The purpose of this ATM is to define the adequacy of the LSPE Timer Control Module Seals in response to the Array E CDR Action Item 382. The control modules contain the timers basic timing mechanism and are specified as having a leak rate of less than $1 \times 10^{-6}$ STD CC/SEC when tested in accordance with MIL-STD-202C, Method 112A, Procedure III. Appendix 1 of this ATM is the leak rate analysis which indicates the seals have adequate design margin to successfully survive the expected environment. The body of the ATM presents the basic design concept of the seals, the volumetric analysis, and details the precautions taken in finishes, materials, inspection, storage and testing.

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## BACKGROUND

During the early design phase of the LSPE Timers it was determined that
certain induced effects could cause earlier than desired time-out of the
timers. It was known that high temperature, low pressure, and/or reduced
G's would cause a watch mechanism to run fast by increasing the balance
wheel amplitude. What was not known was the percentage of amplitude in-
crease contributed by each of the temperature, pressure, and gravity effects.
It was also not known if the effects were additive or not.

Tests were conducted at Bendix, the timer subcontractor, Lewis Research
Center, and Wright-Patterson Air Force Base which have established the
effects as additive and have defined the percentage contribution of each effect.
As a result the parametric definitions affored by the tests, the seal design has
been closely reviewed during a delta CDR at the timer subcontractors facility.
Additionally, at the request of NASA/MSC, a meeting was held on 25 June 1971
at MSC between NASA and BxA R and QA personnel to review the seal require-
ments.

## SEAL REQUIREMENTS

In order for the basic timing mechanism of the timers to function properly in
the defined lunar environment, the control module seal requirement has been
set at a leak rate not to exceed $1 \times 10^{-6}$ STD cc/second. With the seal mechanism
functioning within this requirement, the enclosed watch movement which consti-
tutes the basic timing mechanism will provide a regulated output to a drive train
that in turn provides a controlled time-out function.

## BASIC DESIGN

Figure 1 shows the external view of the Battery and the Safe Arm Slide Timer.

Figure 2 shows the control module design including the o-rings, stem, shaft,
and covers, including a tabulation of those dimensions critical to the seal design.

Figure 3 is a compilation of data relative to the characteristics of the viton o-ring.

During the intended mission time the back cover seal is totally static. The control
shaft seal is essentially static in that the shaft rotates through a five degree

**Bendix**

**Aerospace Systems Division**

LSPE Timer Control Module
Seal Leak Rate Analysis

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arc once every thirty minutes. The stem seal is also effectively static
as it is motionless except for the depressing of the watch stem which
starts the timer when the astronaut pulls the timer start pull-pin during
deployment. In addition, each o-ring groove contains high vacuum Krytox
grease as recommended by the o-ring supplier for high vacuum environments.

The housing, back cover and control shaft covers, and the adapter shown
in figure 3 are all made of 2024 aluminum alloy heat treated to T3, T4,
or T351. The watch stem is made of 416 stainless steel and the control
shaft is made of 303 stainless steel. Based on the tolerances associated
with the critical dimensions of these parts, surface finishes average between
8 and 125 microinches, rms.

In the analysis that follows, the formula used for the o-rings is:

$$\text{Volume - } V = 2\pi^2 C r^2$$

This formula is depicted in figure 2 using the o-ring drawing to define
the terms C and r. It should be noted that in finding the volume of a
torus, the dimension C does not change as a function of tolerance caused
changes in a or d and r. This is because of the theorem of Pappus con-
sidering the juncture of C and r as the center of gravity of a plane passing
along a circular arc and completing a full revolution with the axis of
revolution being theoretically as shown in figure 3 for the o-ring. Conse-
quently, as 2r changes for a given o-ring because of tolerance variances,
2r tends to decrease or increase as a dimension, but the theoretical center
doesn't move. If any other approach is used to calculate the volumetric
fill or worst case tolerance stack-up, then an indeterminate variable is
introduced. In addition, this assumption creates an averaging effect for
the variances in cross-sectional concentricity of the non-rigid body (the
o-ring).

It is also assumed that any shrinkage or swelling of the o-ring is not in
excess of the 2r tolerance given for each size of o-ring.

The calculations for the adapter and housing considers the basic volume
of the seal area less any radii and the stem or shaft volume versus the
o-ring volume.

