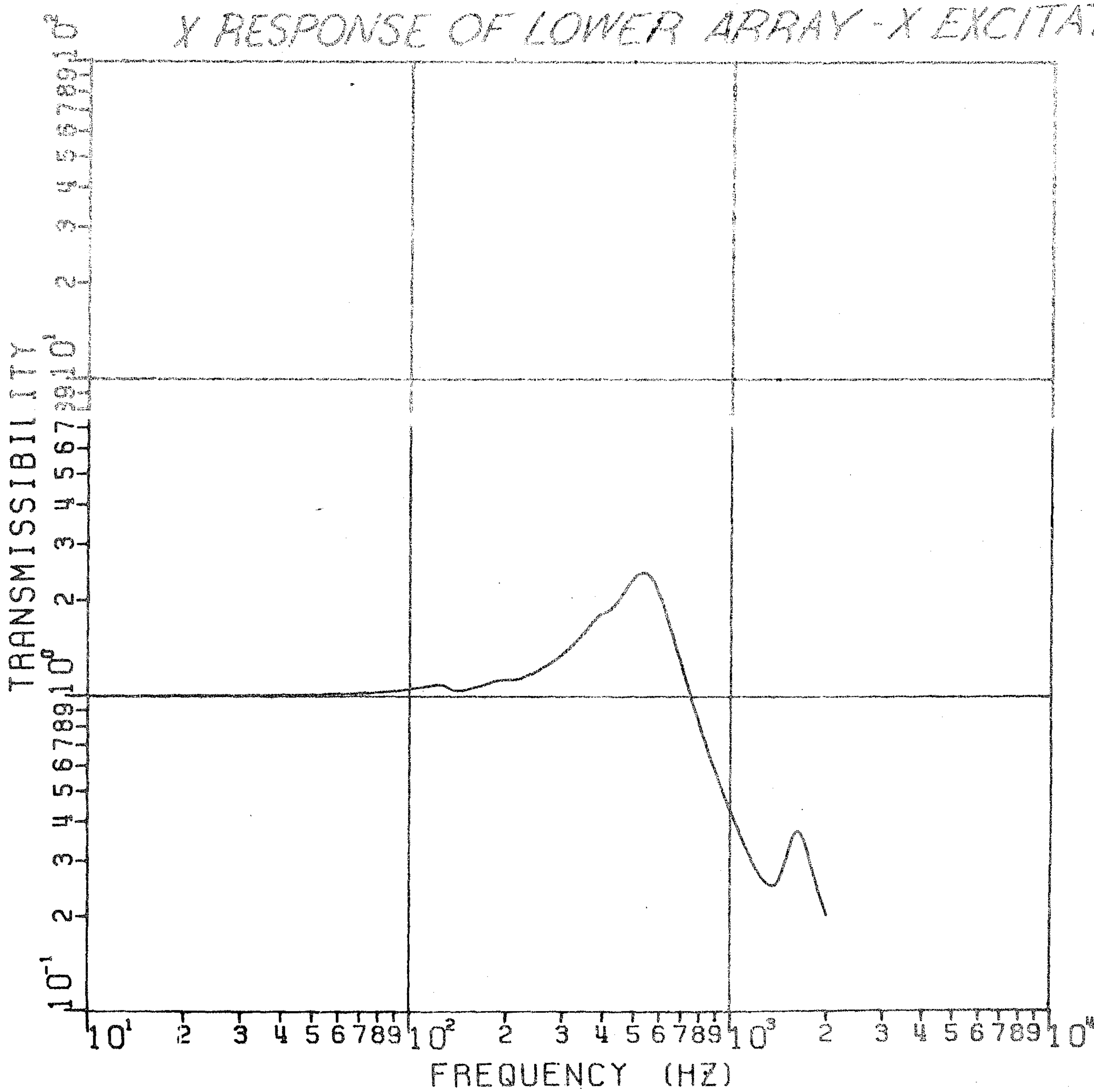


FREQUENCY RESPONSE FØR LR3 300,60.F. MODEL

FIGURE 3.2.1A TRANSMISSIBILITY

LØCATIØN 1

X RESPONSE OF LOWER ARRAY - X EXCITATION

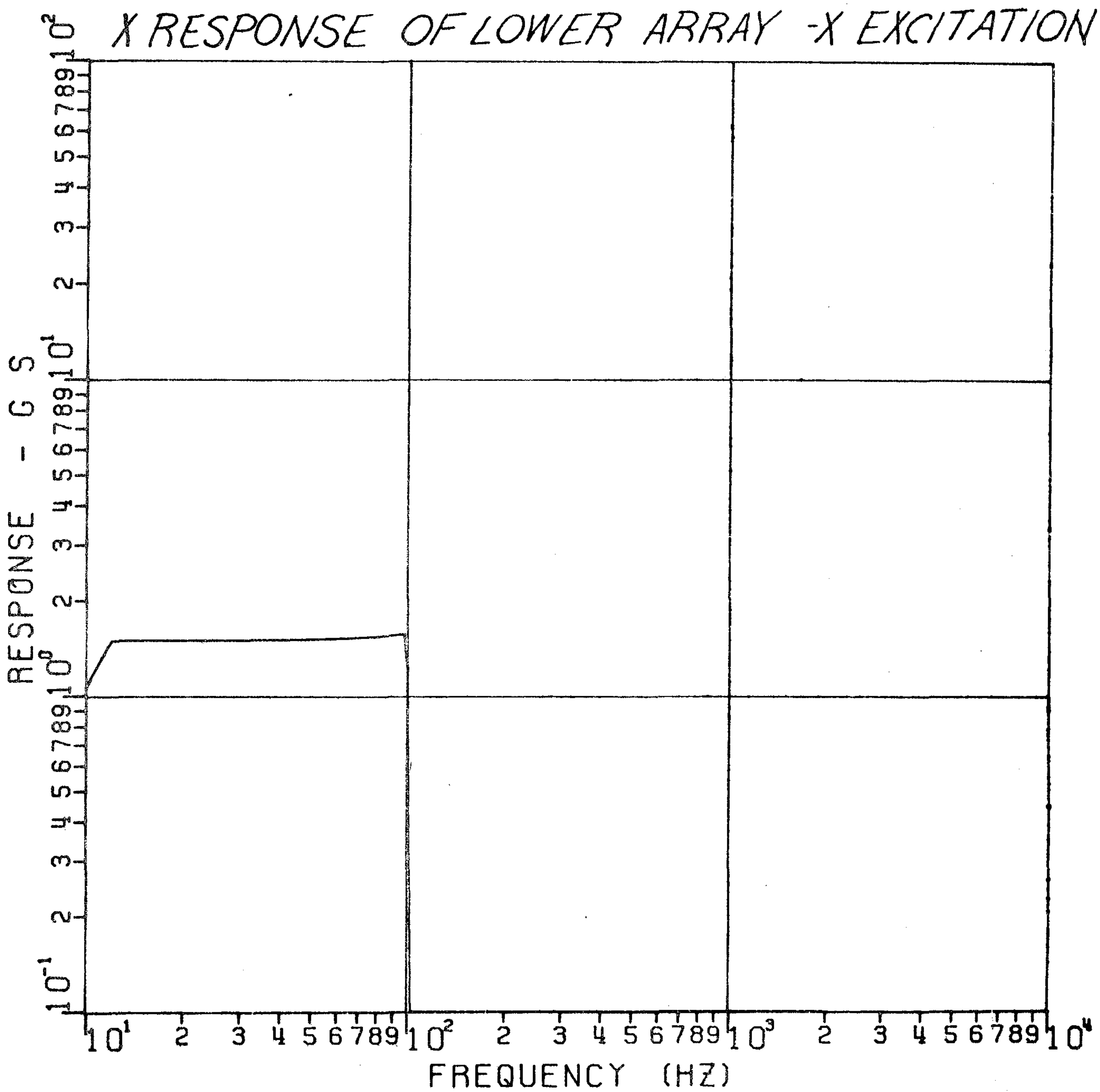


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.1B SINE RESPONSE

LOCATION 1

X RESPONSE OF LOWER ARRAY -X EXCITATION

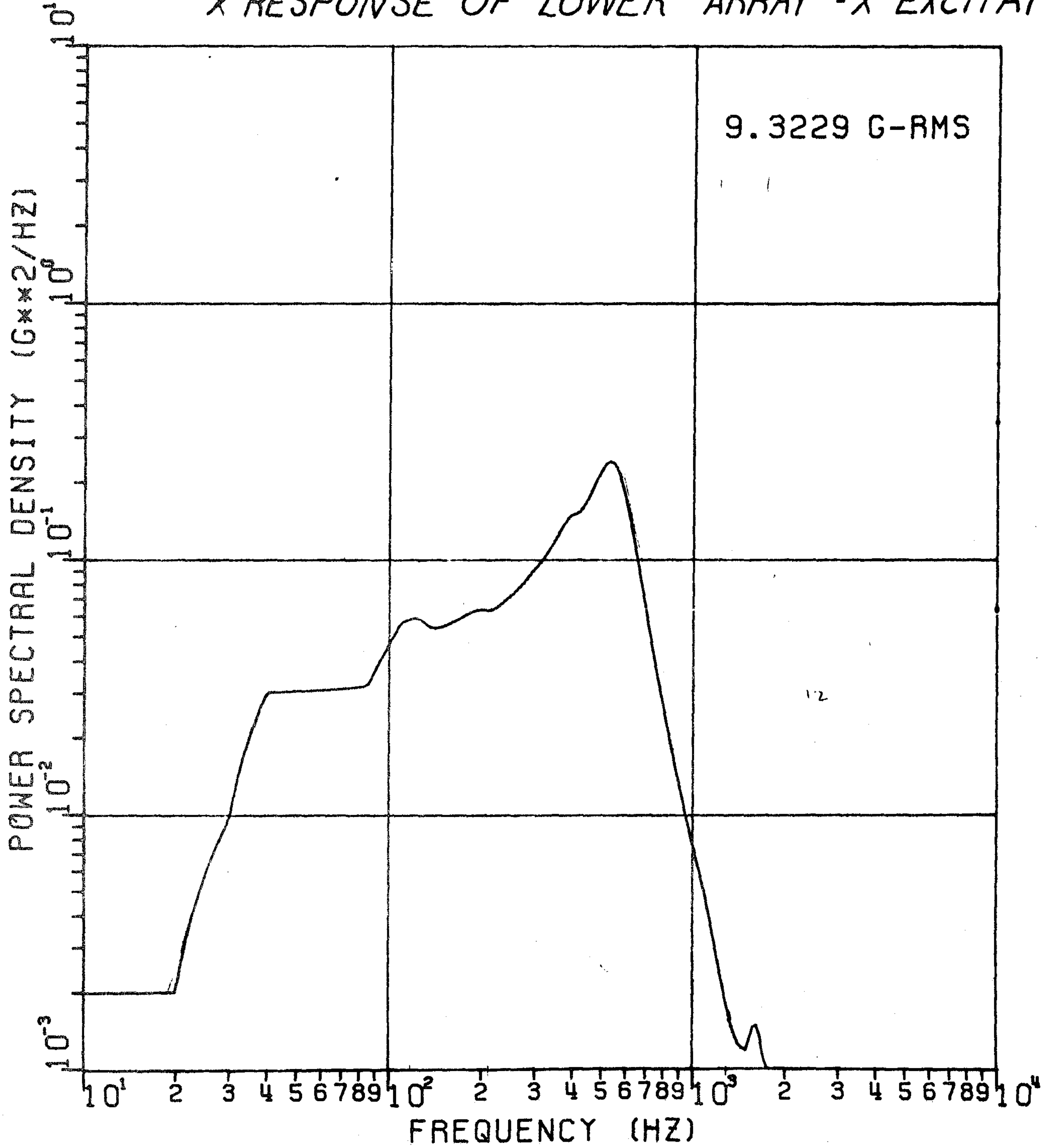


FREQUENCY RESPONSE FOR LR3 300.6D.F. MODEL

FIGURE 3.2.1C RANDOM VIBRATION SPECTRUM

LOCATION 1

X RESPONSE OF LOWER ARRAY -X EXCITATION

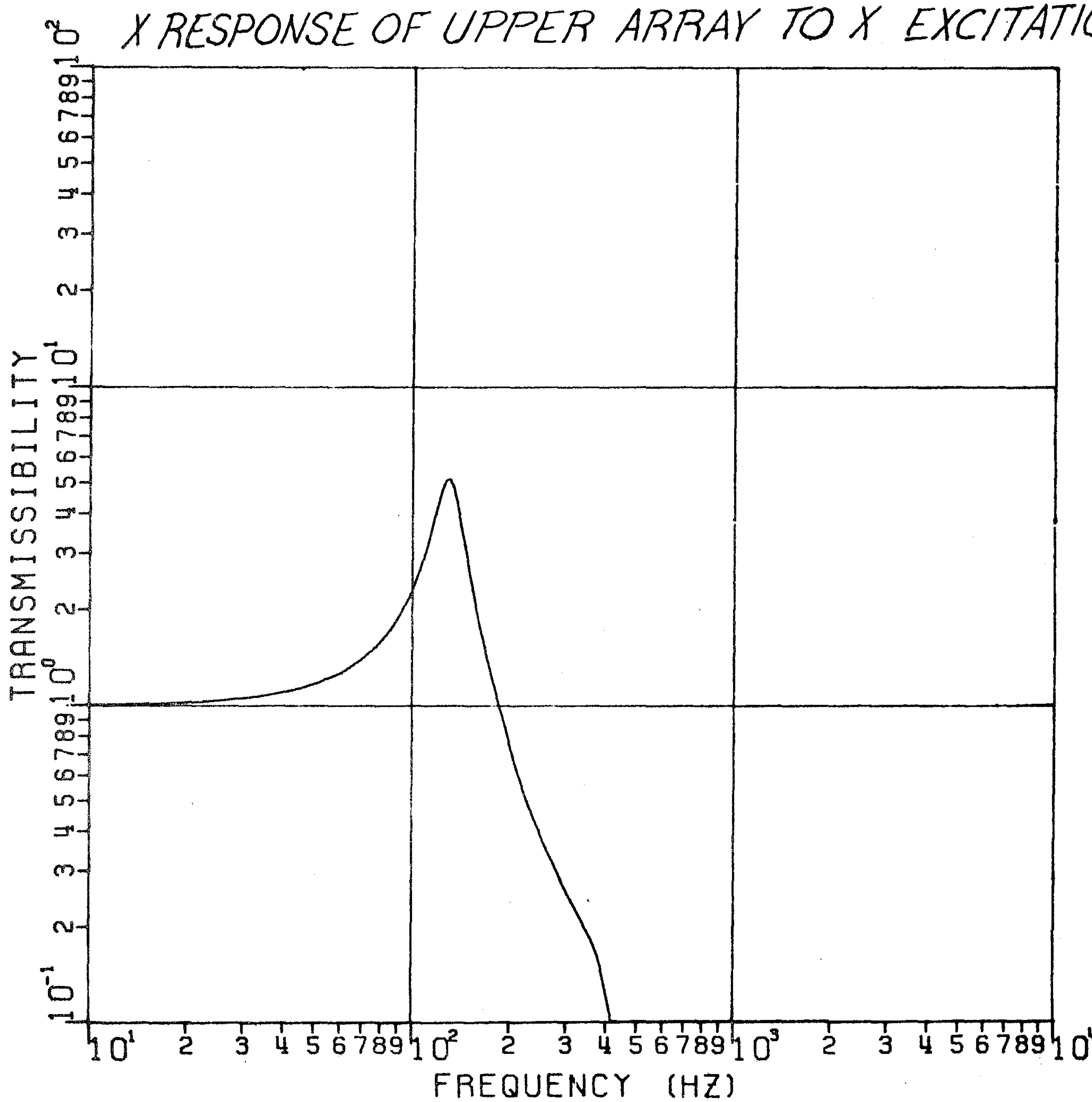


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.2 A TRANSMISSIBILITY

LOCATION 4

X RESPONSE OF UPPER ARRAY TO X EXCITATION

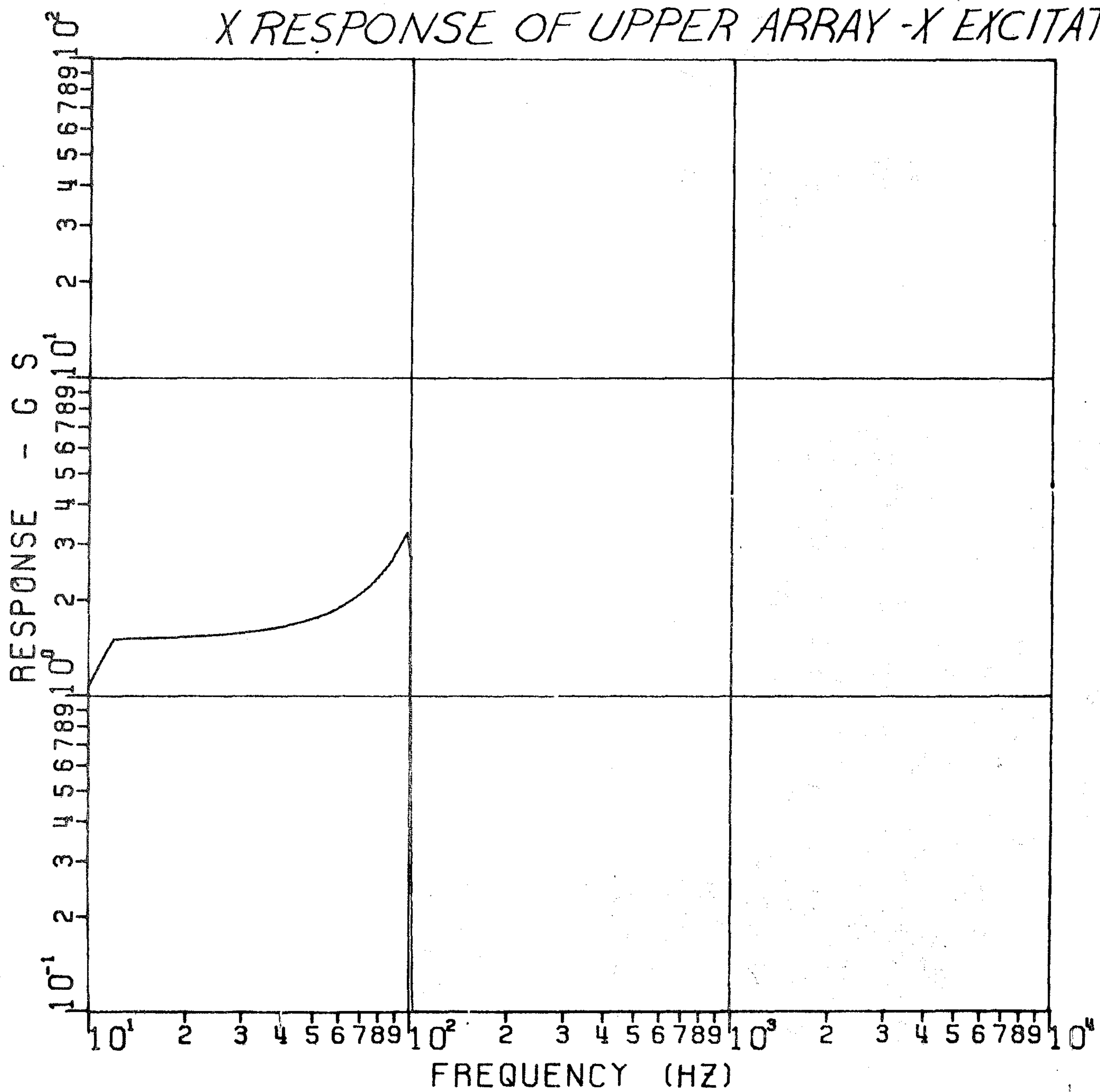


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.2.B SINE RESPONSE

LOCATION 4

X RESPONSE OF UPPER ARRAY -X EXCITATION

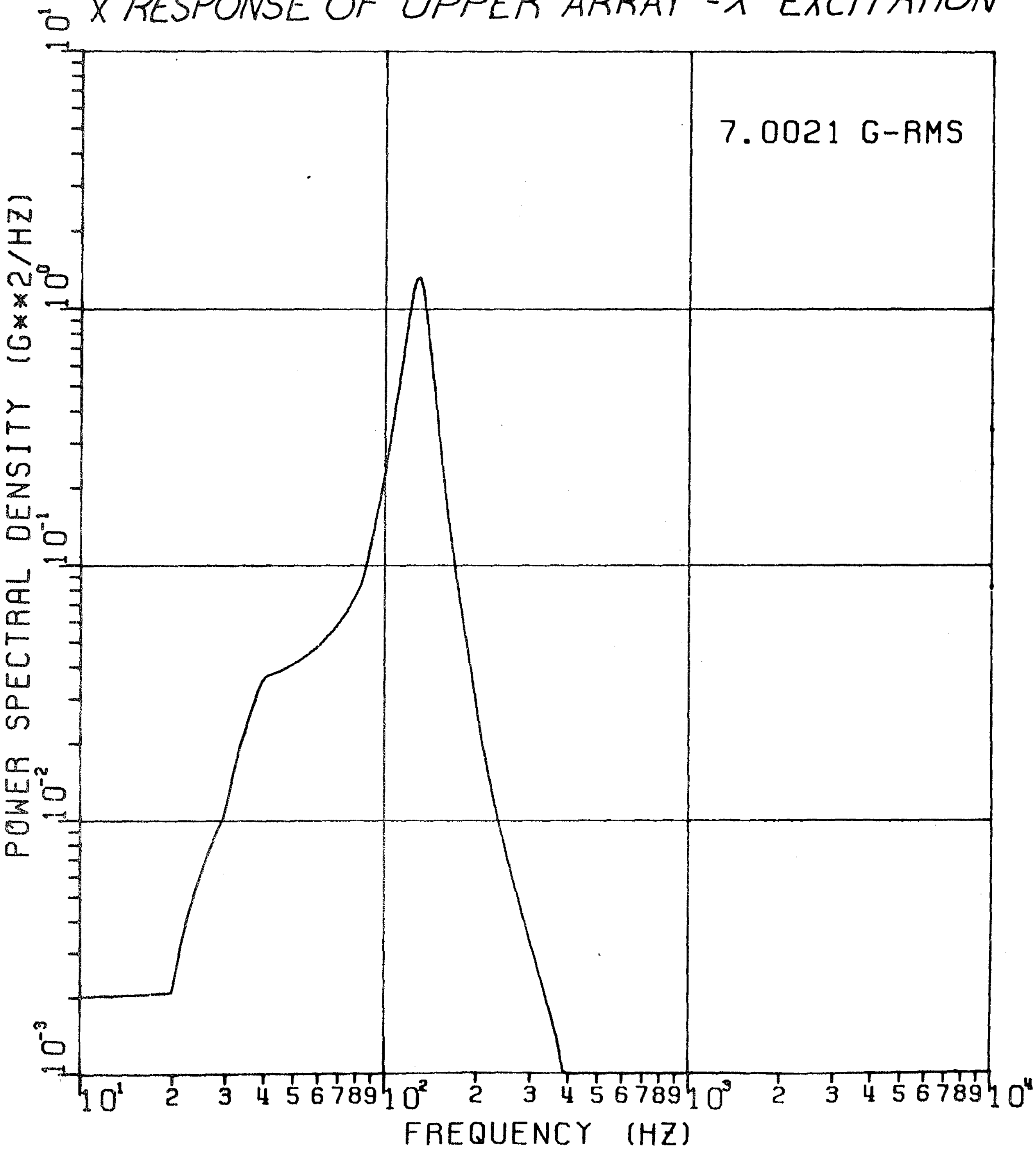


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.2C RANDOM VIBRATION SPECTRUM

LOCATION 4

X RESPONSE OF UPPER ARRAY -X EXCITATION

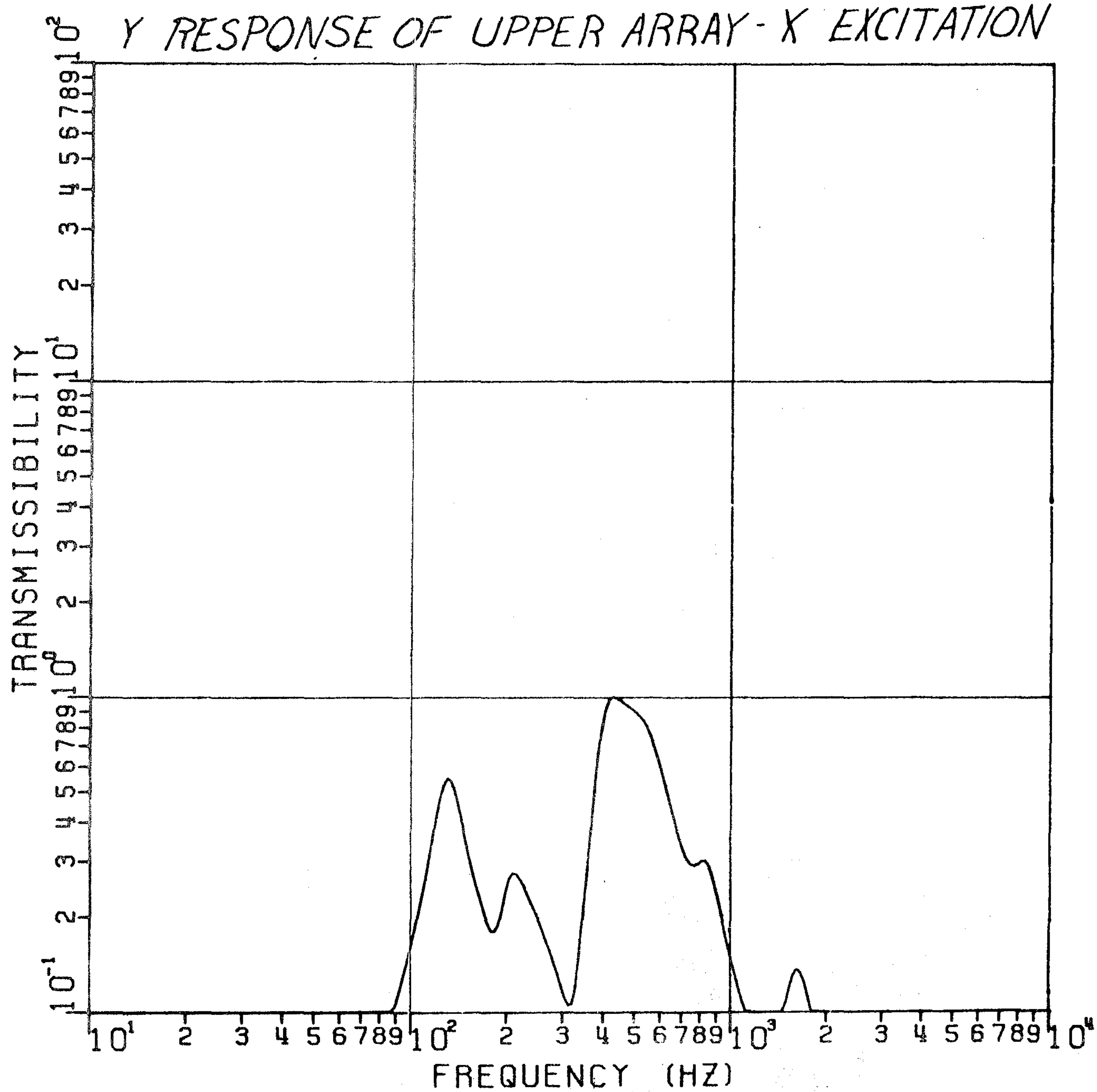


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.3 A TRANSMISSIBILITY

LOCATION 5

Y RESPONSE OF UPPER ARRAY - X EXCITATION

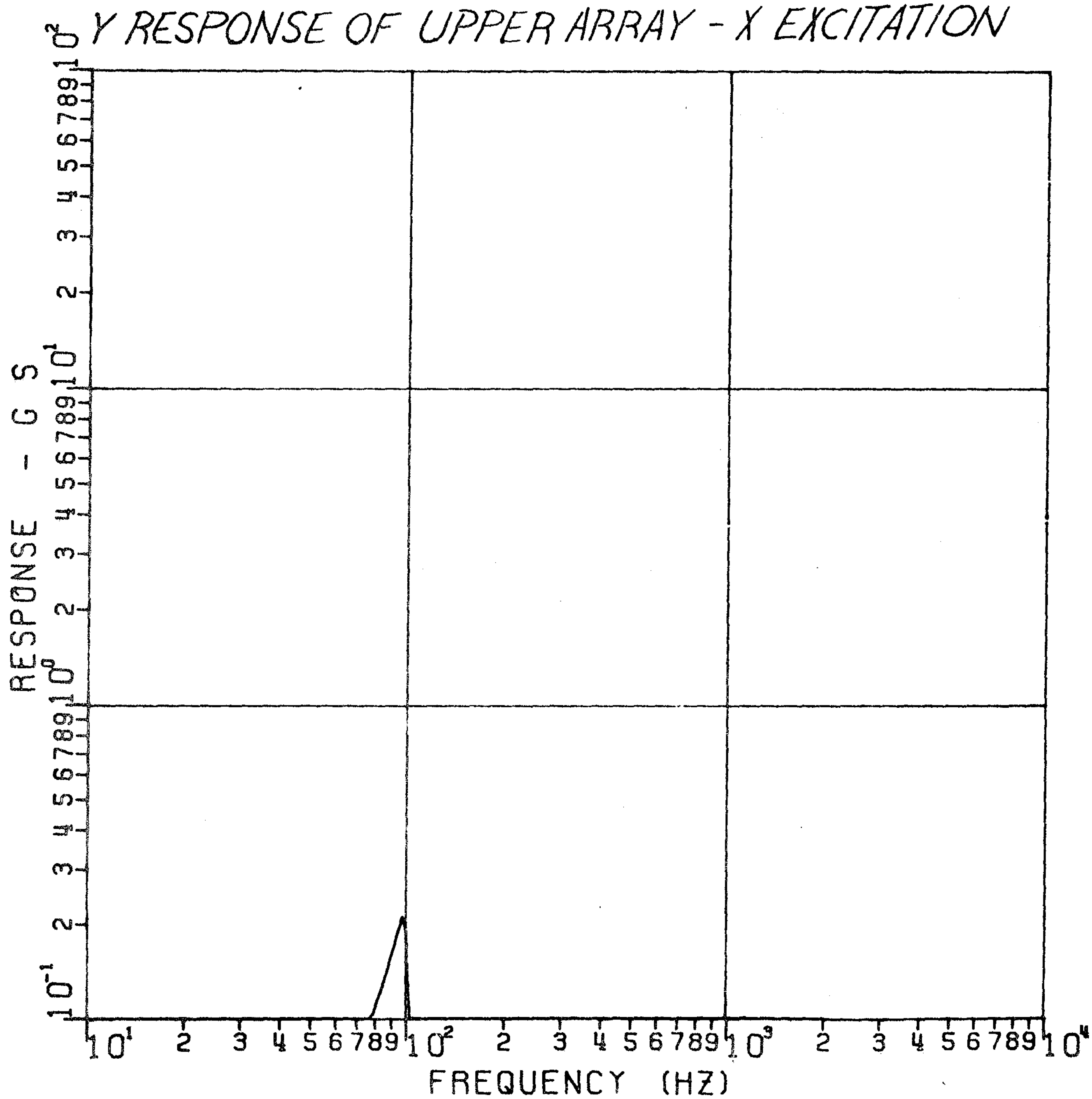


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.3 B SINE RESPONSE

LOCATION 5

Y RESPONSE OF UPPER ARRAY - X EXCITATION

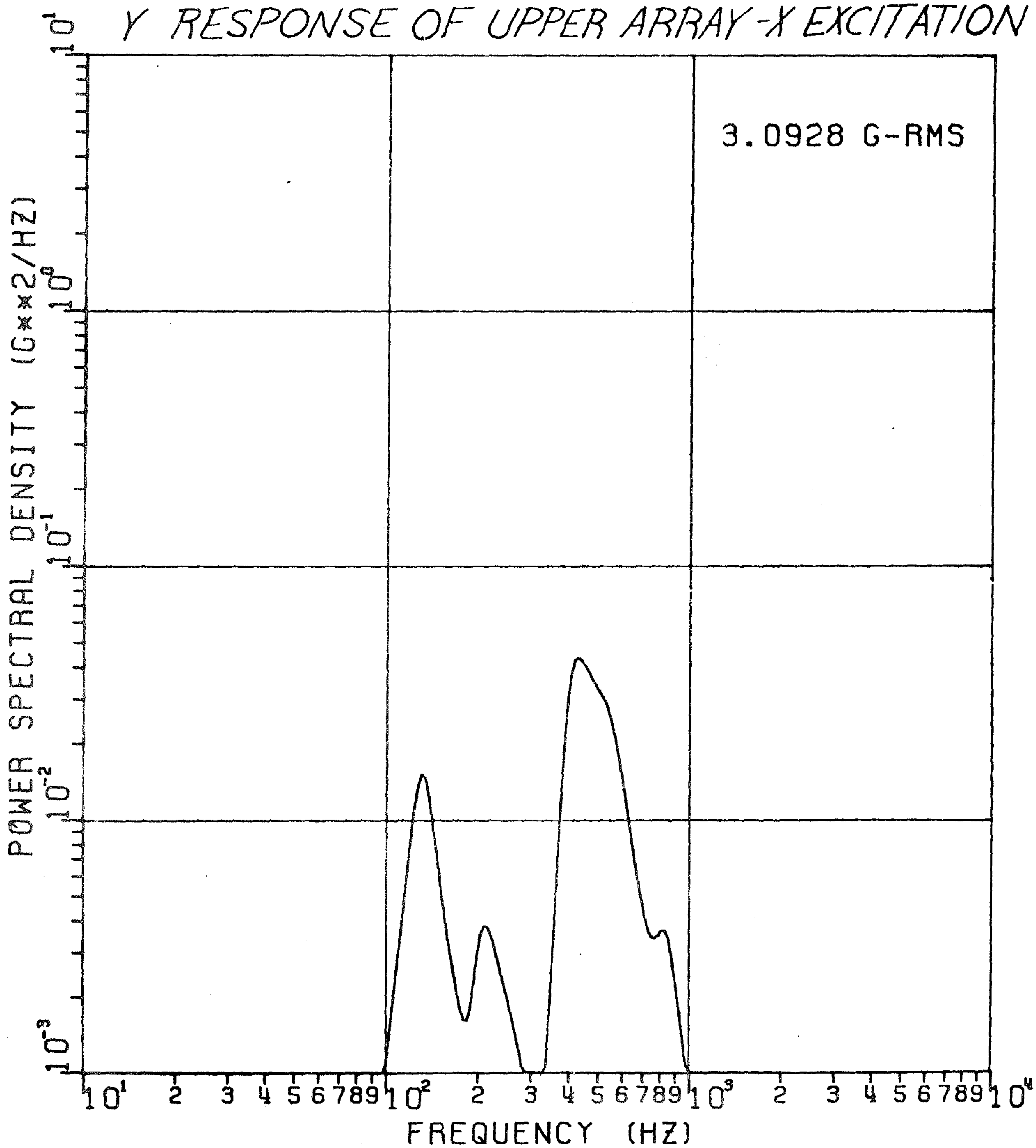


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.3C RANDOM VIBRATION SPECTRUM