## VOLUMETRIC FILL ANALYSIS:

The following formulas were used to calculate volume and cross-sectional area:

O-ring

$$V = 2\pi^2 C r^2$$

$$A = \pi r^2$$

Back cover, Adapter and Housing

$$V = \pi g \left[ 1/2\ e^2 - (1/2\ e\text{-}f)^2 \right] \qquad A = fg$$

$$V = \pi i\ (1/2\ h - 1/2\ k^2) \qquad A = 1/2\ h\ i - 1/2\ k\ i$$

$$V = \pi\ (s\text{-}t)\ (1/2\ \ell - 1/2\ Q^2) \qquad A = 1/2\ \ell\ (s\text{-}t) - 1/2\ Q\,(s\text{-}t)$$

Stem

$$V = \pi\ (1/2\ k)^2\ (i)$$

$$A = k\ i$$

Shaft

$$V = \pi\ (1/2\ Q)^2\ (s\text{-}t)$$

$$A = Q\ (s\text{-}t)$$

Radii

$$V_f = 2\ \pi\ d\ A_f$$

$$A_f = r^2\ (0.2146)$$

In each formula, the terms are derived from the drawings on figure 2. In both volume and area calculations the radii are subtracted out in order to more precisely define the volumetric fill percentages.

The following tabulation gives volume and area in the noted worst case conditions and are derived from the above formulas:

LSPE Timer Control Module
Seal Leak Rate Analysis

### BACK COVER:

Where the groove is maximum size and the o-ring is minimum:

|  | Vol. Fill % | Cross-Sectional Area - % |
|---|---|---|
| (1) $V = 0.0161558 - 0.012416 = 0.0037398$ ∴ | 76.9% | |
| $A = 0.004594 - 0.003526 = 0.001068$ | | ∴ 76.8% |

where the groove is minimum and the o-ring is maximum:

| (2) $V = 0.0135823 - 0.0147399 = -0.011576$ ∴ | 108.5% | |
| $A = 0.003727 - 0.0041854 = 0.0004584$ | | ∴ 112.3% |

### ADAPTER:

Where the groove is maximum, the stem and o-ring are minimum:

| (3) $V = 0.0005038 - 0.0005015 = 0.0000023$ ∴ | 99.5% | |
| $A = 0.0017003 - 0.0017350 = -0.0000347$ | | ∴ 102% |

where the groove is minimum, the stem and o-ring are maximum:

| (4) $V = 0.0003945 - 0.0006376 = -0.0002431$ ∴ | 161.6% | |
| $A = 0.0014751 - 0.0022062 = -0.0006311$ | | ∴ 149.6% |

### HOUSING:

Where the groove is maximum, the shaft, and o-ring are minimum:

| (5) $V = 0.0024764 - 0.0018598 = 0.0006166$ ∴ | 75.1% | |
| $A = 0.0071175 - 0.0051056 = 0.0020119$ | | ∴ 71.7% |

Where the groove is minimum, the shaft and o-ring are maximum:

(6) $V = 0.0021812 - 0.002272 = -0.0000908$ $\therefore$ 104.2%

$A = 0.0062815 - 0.0062345 = 0.000047$ $\therefore$ 99.3%

Using the guide lines set forth in the Parker Seal Company's design handbook, the volumetric fill requirement for a hard vacuum application is centered on an almost completely filled groove, but the fill by the o-ring cross-sectional area should not exceed the cross-sectional area of the groove by more than 1% under any tolerance condition. It therefore becomes apparent that the tolerances for the back cover groove are too wide for this application. The adapter groove is acceptable when the conditions noted in (3) apply, but not in (4). The housing groove is acceptable only when the conditions of (6) apply.

Due to the critical nature of these seals, the goal for volumetric fill has been established by ALSEP Reliability at 100% for cross-sectional area of the grooves as related to the respective o-ring. A practical range is considered to be 90% to 105%.

Based on this analysis of the LSPE timer control module seals, all calculations made have been forwarded to the timer subcontractor. As necessary, tolerances shall be revised to conform to the acceptable range noted above. Surface Finishes in the seal areas are limited to 32 micro-inches rms or better.