LOCATION 5

Y RESPONSE OF UPPER ARRAY -X EXCITATION

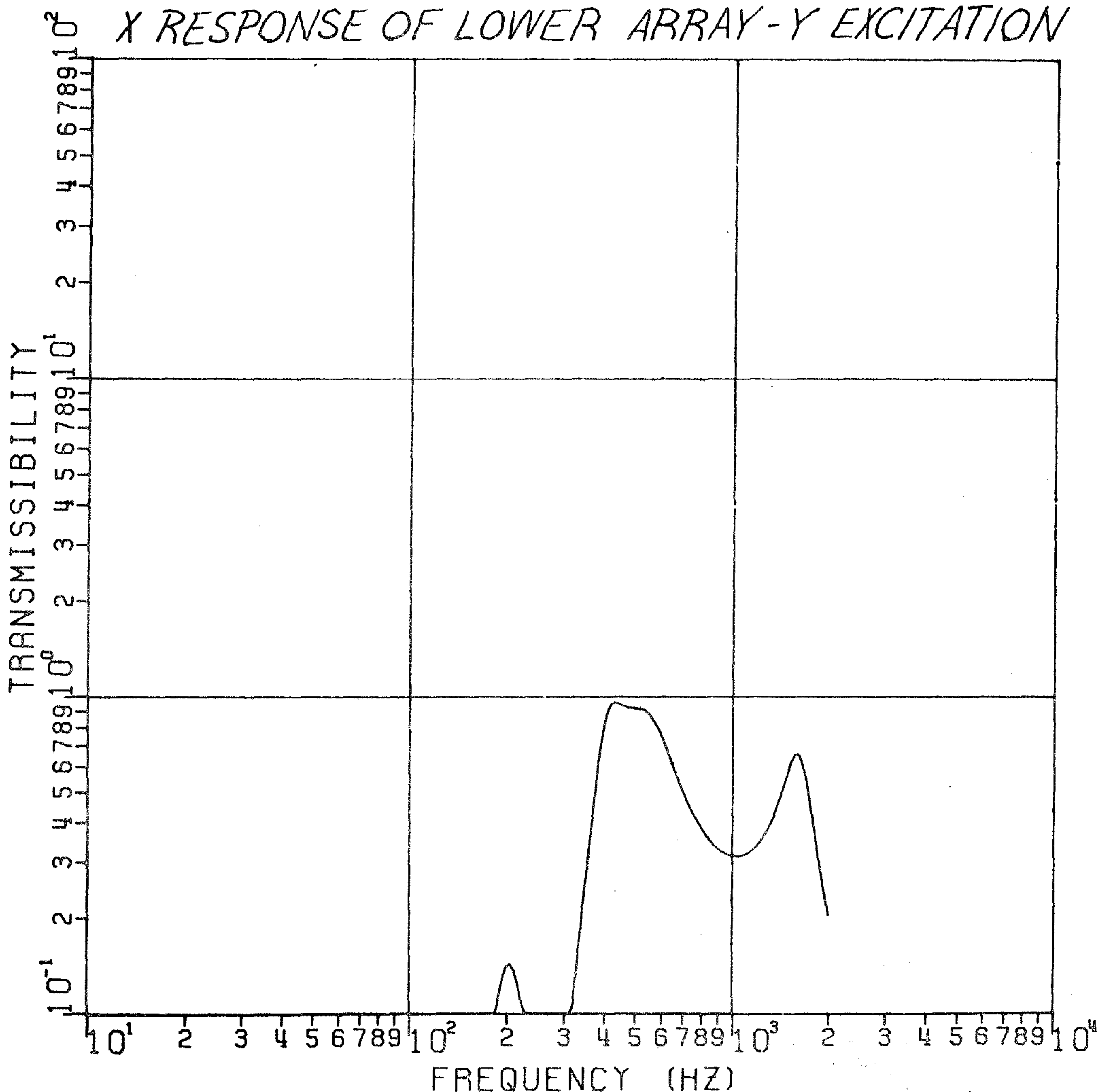


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.4A TRANSMISSIBILITY

LOCATION 1

X RESPONSE OF LOWER ARRAY - Y EXCITATION

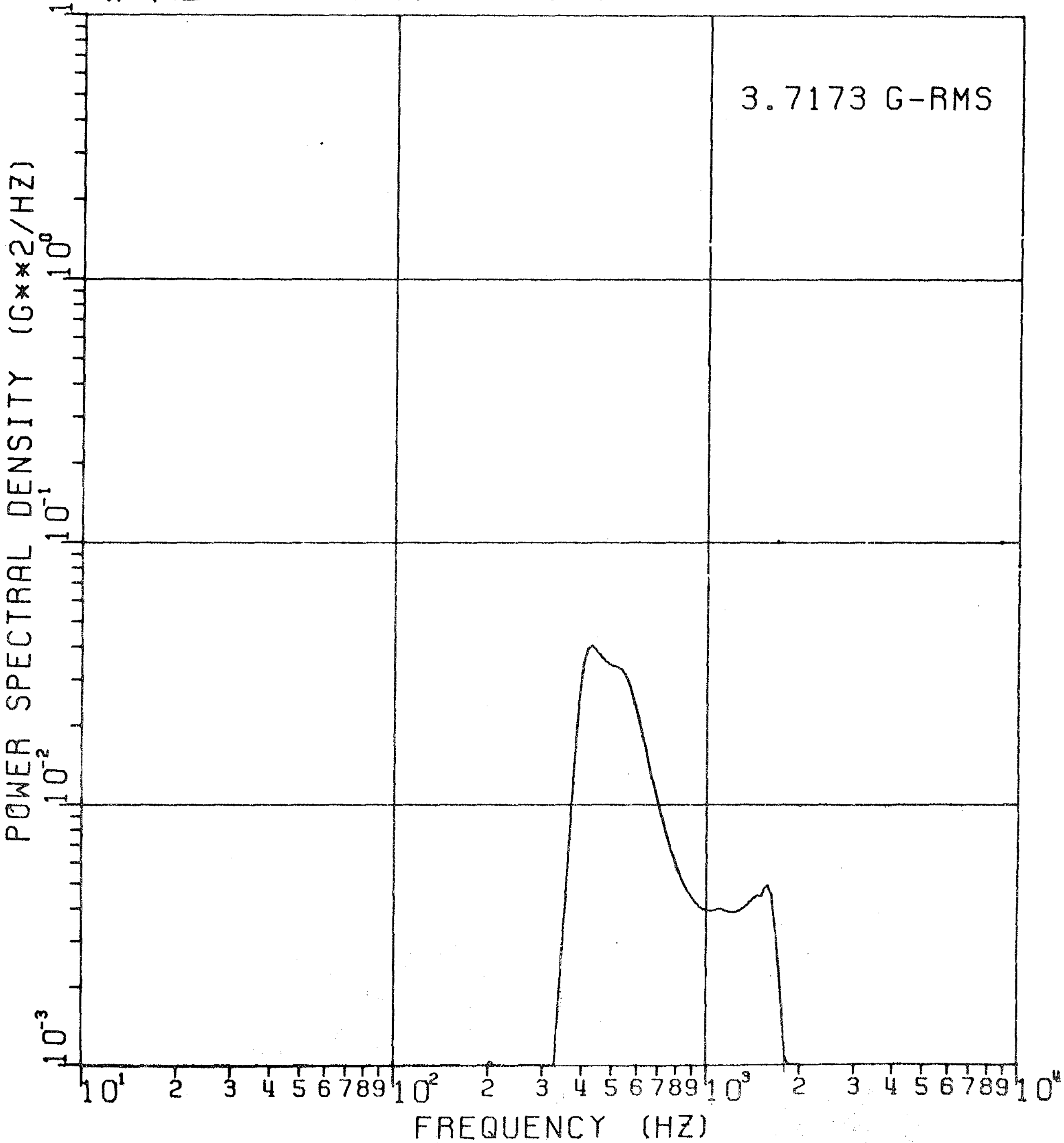


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.4 B RANDOM VIBRATION SPECTRUM

LOCATION 1

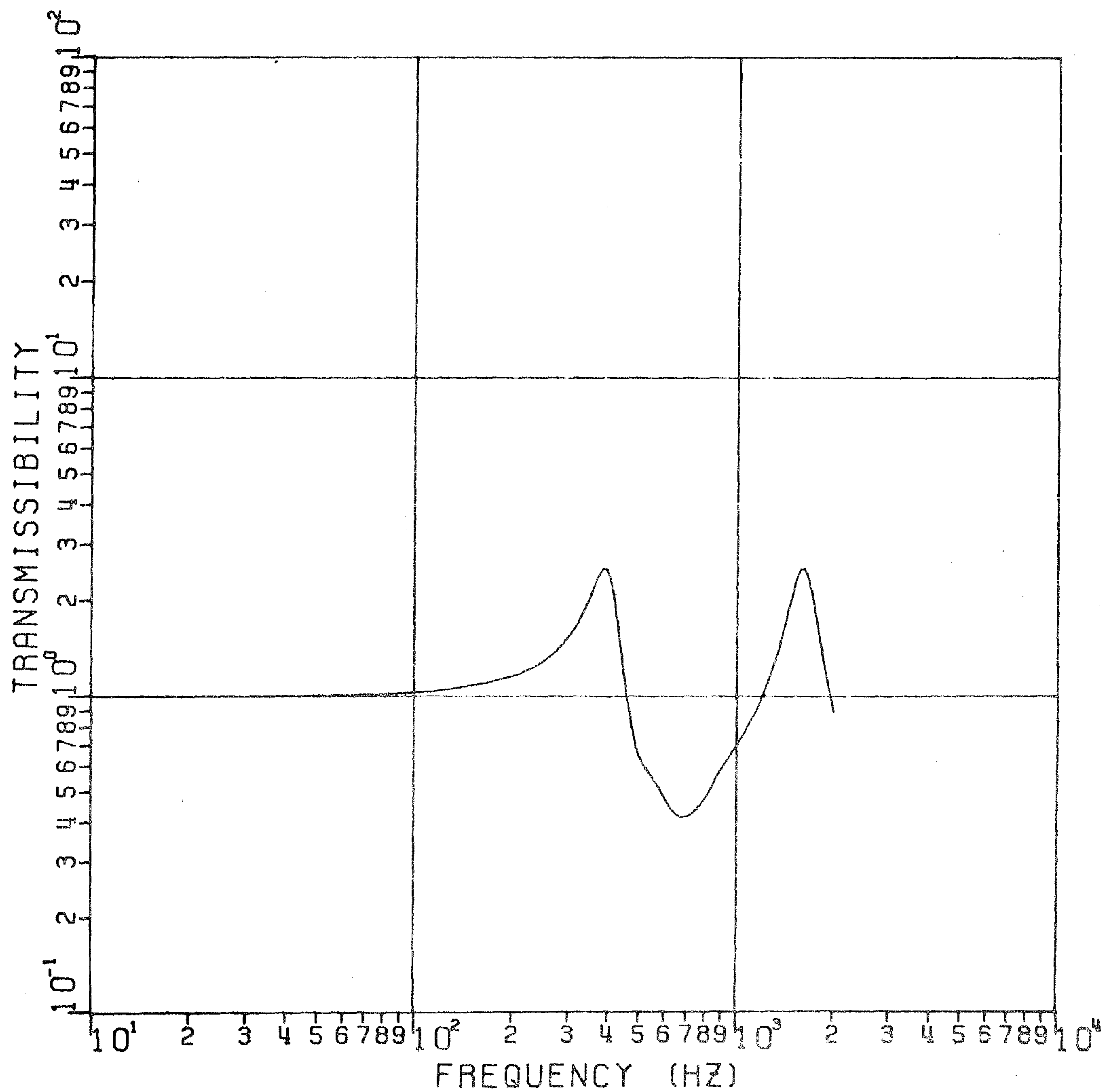
X RESPONSE OF LOWER ARRAY -Y EXCITATION



FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.5 A TRANSMISSIBILITY

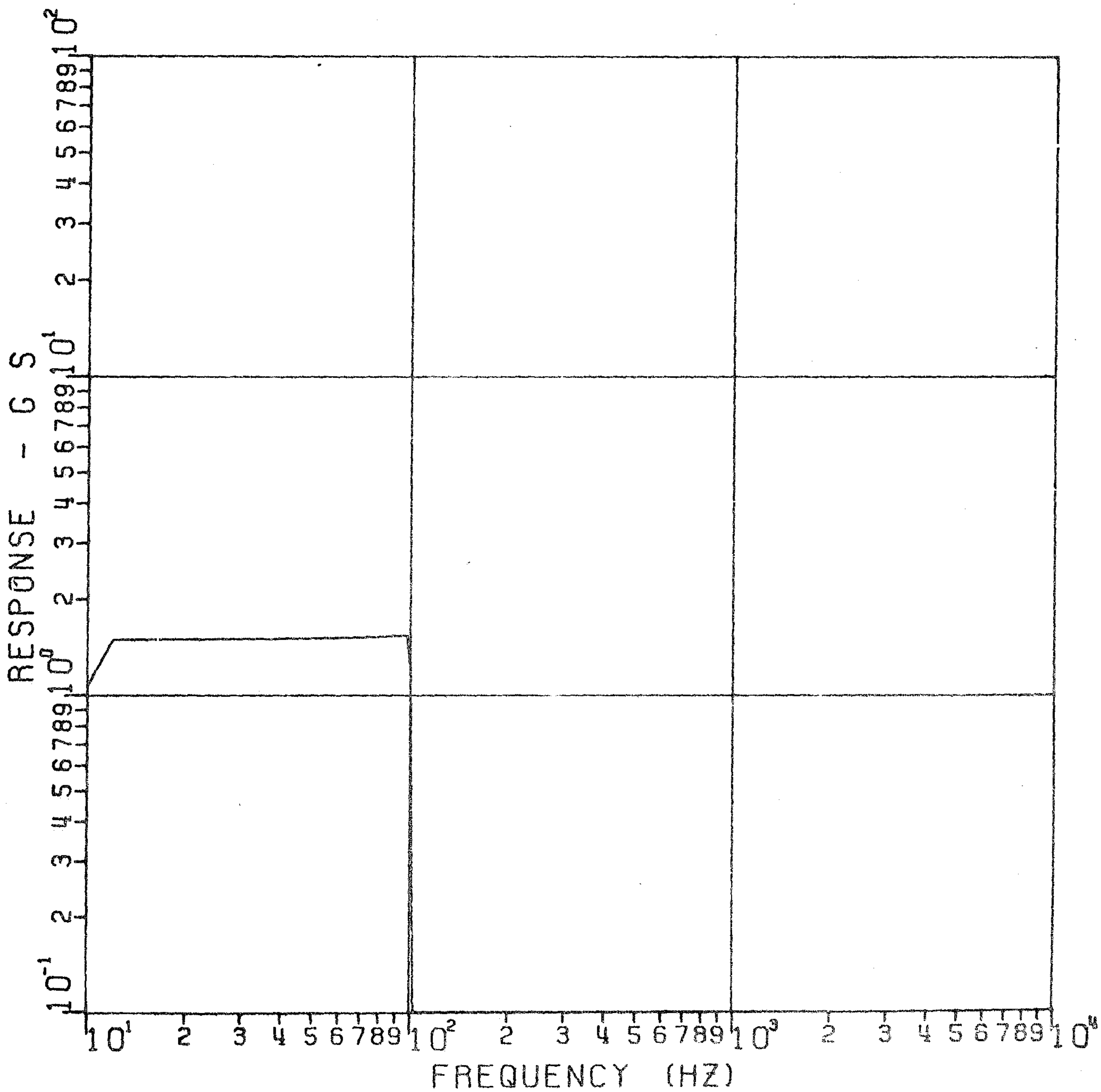
LOCATION 2



FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL,SEPT

FIGURE 3.2.5 B SINE RESPONSE

LOCATION 2

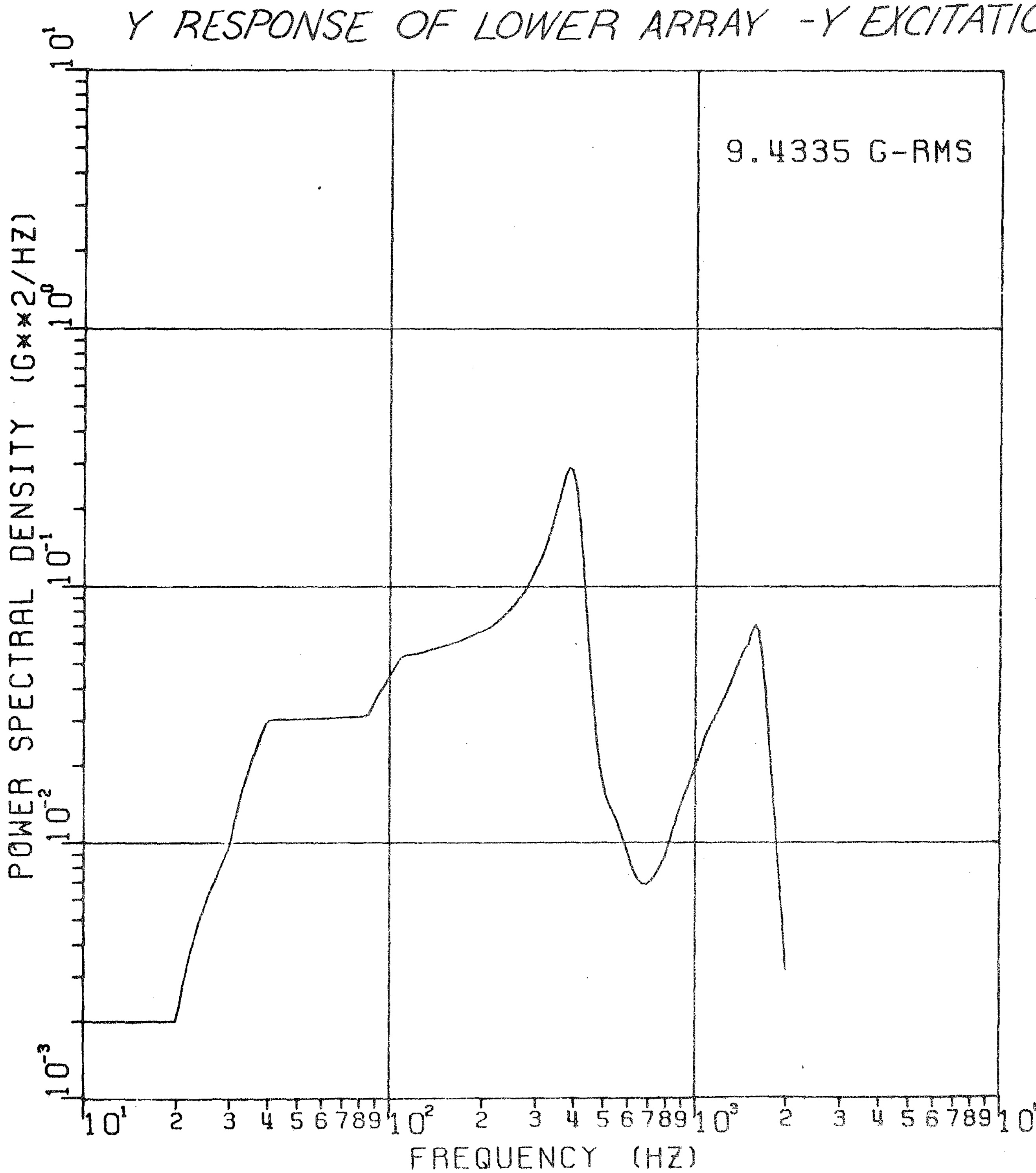


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.5 A RANDOM VIBRATION SPECTRUM

LOCATION 2

Y RESPONSE OF LOWER ARRAY -Y EXCITATION

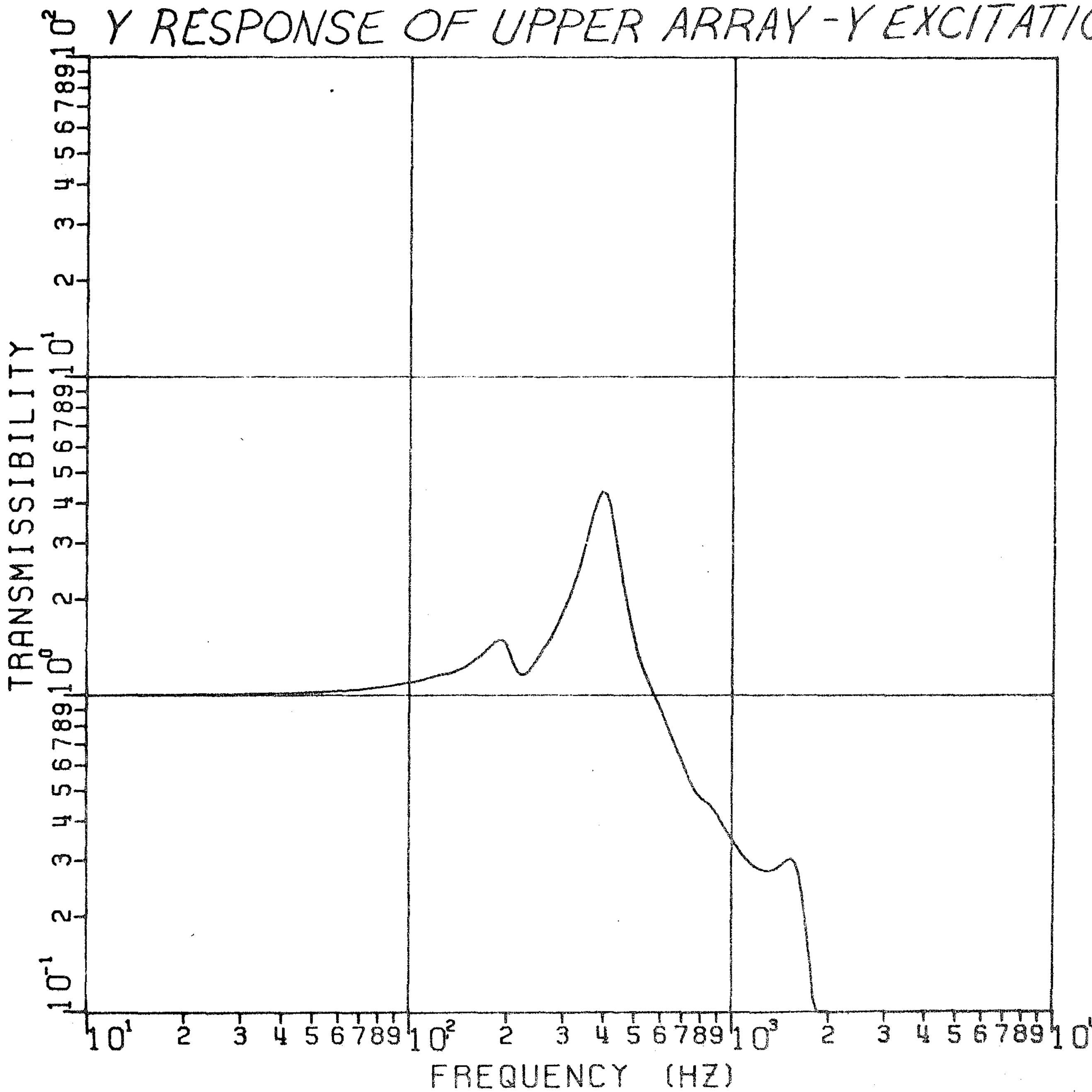


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.6 A TRANSMISSIBILITY

LOCATION 5

Y RESPONSE OF UPPER ARRAY -Y EXCITATION

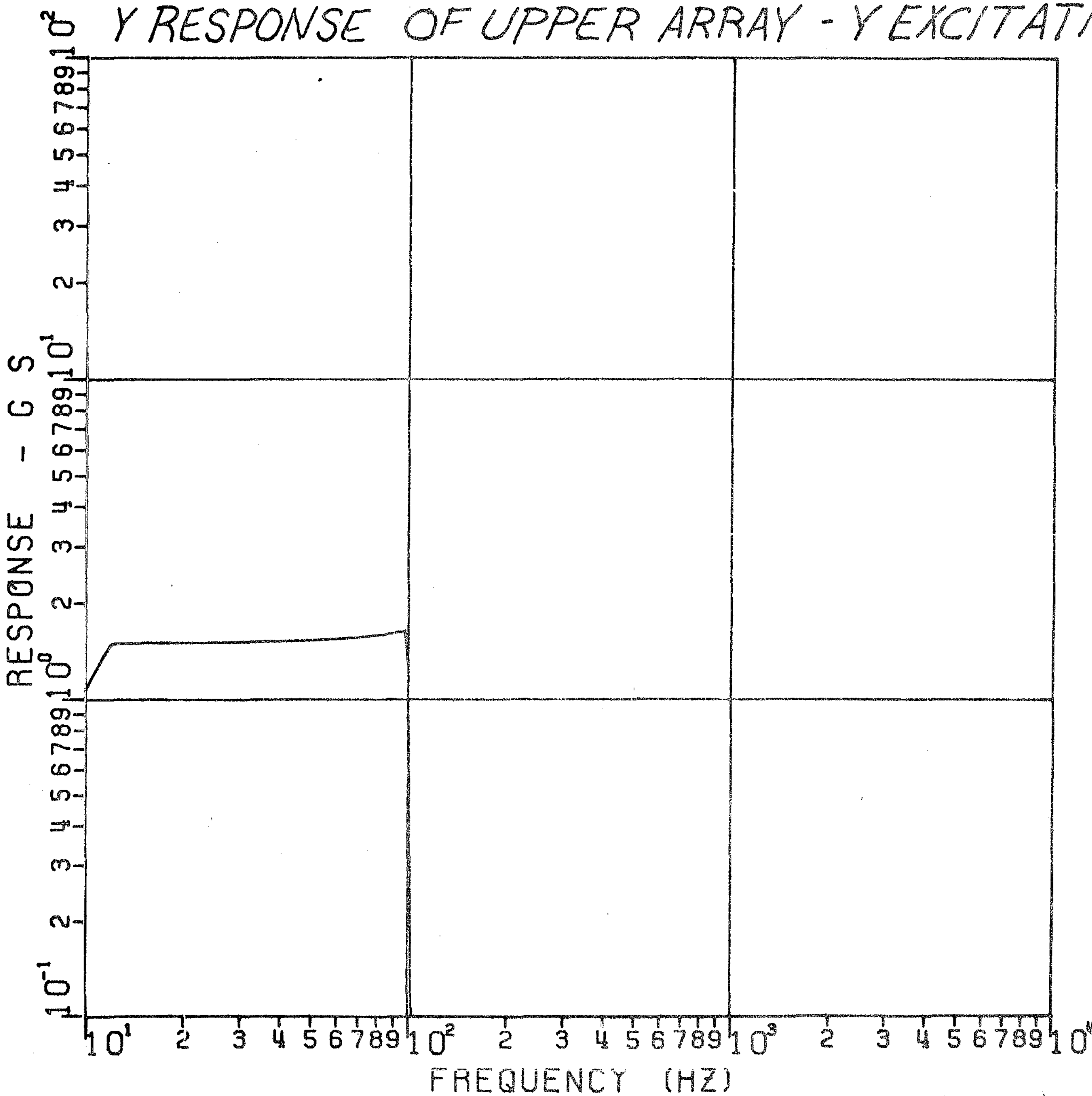


FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL 5EP

FIGURE 3.2.6 B SINE RESPONSE

LOCATION 5

Y RESPONSE OF UPPER ARRAY - Y EXCITATION



FREQUENCY RESPONSE FOR LR3 300,6D.F. MODEL

FIGURE 3.2.6 C RANDOM VIBRATION SPECTRUM

LOCATION 5

Y RESPONSE OF UPPER ARRAY - Y EXCITATION

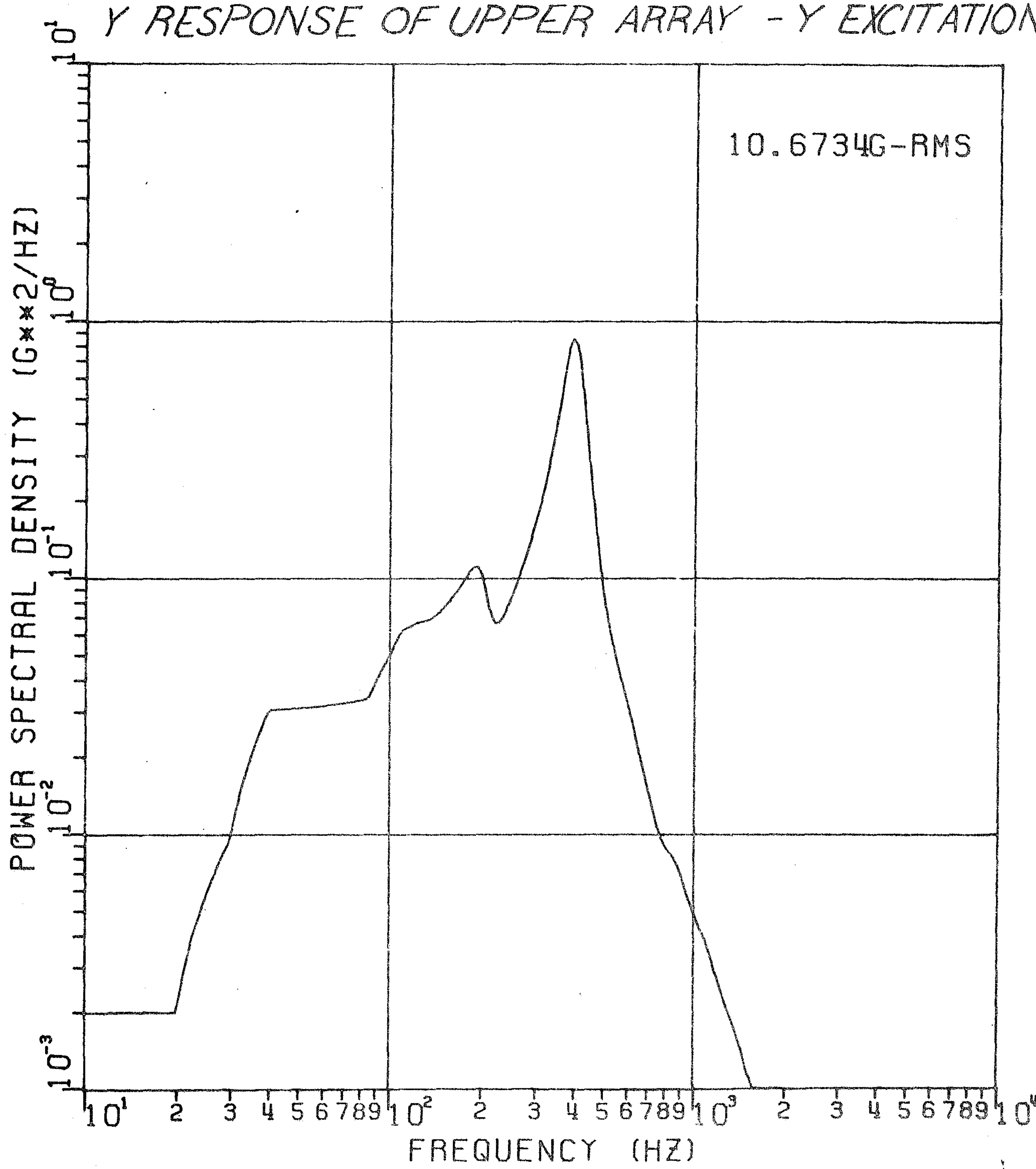


TABLE 3.2.1
SUMMARY OF IN PLANE RESPONSES

Excitation Direction	Array	Coordinate Direction	Max Sine Resp. (g's peak)	Ramdom Resp. (g's RMS)
X	Upper	X	3.28	7.00
		Y	.215	3.09
		θ	1.84×10^{-2} pseudo g's	.235 pseudo g's
	Lower	X	1.57	9.32
		Y	.512	2.84
		θ	1.91×10^{-3} pseudo g's	.247 pseudo g's
Y	Upper	X	7.31×10^{-2}	.653
		Y	1.64	10.67
		θ	1.56×10^{-2} pseudo g's	.611 pseudo g's
	Lower	X	1.55×10^{-2}	3.71
		Y	1.54	9.43
		θ	3.86×10^{-3} pseudo g's	.859 pseudo g's

TABLE 3.2.2

Excitation Direction	Array	Translational Response (grms)	Rotational Response $\frac{rad}{sec^2 \, 386}$	Distance To Edge (in)	Total* Resp. (grms)
X	Upper	7.00	.235	6.5	7.16
	Lower	9.32	.248	12.9	9.82
Y	Upper	10.67	.611	10.9	12.5
	Lower	9.43	.859	10.9	13.3

*Total Response $= \sqrt{\text{Translational Response}^2 + (\text{Rotational Resp.} \times \text{Dist. to Edge})^2}$