TEST RESULTS:

The timer subcontractor has conducted repeated leak tests on engineering models and has completed leak tests on several prototype timers before and during acceptance tests. All timers have exhibited leak rates of less than

$1 \times 10^{-6}$ STD cc/second and the majority have exhibited leak rates less than $1 \times 10^{-8}$ STD cc/second. One of the engineering model tests was conducted to determine if the stem seal could be upset by repeated movement of the stem. This unit was leak tested at the start of the test to $1 \times 10^{-8}$ STD cc/second and had no detectable leak. The stem was then actuated - up and down - one hundred (100) times and the same leak rate test conducted. There was again no detectable leak. The stem was then actuated three hundred (300) more times with the same leak rate test conducted after every one hundred cycles. At no time, including after the four hundred actuations, did the timers leak rate exceed $1 \times 10^{-8}$ STD cc/second.

Comparing the above leak test results with the Appendix 1 leak rate analysis, there is every reason to believe the timer seals will provide adequate sealing for $2.724 \times 10^3$ hours or 113.5 days while the specified mission life is three (3) weeks from launch or twenty-one days (504 hours). In the leak rate analysis, it was assumed that regardless of leak rate during earth storage there would be a minimum of one atmospheric internal pressure at $+80°F$ at the time of launch.

## INSPECTION AND STORAGE CRITERIA:

The timer subcontractor has established an inspection criteria whereby each o-ring is visually inspected under 30X optics for cuts, cracks, voids, or abrasions using a "no defect" rejection requirement. Upon being accepted the o-rings are stored in their individual plastic bags in a closed cardboard container in bonded stores. Maximum time from being removed from the cardboard box to finished assembly in a timer is three days. During this time the o-rings are in closed plastic boxes and there is no evidence that the surface crazing effect caused by ultraviolet radiation is occurring. The UV emitted by local neon sources is not considered high enough to create the effect in a three day time period. In addition, the supplier claims indefinite shelf life for the o-rings without special storage conditions.

Once the o-rings are installed in the timers there is again no special storage requirement to enhance the life expectancy of the viton. Even a $-45°F$ temperature - where brittleness begins - is no problem in that recovery of the viton when warmed up to temperatures above $-45°F$ is rapid and complete to the previously established compression set level.

A review of the data listed in figure 3 indicates vitron has the necessary properties and immunities to function better than other available o-ring materials and has very acceptable thermal-vacuum characteristics. It is therefore not only an acceptable Apollo material but a good choice for vacuum sealing.

CONCLUSION:

The test results achieved during engineering and prototype model leak testing and the conclusion of the leak rate analysis have verified the validity of the specification limit of a leak rate not to exceed $1 \times 10^{-6}$ STD cc/sec for the control modules. The dimensional tolerance review and modifications relative to piece part tolerance stack-up of the hardware give the assurance necessary of surviving not only the potential two year storage period, but also the three weeks of lunar environment exposure. It is therefore concluded that the control module seal design approach is adequate for the intended mission.