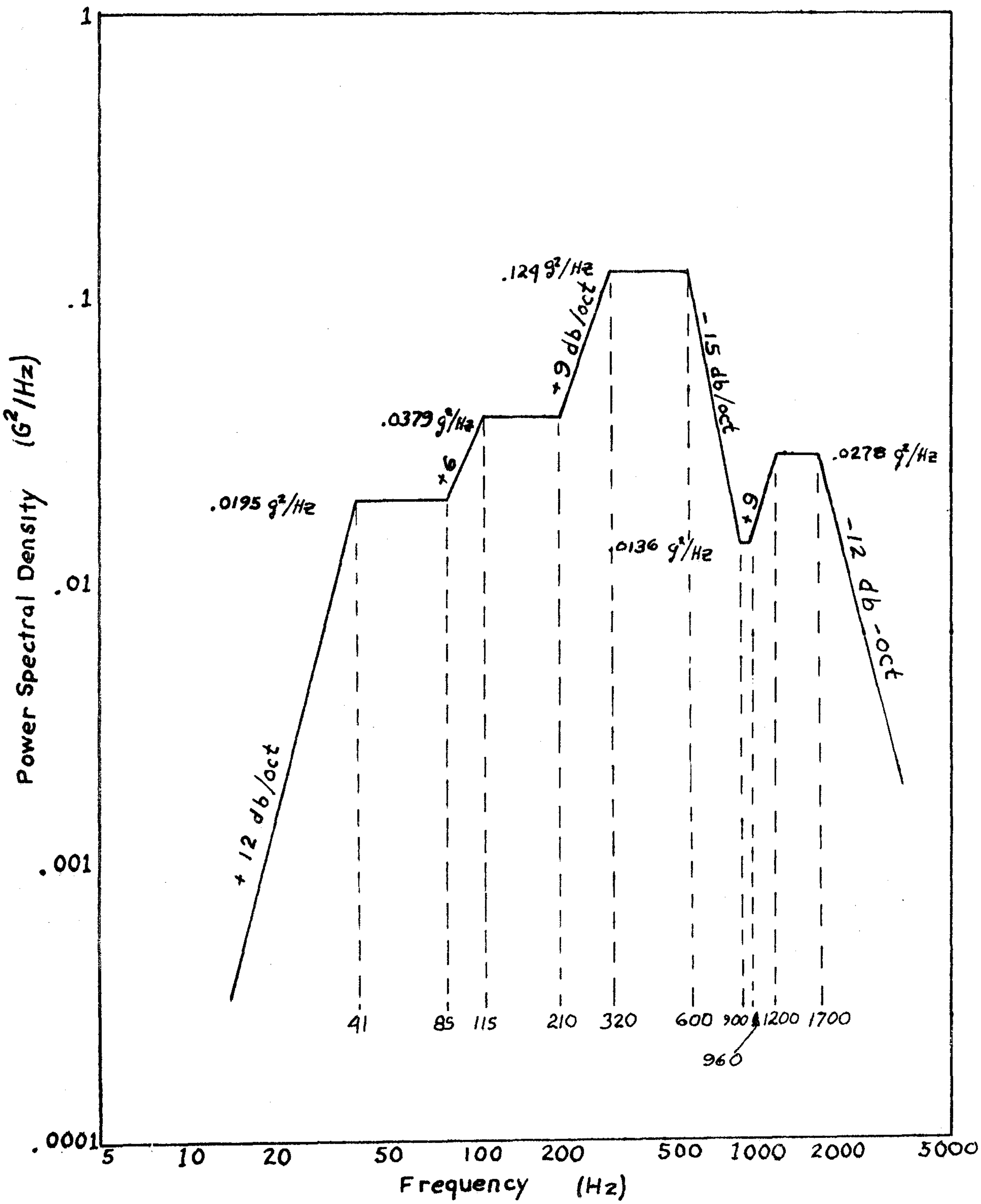
This table assumes that maximum responses in translation occur at the same frequency as maximum rotation responses, a conservative assumption. The table indicates that the maximum responses at a corner occurs along the front and rear edges of the large array. The root mean square response along these edges is 13.3 grms giving a 3 sigma peak value of 39.9 g. The peak power spectral density of the response occurs at about 400 Hz. Thus a corner mount design that will survive a sinusoidal vibration environment well in excess of 40 g peak in this frequency range is an adequate design.

In order to test the upper array alone, vibration levels in the plane of the arrays at the attachments between the two arrays were calculated. Since the test set up required the upper array to be tested with translational vibration only, the vibration spectrum on the lower array under the c.g. of the upper array was found. Due to the proximity of the centers of gravity of the two arrays and the relatively low levels of the rotational responses at the lower array c.g., the response under the c.g. of the small array due to rotation of the large array could be neglected. In order to simplify the test set up, the responses due to X and Y excitation were superimposed to give one set of test vibration levels for both the X and Y axes. The test envelope levels, reduced by a factor of 1.69 to acceptance levels, are shown in Figure 3.2.7.

Figure 3.2.7

UPPER ARRAY ACCEPTANCE RANDOM VIBRATION LEVELS

X & Y AXES IN PLANE OF ARRAY



3.3 Responses In Off-Loaded Condition

Response to the LM Quad III vibration environment for the LRRR in the "off-loaded condition" were calculated perpendicular to the plane of the array using the model of section 2.3. Responses of several locations on this model are shown in Figure 3.3.1 - 3.3.5. The responses for the experiment in this condition are seen to be slightly reduced compared to out-of-plane responses of the lower array with the upper array in place.

In-plane responses of the lower array alone were not calculated. This is justified since in the plane of the arrays the upper array adds no stiffness to the lower array but does mass load the lower array. Since the random excitation spectrum is almost flat in the region of the natural frequencies of both cases, larger loads on the LM interface fittings will be experienced with the upper array connected.

4.0 Dynamic Loads & Deflections

4.1 Method of Calculation

Dynamic loads on the structure were calculated using the out-of-plane model of the two connected arrays. The in-plane model gives dynamic loads on the interface fittings, hinges, and clevis fittings. The loads used in designing these fittings were so conservative that any conceivable dynamic load produced by the dynamic environment will be easily survived. Dynamic loads with the upper array removed will be lower than with it attached since mass loading induced by the small array is large compared to the amount of stiffness the small array contributes to the lower array.

The acceleration response at each lumped mass location, the stiffness matrices for each array, and the transformation between coordinates local to each array and coordinates of the whole system determine the dynamic loads on the system.

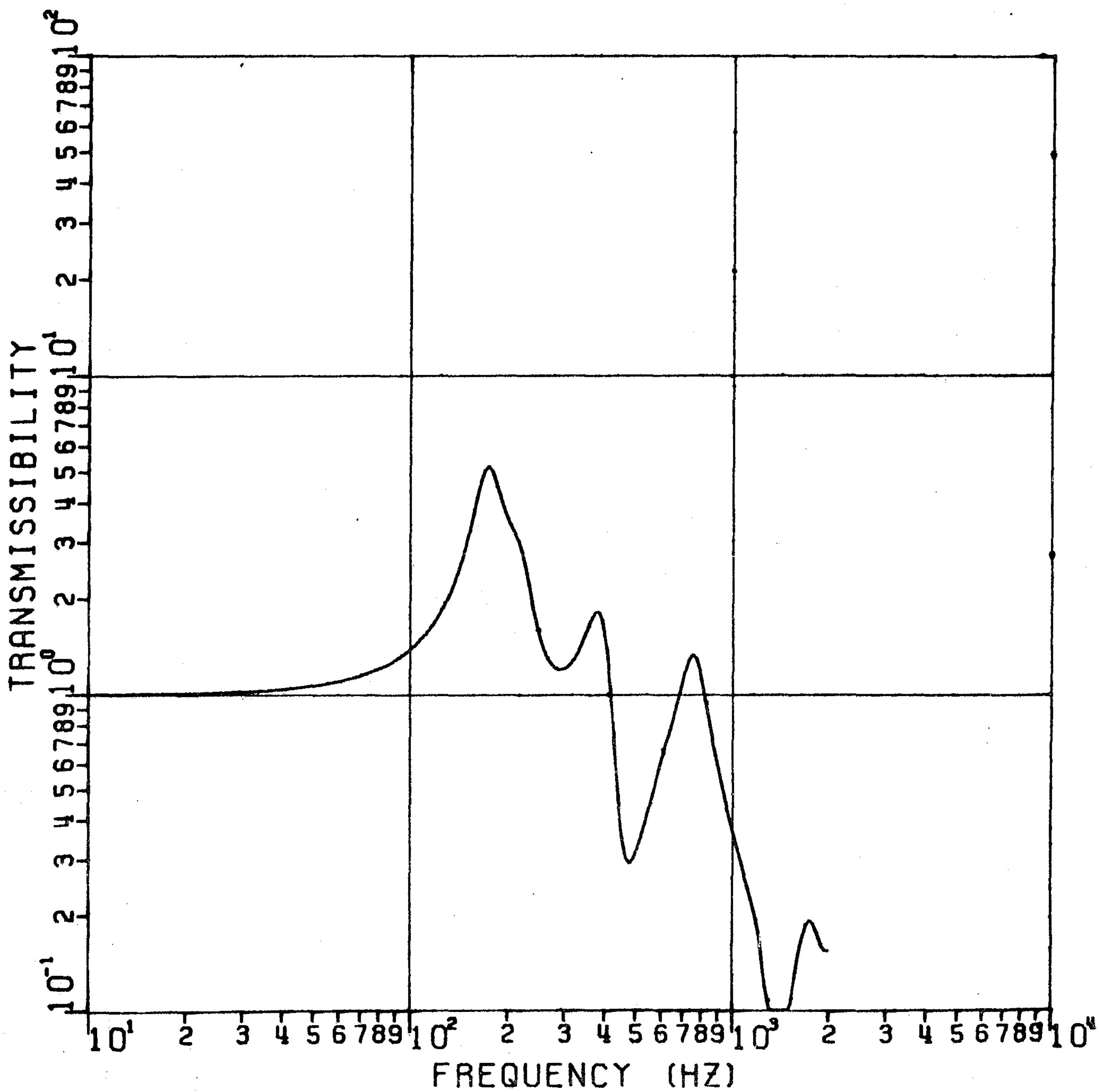
$$\text{Let} \quad [K] = \underbrace{\begin{bmatrix} [K_L] & & O & \\ & [K_u] & & [-K_u] \\ O & & & \\ & [-K_u] & & [K_u] \end{bmatrix}}_{38} \Bigg\updownarrow 38$$

Where $[K_L]$ & $[K_u]$ are as previously defined

LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.1A TRANSMISSIBILITY

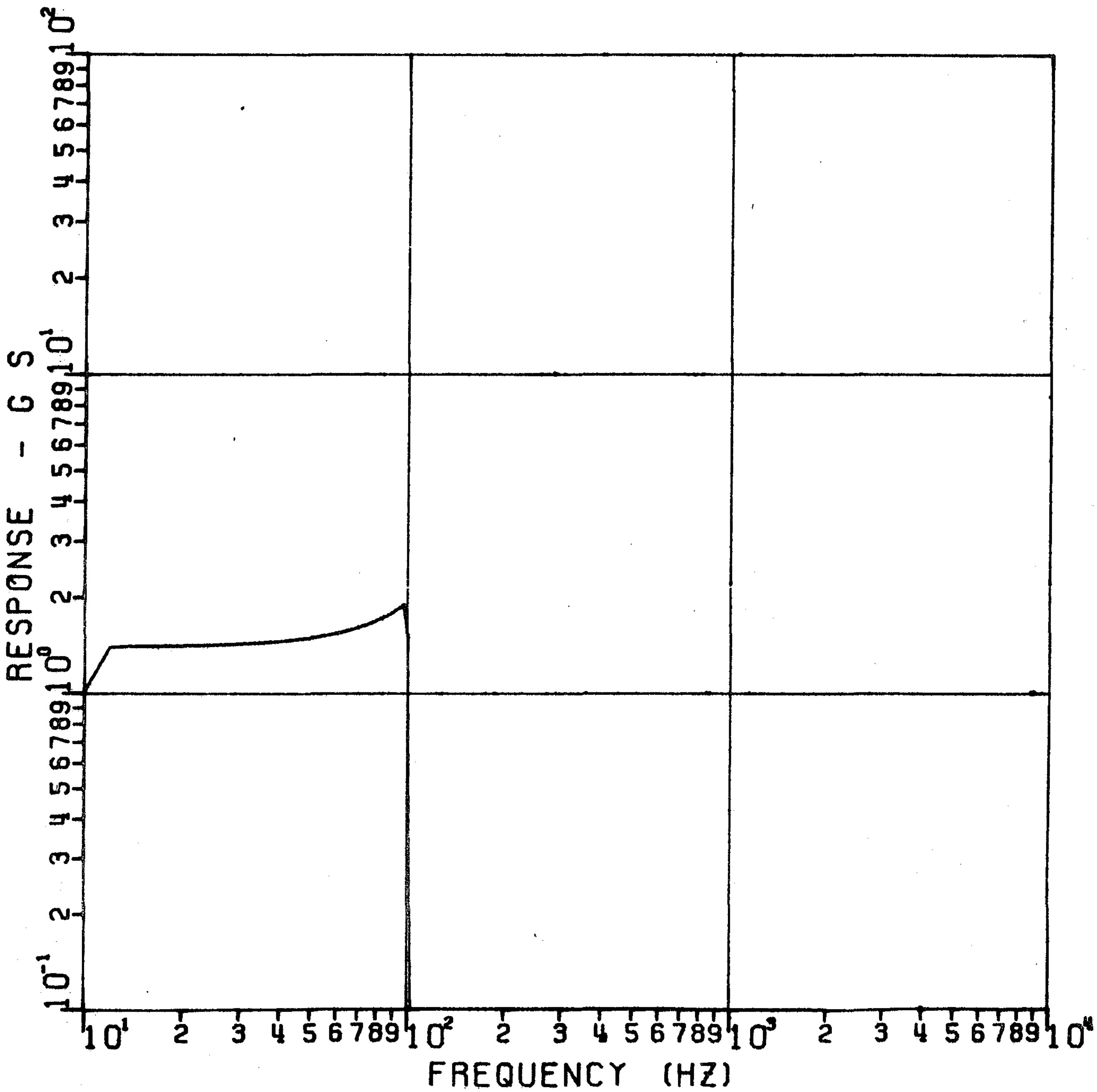
LOCATION 3



LRRR 300 IN THE OFF LOADED CONDITION 23 D.I

FIGURE 3.3.1 B SINE RESPONSE

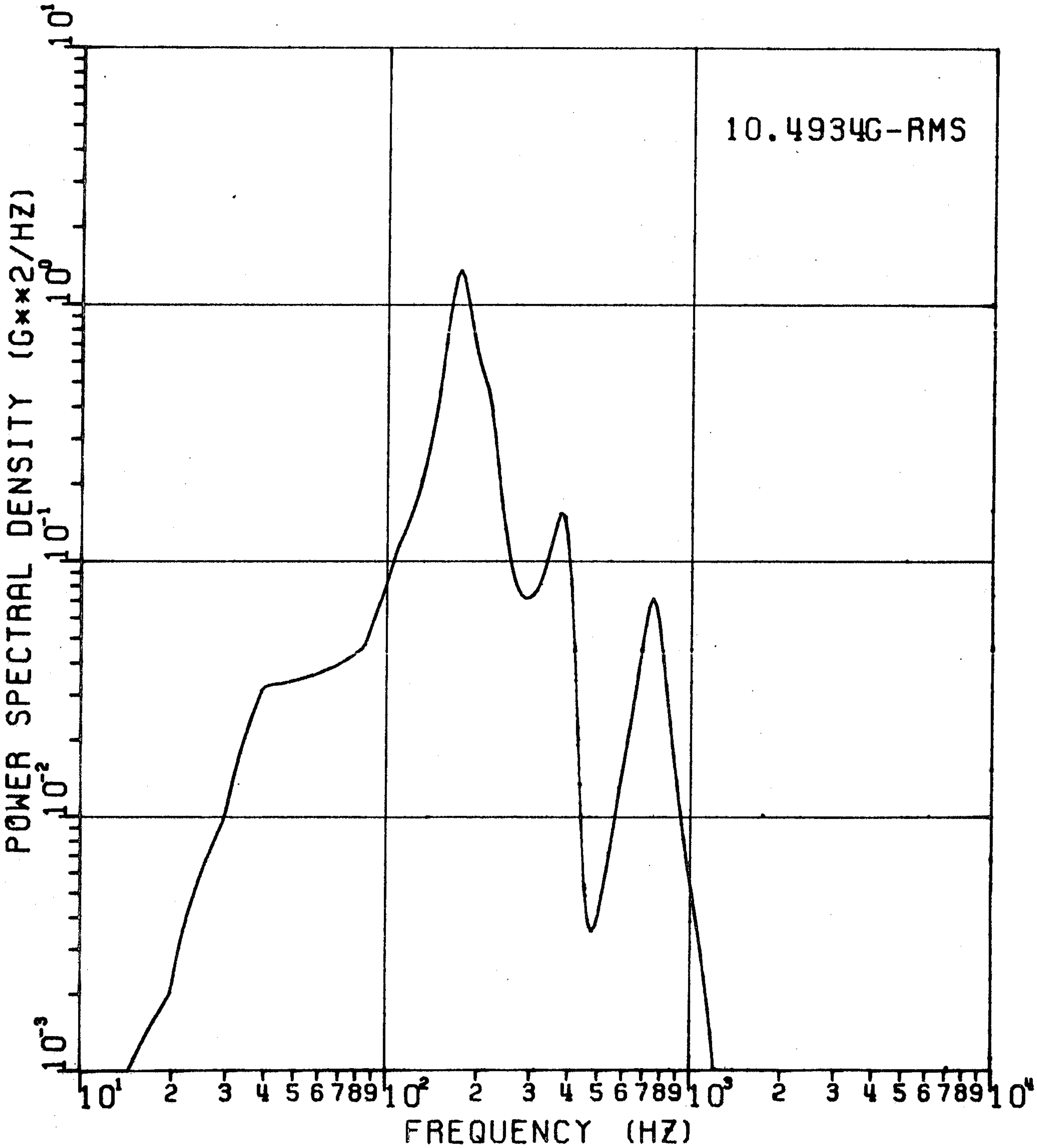
LOCATION 3



LRAA 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.1C RANDOM VIBRATION SPECTRUM

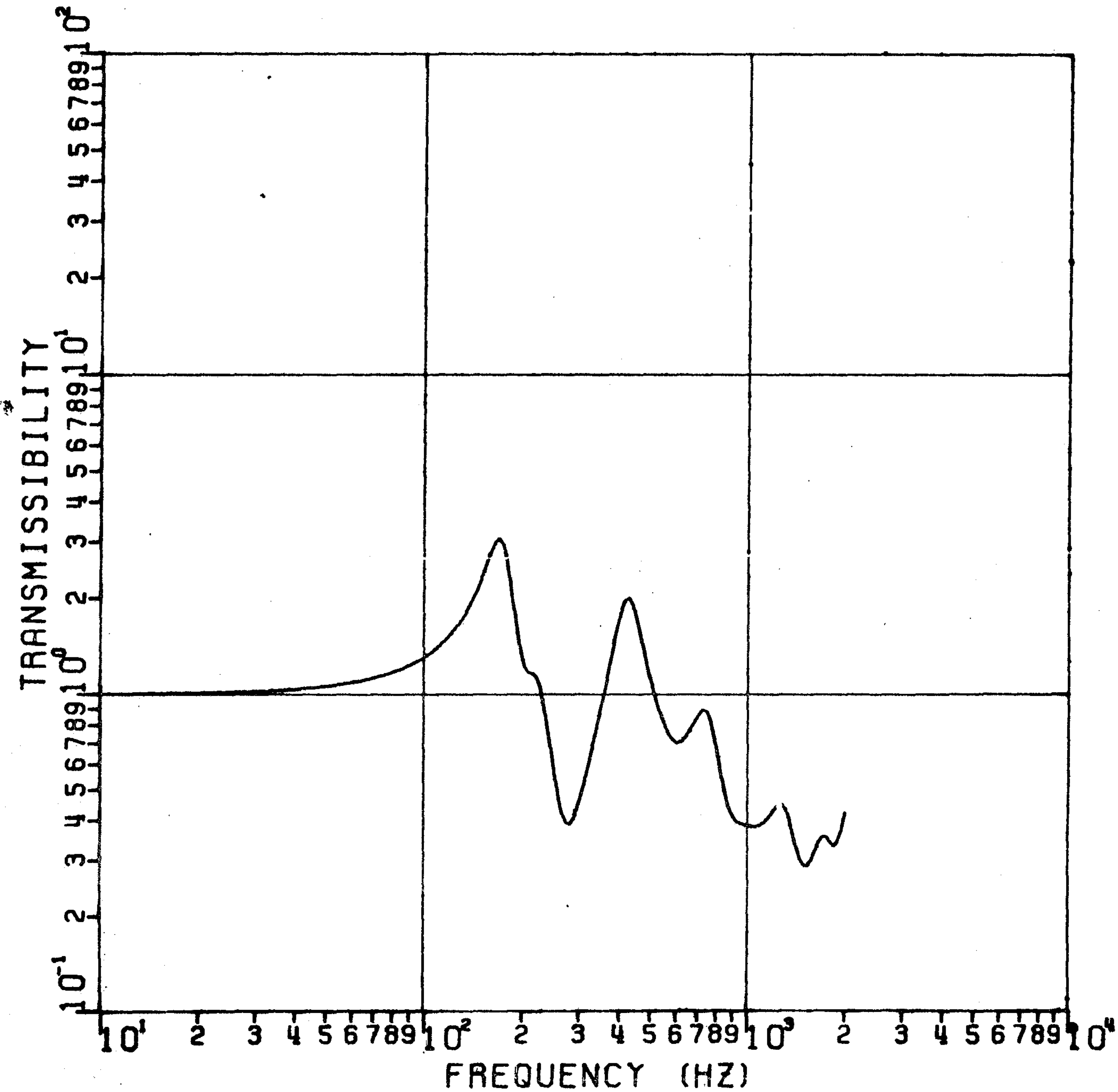
LOCATION 3



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.2 A TRANSMISSIBILITY

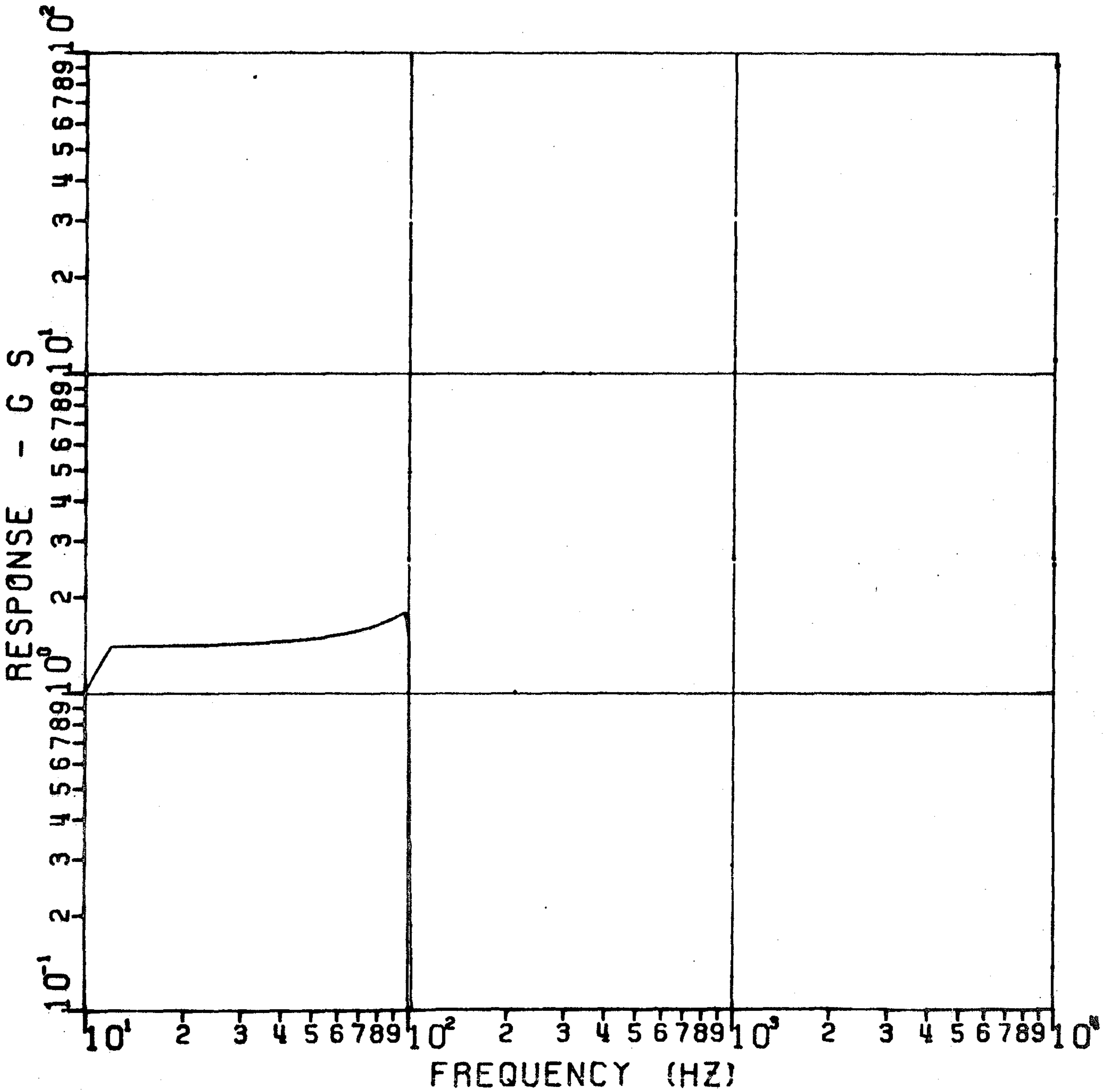
LOCATION 8



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.2B SINE RESPONSE

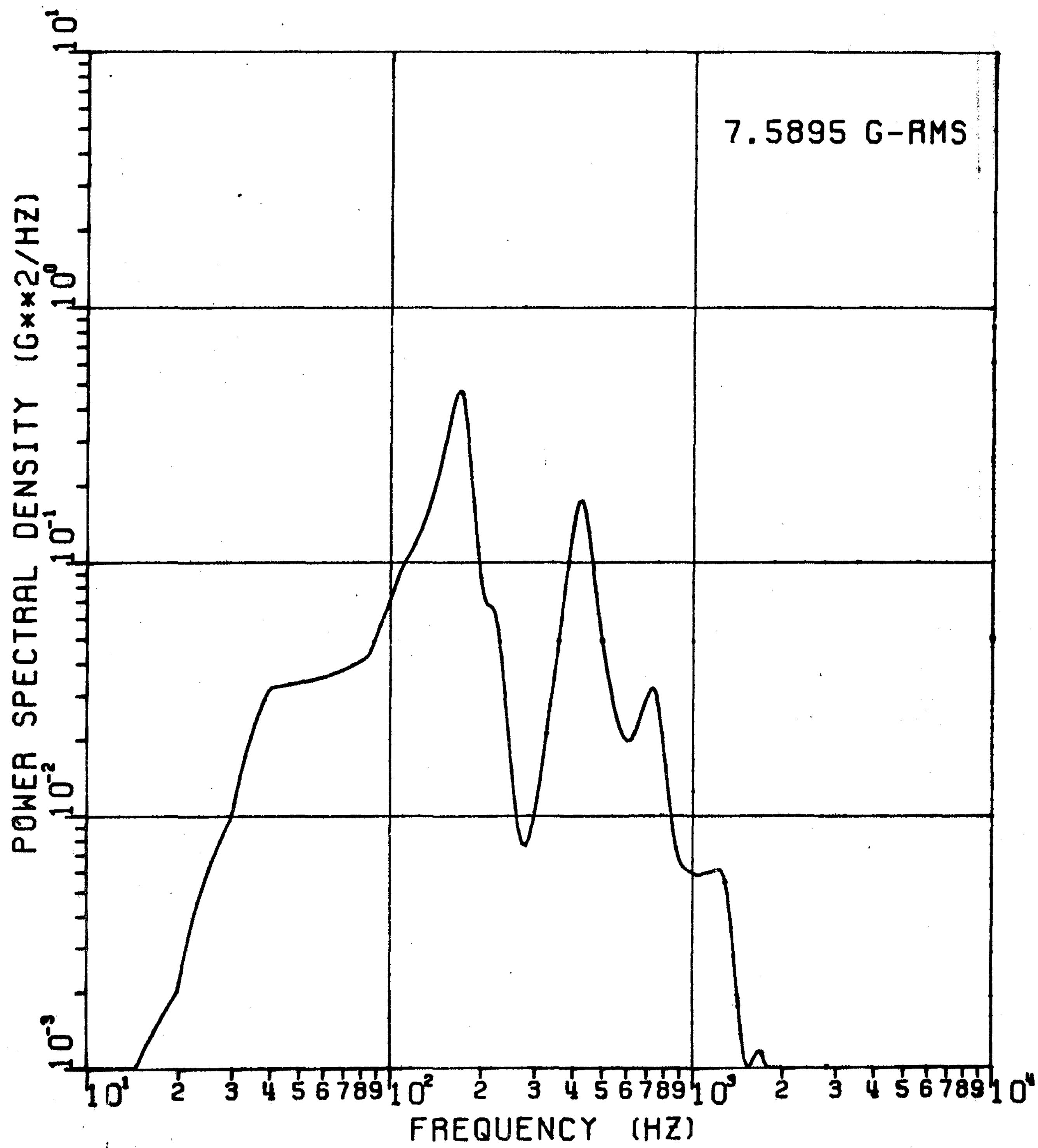
LOCATION 8



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.2C RANDOM VIBRATION SPECTRUM