LSPE Timer Control Module Seal
Leak Rate Analysis

Figure 1

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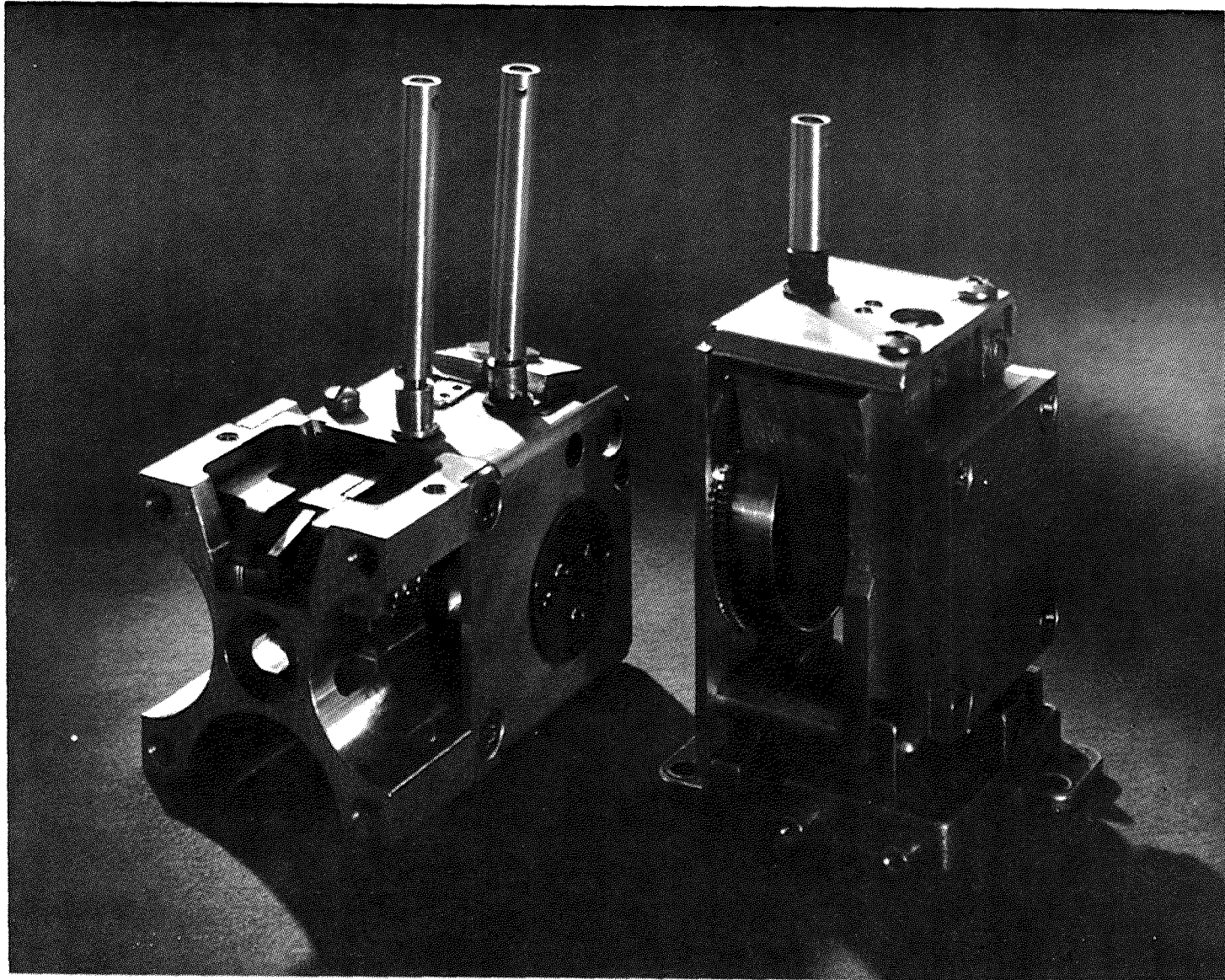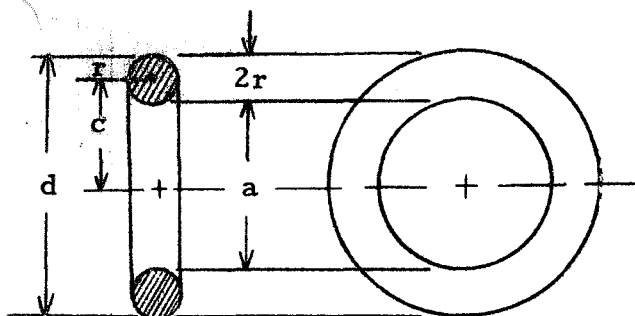**Aerospace**
**Systems Division**

## O-RINGS



**87190-01**
d = Not given in suppliers catalog. Computed from a & 2r plus their tolerances.
a = 0.042 ± .006
2r = 0.050 ± .003
∴ = d = 0.142 ± .012
c = 0.046

**87190-02**
d = same as -01
a = 0.056 ± .006
2r = 0.060 ± .003
∴ = 0.176 ± .012
C = 0.058

**87190-03**
d = same as -01
a = 1.051 ± .010
2r = 0.070 ± .003
∴ d = 1.191 ± .016
c = 0.5605

Radius = 0.020 ± .005 Both Corners

-03

e = 1.190 ± .004
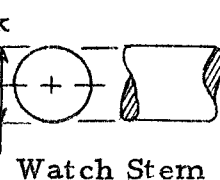f = 0.090 + 0, -.005
g = 0.052 + 0, -.005

87157

g | Back Cover

Radius = 0.010 ± 0.005

-01

k

h

j

i

Watch Stem

87138
Stem Seal Adapter

h = 0.140 ± .001
i = 0.035 ± .002
j = 0.052 ± .003
                  - .001
k = 0.0492 ± .0004

l(hole)
m(cover)

p

n

o

t

s

87132 Control Shaft

-02(2)

87140
Cover

87158
Housing

l = 0.1590 ± .0005
m = 0.1575 ± .0005
n = 0.0937 ± .0002
p = 0.0635 + .0005
                  - 0
q = 0.0625 + .0001
                  - .0005
s = 0.156 ± .005
t = 0.015 + .005
                  - 0

Figure 2

# LSPE Timer Control Module Seal
## Leak Rate Analysis

### Figure 3

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## SUMMARY OF VITON DATA

$$\left\{ \begin{array}{c} H\text{-}F\text{-}CF_3\text{-}F \\ C\text{-}C\text{-}C\text{-}C \\ H\text{-}F\text{-}F\text{-}F \end{array} \right\}$$

**GENERIC TYPE - COPOLYMER OF HEXAFLUOROPROPYLENE AND VINYLIDENE FLUORIDE**

### PERMEABILITY AIR $10^{-7}$ CM$^3$/SEC/CM$^2$ CM/ATM

| TEMP | VALUE |
|------|-------|
| $75^0$ | 0.0099 |
| $176^0$ | 0.88 |
| $250^0$ | 3.6 |

**PERMEABILITY**

$CO_2$ - $5.9 \times 10^{-7}$ @ $86^0$F

$HE_2$ - $0.892 \times 10^{-7}$ @ $75^0$F - $17.4 \times 10^{-7}$ @ $250^0$F - $67.0 \times 10^{-7}$ @ $400^0$F

Nitrogen - $0.0054 \times 10^{-7}$ @ $75^0$F

$O_2$ - $0.11 \times 10^{-7}$ @ $86^0$F

### COMPATIBILITY VARIOUS FLUIDS

**SATISFACTORY**

MIL-H-5606
ENGINE OIL
OS-45 (SILICATE)
MLO-820 (SILICATE)
FUEL, GASOLINE

**UNSATISFACTORY**

SKYDROL-7000 (PHOSPATE)

SKYDROL - 5000 (PHOSPATE)

LOW MOLECULAR WEIGHT, ESTERS

ETHERS, KETONES

CERTAIN AMINES

### RADIATION EFFECTS $70^0$F (GAMMA)

| DOSAGE | TENSILE | ELONG | HARD |
|--------|---------|-------|------|
| $5 \times 10^6$ | | -25% | SH84 |
| $1 \times 10^7$ | | -50% | |
| $5 \times 10^7$ | NONE | | |
| $8 \times 10^7$ | +25% | | |
| $1 \times 10^8$ | +50% | | SH100 |
| NONE | | | SH80 |

### RADIATION EFFECTS ULTRA-VIOLET (SURFACE ONLY)

CRAZING, SURFACE CRACKING, WHEN DIRECTLY EXPOSED TO RADIATION

### VACUUM EFFECTS VOLATILE CONDENSABLE MATERIALS $125^0$C - $10^{-6}$ TORR

| | HOURS | | | |
|---|---|---|---|---|
| | 24 | 48 | 96 | 336 |
| WEIGHT LOSS % | .44 | .46 | .53 | .61 |
| VCM WT LOSS % | .02 | .02 | .01 | .01 |

**PRESSURE (10-8 TORR)**

| TEMP | WT LOSS % | PRESSURE |
|------|-----------|----------|
| $77^0$F | 2.1 | 0.18 (TORR) |

### OFF GASS PRODUCTS $125^0$C & $10^{-6}$

(VITON A4411A-990)

MAJOR - BENZL ETHER

MINOR CARBON DIOXIDE + $(CF_2)$ N AND $C_N$ $H_N$ $F_N$ DIOCTYL PHTHALATE

### OTHER MECHANICAL AND PHYSICAL PROPERTIES

| Property | Value |
|----------|-------|
| SPECIFIC GRAVITY | 1.90 |
| SPECIFIC HEAT | 0.395 |
| THERMAL CONDUCTIVITY BTU/IN/FT$^2$/$^0$F/HR @ $100^0$F | 1.58 |
| COEFFICIENT OF EXP. PER $^0$FX$10^{-6}$ | 88 |
| HARDNESS DUROMETER A | 70-90 |
| TENSILE STRENGTH P.S.I. | 1500 - 2500 |
| ELONGATION % ($75^0$F) | 150-300 |
| COMPRESSION SET * % @ $400^0$F 70 HR | 30 |
| BRITTLE TEMP. | -45 TO -65$^0$F |
| LOW TEMPERATURE STIFFNESS P.S.I. @ (CLASHBERG-ASTM-D1043 | 8600 |