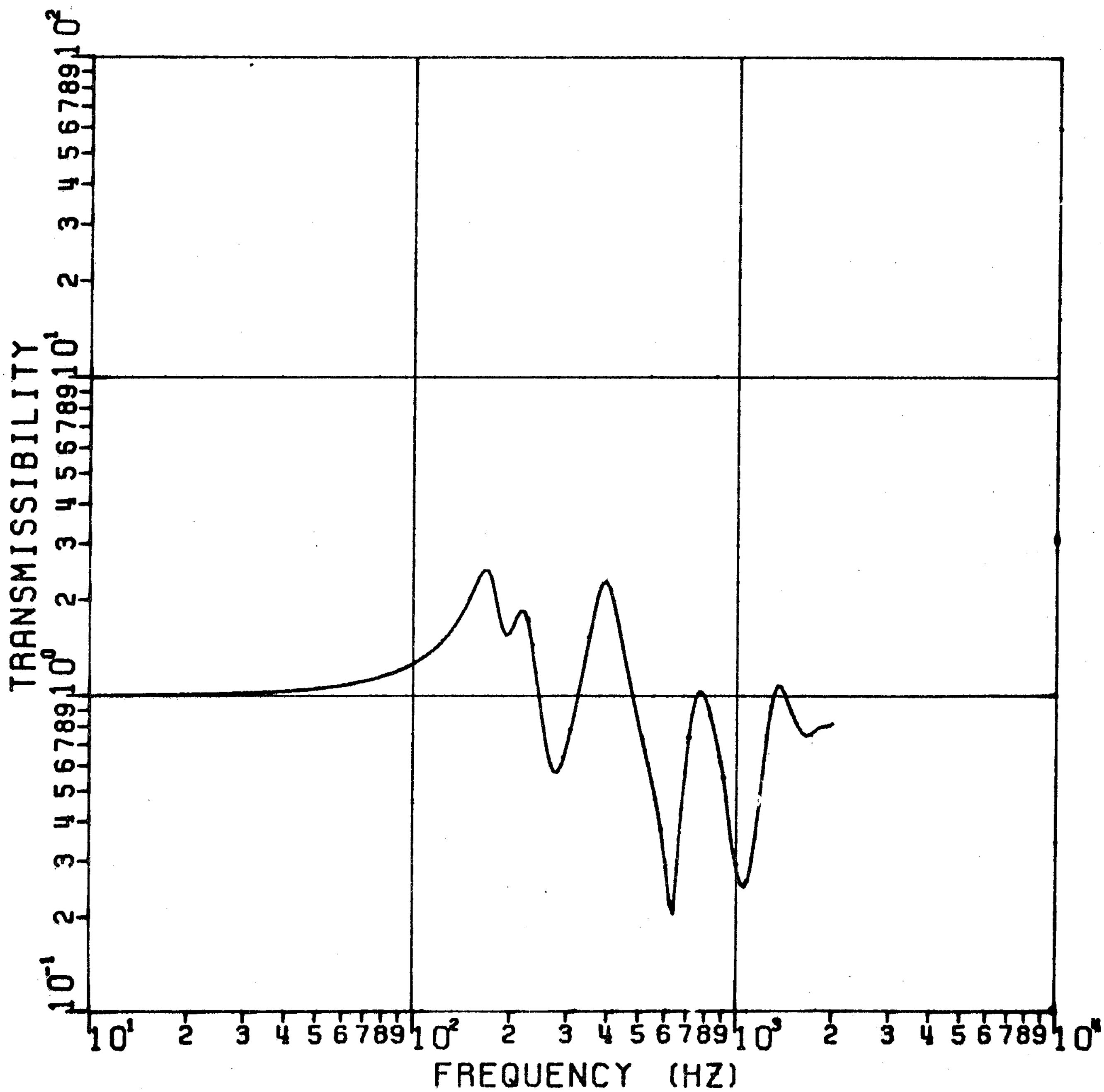
LOCATION 8



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 33.3A TRANSMISSIBILITY

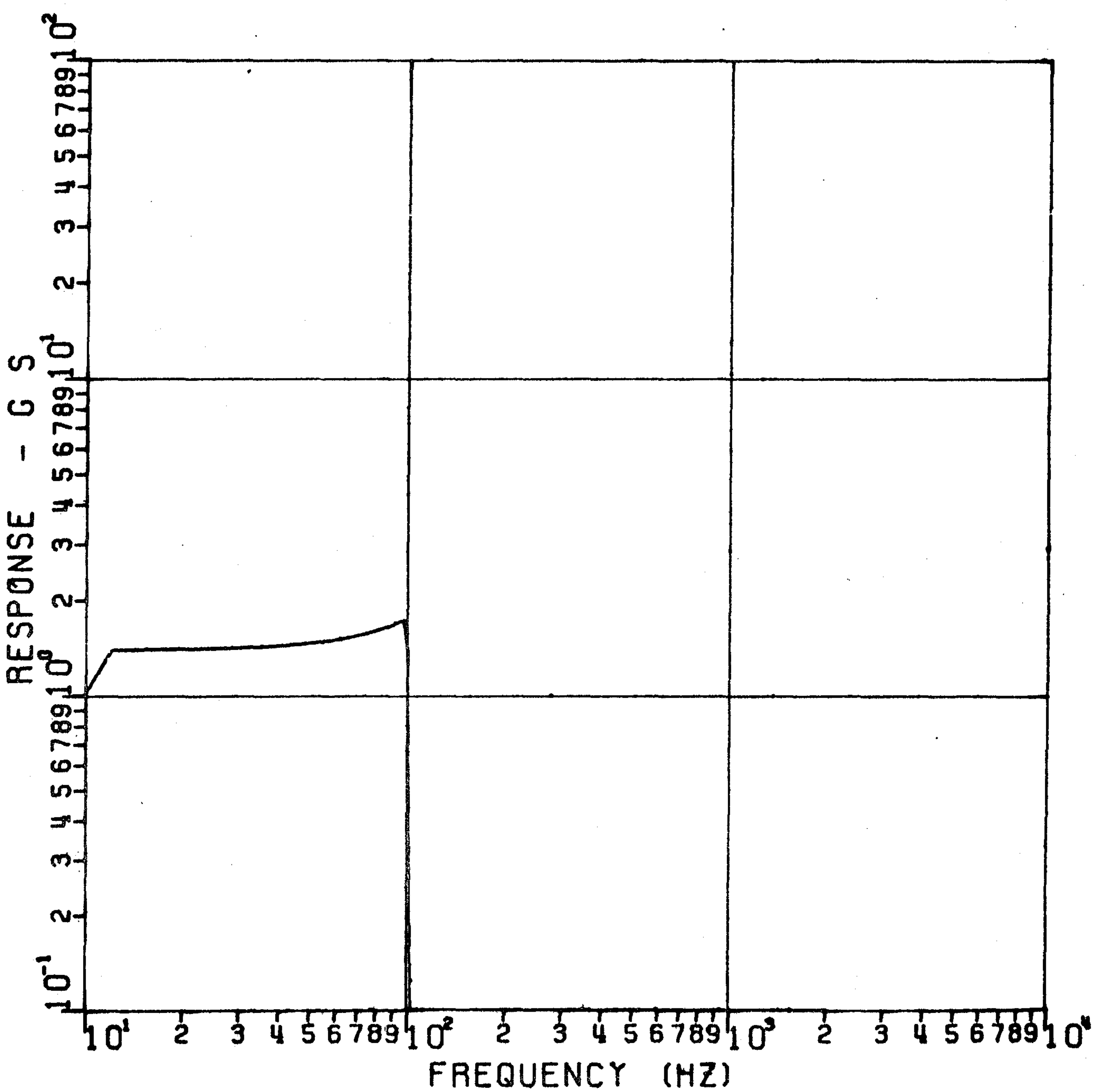
LOCATION 10



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.3 B SINE RESPONSE

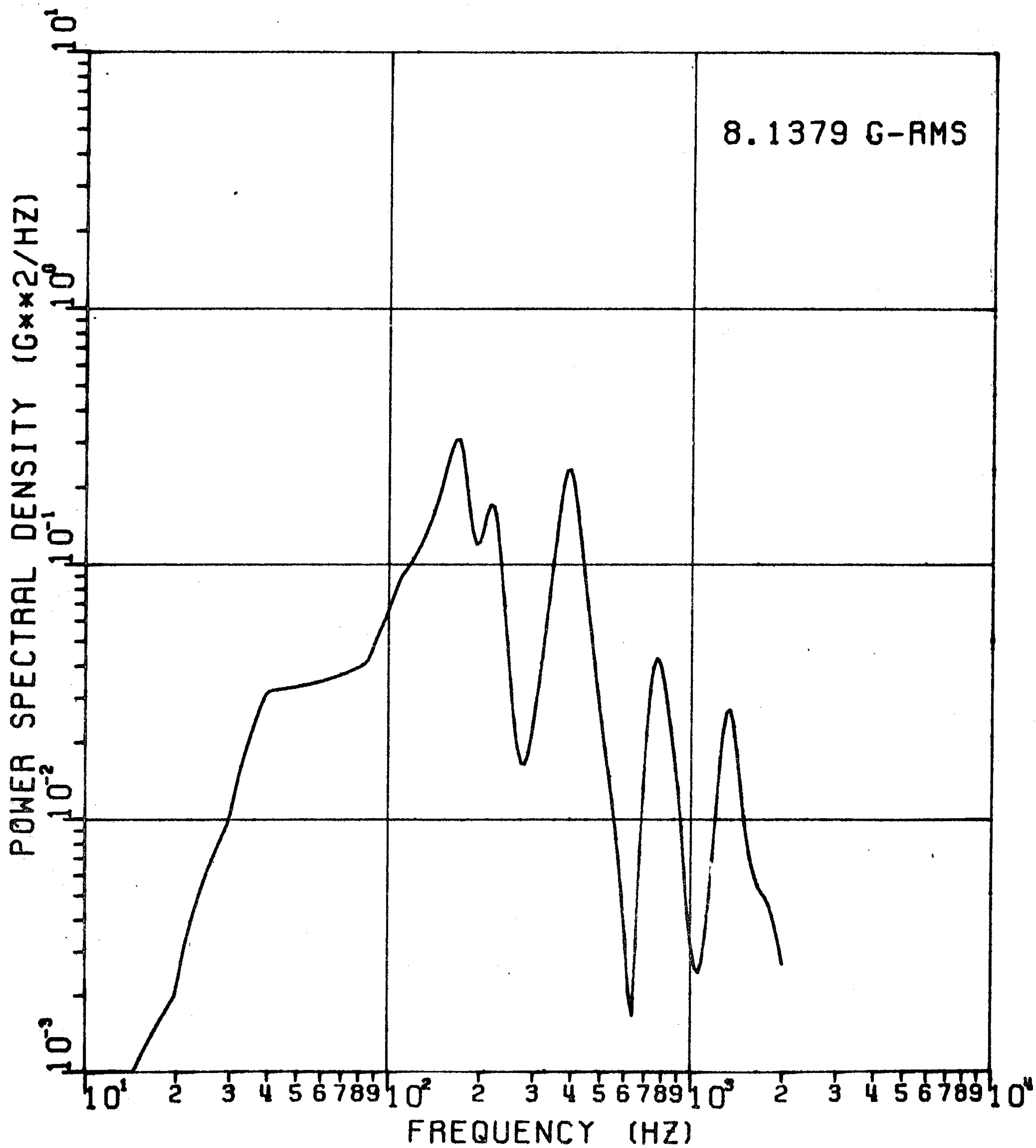
LOCATION 10



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.3C RANDOM VIBRATION SPECTRUM