*ASTM METHOD D-395

**COMPRESSION SET* - "O" RINGS 1" X 0.139**

| | |
|---|---|
| 70 HRS - $75^0$F | 6 |
| 166 HRS/$350^0$F | 20 |
| 22 HRS/$400^0$F | 12 |
| 70 HRS/$400^0$F | 24 |
| 168 HRS/$400^0$F | 38 |
| 336 HRS/$400^0$F | 51 |
| 22 HRS/$450^0$F | 26 |
| 70 HRS/$450^0$F | 65 |
| 165 HRS/$450^0$F | 82 |
| 22 HRS/$500^0$F | 56 |
| 70 HRS/$500^0$F | |

*ASTM METHOD B AND MIL-R-83248

### EFFECTS OF THERMAL-VACUUM ENVIRONMENT ON VITON A-4411A 990   500 HRS @ $135^0$C AND $10^{-6}$ TORR

| MATERIAL | DIMENSIONAL CHANGE | WEIGHT CHANGE | SHORE HARDNESS "A" CONTROL | SHORE HARDNESS "A" TEST | TENSILE P.S.I. CONTROL | TENSILE P.S.I. TEST | % ELONGATION CONTROL | % ELONGATION TEST |
|----------|--------------------|--------------|-----------|------|---------|------|---------|------|
| VITON A-4411A-990 | L.N.C. | -0.54 | 85.4 | 86.7 | 2030 | 2220 | 358 | 273 |

9270-1027

## APPENDIX I

### (Prepared by: T. W. Fox, ALSEP Reliability)

Contained herein is a leak-rate analysis for the LSPE Timer Control Module. This analysis assumes worst case conditions for both earth storage and post-launch environments.

---

### List of Symbols

$p_o$    = pressure of gas in sealed container (module)

$T_o$    = temperature of gas in sealed container

$\rho_o$    = mass density of gas in sealed container

$p_e$    = static pressure in orifice

$\rho_e$    = mass density in orifice

$A_e$    = cross-sectional area at orifice

$\dot{m}_e$    = mass efflux thru orifice

$M$    = Mach number in gas flow

$\gamma$    = ratio of specific heats of gas

$V$    = Volume of sealed container

$t$    = time

$R$    = gas constant (ideal gas)

$u_e$    = gas velocity at orifice

$\tau$    = time integration dummy variable

$(\ )_{ref}$ = conditions when leak rate was measured.

$n_o$    = molecular number density in sealed container

$m_{N_2}$  = mass of one $N_2$ molecule

$l_{char}$ = characteristic dimension

$k$    = boltzman's constant

## LEAK RATE ANALYSIS - LSPE TIMER CONTROL MODULE

The objective of this analysis is to determine the adequacy of the
seals in the LSPE Timer Control Module under worst case conditions
established in the specifications.  It has also been established that
as long as the internal pressure is maintained at 1 torr or more, the
timer will function properly.*

This module is initially pressurized to 16.0 psia with dry nitrogen
gas, $N_2$, at a temperature of 70 °F.  During acceptance testing, it is
verified that the leak rate in vacuum is less than $1 \times 10^{-6}$ STD cc/sec.
Following timer acceptance the timer module must be capable of surviving
a three year earth storage and then three weeks in a thermal vacuum
environment.

The storage environment is specified at 14.7 psia and +80°F max.
Under these conditions the minimum pressure inside the sealed module
will be 14.7 psia at 80°F at the time of launch, irregardless of the storage
leak rate.  In the analysis that follows, it is assumed that at launch the
pressure is 14.7 psia (2116 lb f/ft$^2$) and the temperature is 80°F
(540°R).  After launch the timer is exposed to a thermal vacuum environment
and the maximum temperature is increased to 185°F (645°R).

Now, noting that nitrogen gas is a compressible fluid and assuming
that the conditions for a continuum flow analysis are satisfied, the
variation of pressure with time inside the sealed volume can be expressed: **

$$p_o(t) = p_o (t=0) \, e^{ -\left( \dfrac{\dot{m}_e}{p_o V/RT_o} \right)_{ref} \left( \dfrac{T_o}{T_{o_{ref}}} \right)^{1/2} \cdot t}$$

where $\dot{m}_e$ is the initial measured leak rate and ( )$_{ref}$ indicates the con-
ditions when the leak rate was originally determined.  This expression
assumes that the mass flow through the leak orifice is choked.