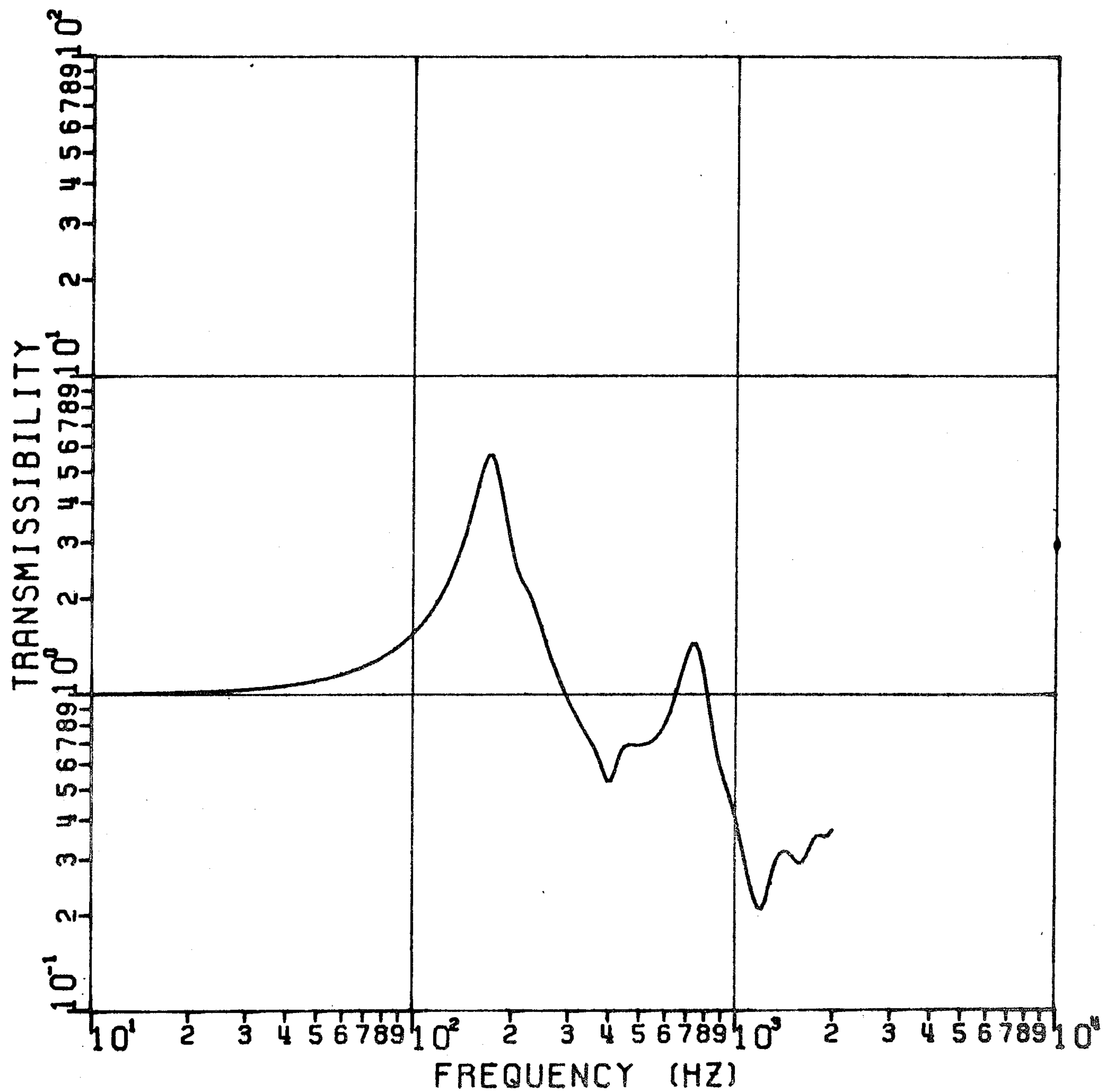
LOCATION 10



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.4 A TRANSMISSIBILITY

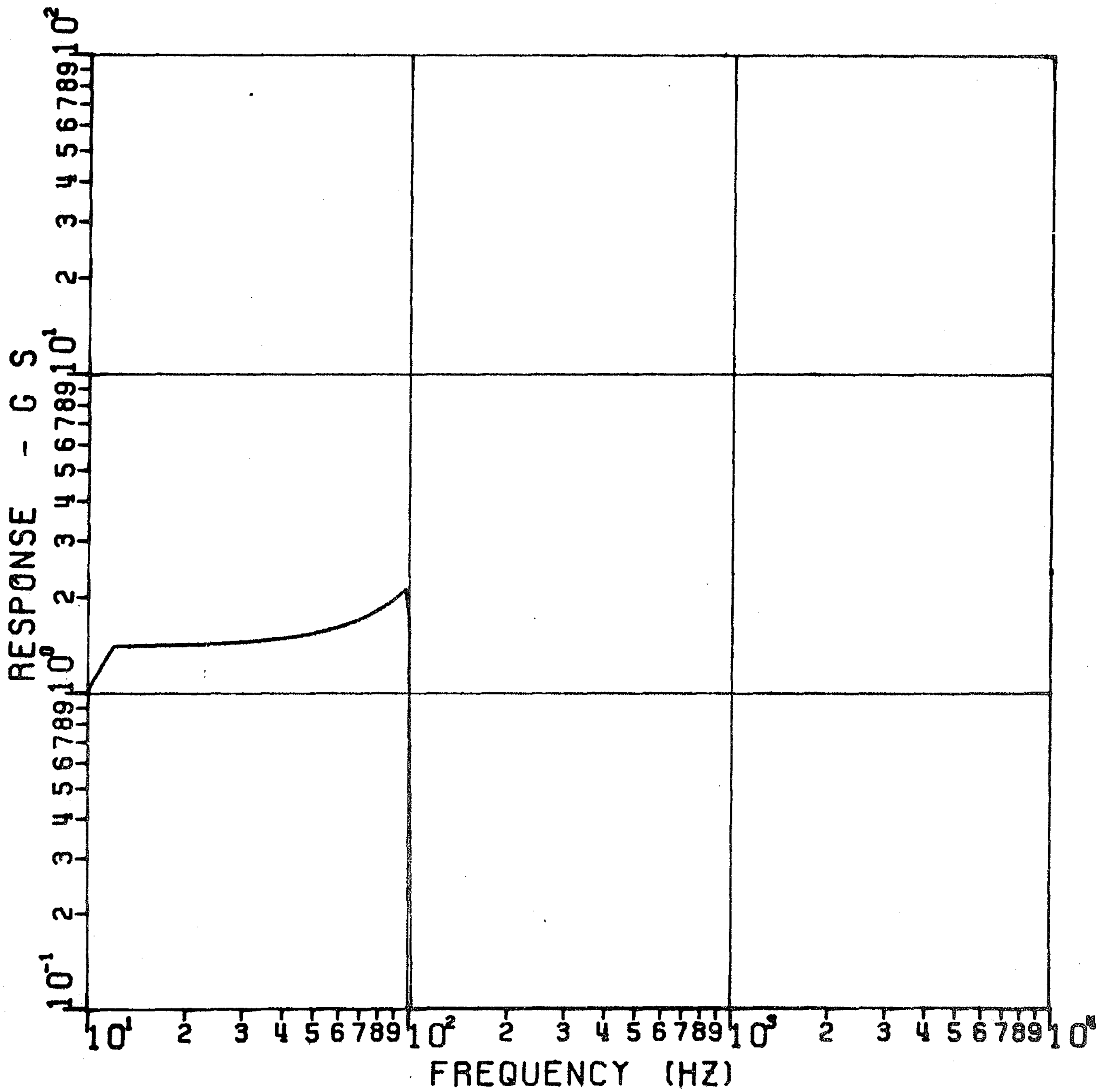
LOCATION 15



LARR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.34B SINE RESPONSE

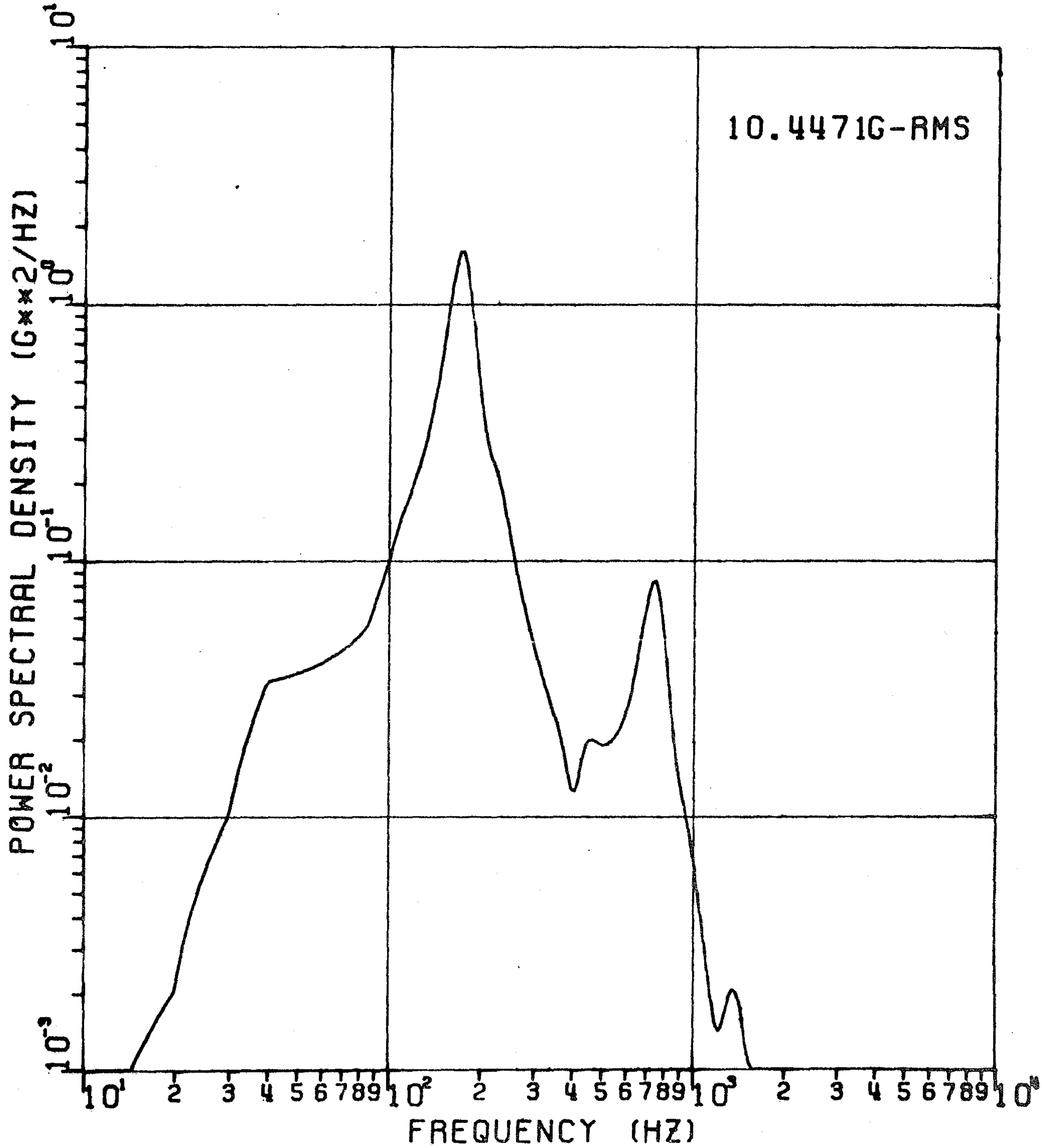
LOCATION 15



LRRR 300 IN THE OFF LOADED CONDITION 23 D.I

FIGURE 3.3.4C RANDOM VIBRATION SPECTRUM

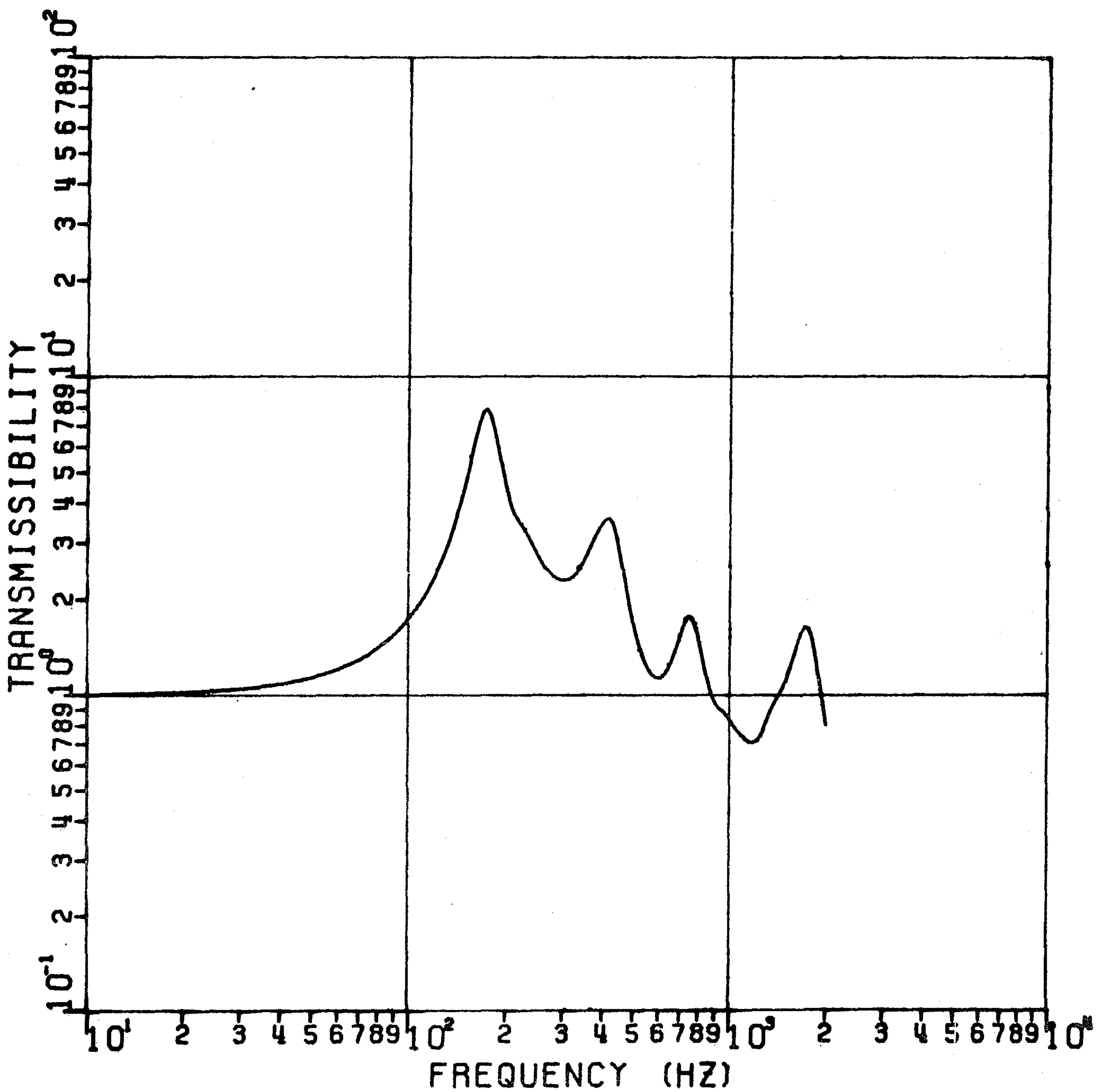
LOCATION 15



LARR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.5 A TRANSMISSIBILITY

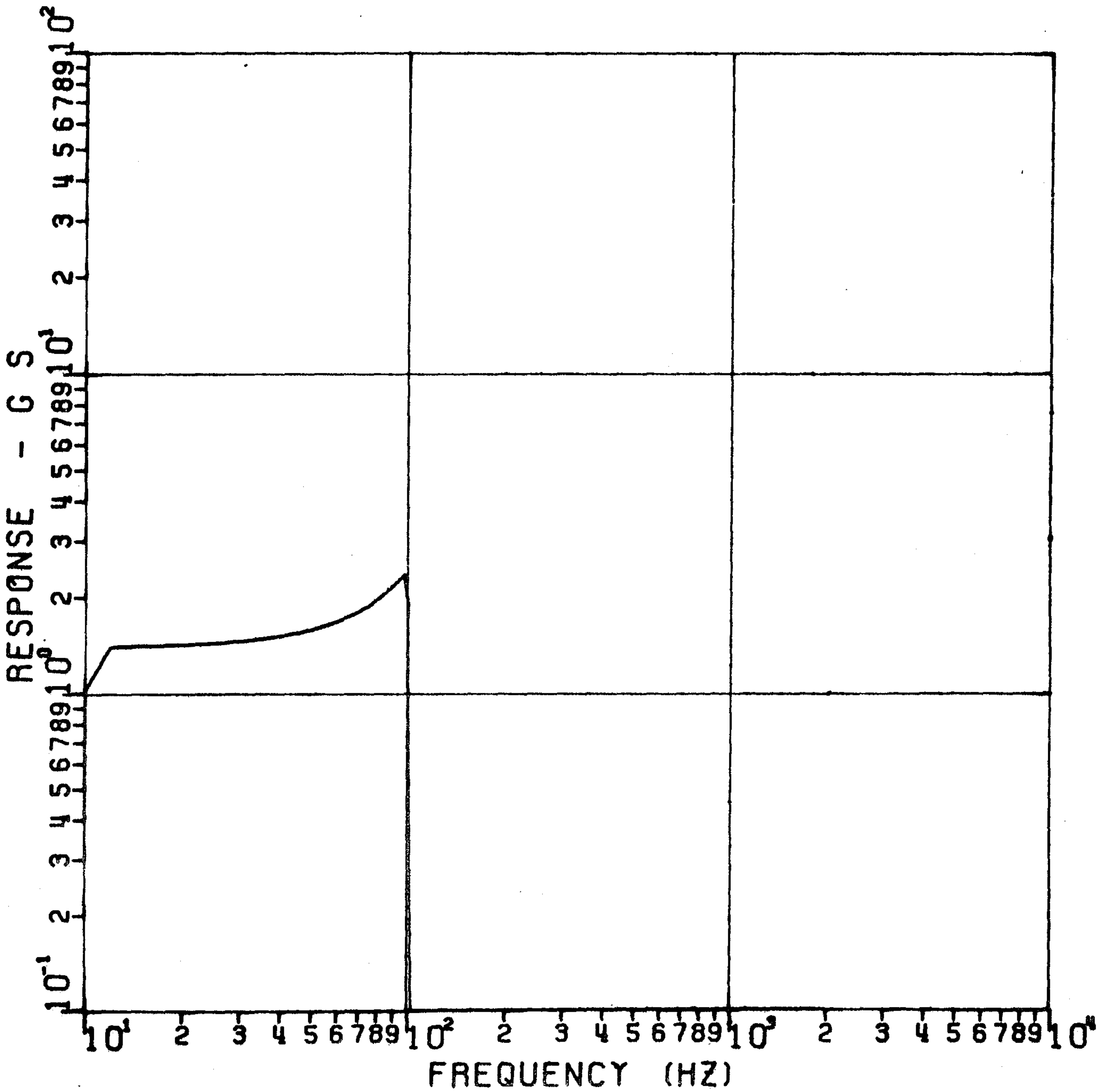
LOCATION 22



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 33.5 B SINE RESPONSE

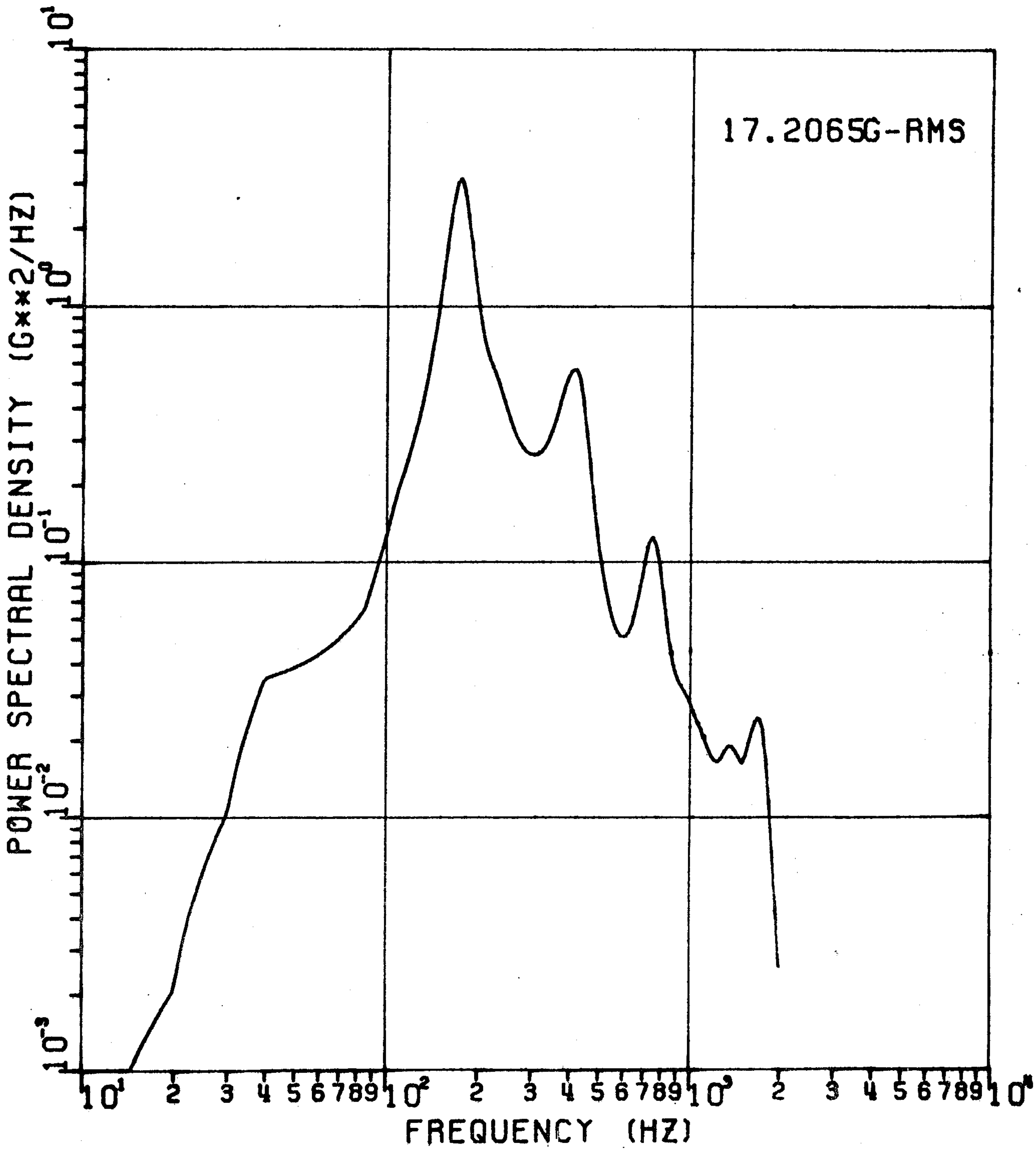
LOCATION 22



LRRR 300 IN THE OFF LOADED CONDITION 23 D.F

FIGURE 3.3.5c RANDOM VIBRATION SPECTRUM

LOCATION 22



The K matrix is 38 × 38. As explained in Section 2.1 the K matrix is reduced in size to correspond to the selected 30 coordinates by the transformation

$$K_s = [\beta]^T [K] [\beta]$$

K_s is shown in Figure 2.1.8.

$[\beta]$ = The coordinate transformation matrix as previously defined

The internal loads on the structure and associated displacements can be determined from the following relations:

$$\{P\} = [K][\beta][K_s]^{-1} \{Q\} \qquad (2)$$

$$\{S\} = [\beta][K_s]^{-1} \{Q\} \qquad (3)$$

Where $\{P\}$ is the internal load vector, $\{S\}$ is the displacement vector

The next step is to compute the load vectors, $\{Q\}$, resulting from the dynamic environment.

4.2 Load Vectors

The expression for the load vectors, Q, for the random environment is

$$[Q]^T = [M][A]$$

$[M]$ = Mass matrix (Figure 2.1.5)

$[A]$ = A diagonal matrix composed of local rms accelerations as previously computed

Random Loads

When the $\{Q\}$ column vectors have been determined, they are substituted into equations (2) and (3) to compute the internal loads and displacements on the arrays. For the random case the rms internal loads and displacements are computed for each mode giving 30 sets of rms values at the 38 coordinate locations. Then for each location the 30 modal rms values are combined by taking the square root of the sum of the squares of the 30 values, thus reducing the number to 30 load and 38 displacement values.

Sine Loading

The load vectors, $\{Q\}$, are again formed by equation (4), but real and imaginary steps are computed separately and combined into one load vector for each frequency where a relative maximum sine response at any location occurs.

For the sinusoidal environment case, there is only one $\{Q\}$ vector per frequency selected and each one will produce a set of internal loads and displacements.

4.3 Discussion

Application of these procedures to the out-of-plane model results in load and deflection sets due to the random excitation shown in Table 4.3.1 Loads due to sinusoidal environment are all less than the 3-sigma random values obtained by multiplying the Table loads by 3, and are therefore not shown.

To calculate reactions at the LM interface fittings, the dynamic loads at each mass location on the lower array were applied to the "rigid grid" model of the structure as a static load case. The reactions at the fittings were calculated by the program. The largest reactions were obtained for the random excitation case. These reactions are listed in Table 4.3.2.

Table 4.3.1

ROOT MEAN SQUARE--COMBINED LOADS

	DEFLECTIONS	INTERNAL LOADS
1	0.2760716D-03	0.2524730D 02
2	0.1193546D-02	0.1592200D 02
3	0.5115325D-03	0.8152104D 02
4	0.4056556D-03	0.3054571D 02
5	0.3663186D-03	0.1666623D 02
6	0.7439323D-03	0.2241168D 02
7	0.6873942D-03	0.1842885D 02
8	0.8720670D-03	0.1747689D 02
9	0.8662774D-03	0.1326628D 02
10	0.1071332D-02	0.3110631D 02
11	0.1112881D-02	0.1893863D 02
12	0.9947329D-03	0.9884547D 01
13	0.1116653D-02	0.2161609D 02
14	0.1637615D-02	0.2079703D 02
15	0.1204343D-02	0.1591713D 02
16	0.4145465D-03	0.8285710D 02
17	0.1982792D-02	0.2976196D 02
18	0.2008225D-02	0.1855285D 02
19	0.4412068D-03	0.8584821D 01
20	0.1422538D-02	0.1705158D 02
21	0.2962408D-02	0.9622102D 02
22	0.1426417D-02	0.2203628D 02
23	0.1876504D-02	0.1130716D 02
24	0.3396223D-02	0.2528912D 02
25	0.1999994D-02	0.3555646D 02
26	0.3365360D-02	0.2827921D 02
27	0.3621811D-02	0.3207238D 02
28	0.1899019D-02	0.2839161D 02
29	0.3236117D-02	0.2307563D 02
30	0.1590335D-02	0.1625034D 02
31	0.1658224D-02	0.1130716D 02
32	0.3216398D-02	0.2528912D 02
33	0.1759846D-02	0.3555646D 02
34	0.3286494D-02	0.2827921D 02
35	0.3105241D-02	0.3207238D 02
36	0.1665940D-02	0.2839161D 02
37	0.3007676D-02	0.2307563D 02
38	0.1577347D-02	0.1625034D 02

TABLE 4.3.2
VERTICAL REACTIONS
AT LM INTERFACE FITTINGS

Fitting	RMS Load (lb)	Peak Load (lb)
Rt. Front	142	426
Rt. Rear	156	468
L. Front	226	678
L. Rear	144	432