---

*ATM 1038
** See attachment for deriviation

The flow through an orifice is controlled by the pressure ratio across the orifice; for a compressible flow this pressure ratio is expressed:

$$\frac{P_e}{P_o} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\left|\frac{-\gamma}{\gamma-1}\right|}$$

When $M = 1$ the flow is choked and the mass flow is a maximum, then the ratio reduces to:

$$\frac{P_e}{P_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

If the actual pressure ratio is less than this limiting value, $M = 1$ and the flow remains choked. For nitrogen, $\gamma = 1.400$ and the ratio is 0.5283. Clearly the mass flow from a pressurized container into a vacuum $(p_o \approx 0)$, satisfies this condition.

Treating the nitrogen gas, $N_2$, as a perfect fluid (which is a valid assumption over the range of pressures of interest), the pressure at time $t = o$ inside the sealed container is expressed:

$$P_o (t = o) = P_{o_{launch}} \cdot \left(\frac{T_o \text{ vacuum at } t = o}{T_o \text{ at launch}}\right)$$

Assuming that the temperature of the gas suddenly rises to the maximum of $+185\,°F$ immediately after launch the pressure at $t = 0$ is:

$$P_o (t = o) = 14.7 \left(\frac{645}{540}\right)$$

$$= 17.558 \text{ psia}$$

$$= 907.8 \text{ torr}$$

To compute the time coefficient in the exponential term, the initial mass flow rate, the volume of the sealed container, and the gas constant must be given, and the reference pressure and temperature must be known. Assuming the specified maximum leak rate of $1 \times 10^{-6}$ STD cc/sec ($2.556 \times 10^{-12}$ lbm/sec), and knowing that the volume is 0.1075 in$^3$ ($62.2 \times 10^{-6}$ ft$^3$) and that the R, the gas constant for nitrogen is 55.15 $\frac{\text{lbf} \cdot \text{ft}}{\text{lbm} \cdot {}^\circ\text{R}}$, the coefficient is computed as:

$$\left(\frac{T_o}{T_{o_{ref}}}\right)\left(\frac{-\dot{m}_e}{P_o V / R T_o}\right)^{\frac{1}{2}}_{ref} = 5.751 \times 10^{-7} \cdot \text{sec}^{-1}$$

The time required for the pressure to be reduced to 1 torr can now be determined from the time-pressure relationship given above:

$$\frac{P_o(t)}{P_o(t = o)} = \frac{1}{907.8} = e^{-5.751 \times 10^{-7} \cdot t}$$

or
$$-5.751 \times 10^{-7} \cdot t = -6.867$$

and the time elapsed is:

$$t = 1.194 \times 10^7 \text{ sec}$$

or

$$t = .1382 \times 10^3 \text{ days}$$

Therefore if the continuum assumption is valid the seals are adequate.

To assure the validity of the continium assumption, the characteristic physical dimension restricting the flow must be compared with the mean free path of the $N_2$ molecules in the gas. For continium assumptions to hold, $d \gg \lambda$. This requirement is satisfied if $d \approx 10\lambda$ or greater; where d is the diameter of the orifice and $\lambda$ is the mean free path length.

The cross-sectional area of the orifice is assumed constant and is computed based on the measured leak rate:

$$A_e = \left( \frac{\dot{m}e}{\frac{p_o}{RT_o} (.6339) (1.1662 \, RT_o)^{1/2}} \right)_{ref} \qquad *$$

where

$$p_o = 16.0 \text{ psia} = 2304 \quad \text{lbf/ft}^2$$

$$T_o = 70\,°F = 530\,°R$$

$$\dot{m}_e \quad \leq 1 \times 10^{-6} \text{ STD cc/sec}$$
$$\leq 2.56 \times 10^{-12} \text{ lbm/sec}$$

Therefore,

$$A_e = .276998 \times 10^{-12} \text{ ft}^2$$

For a circular orifice, the diameter would be

$$1.87799 \times 10^{-6} \text{ ft}$$

or

$$5.7241 \times 10^{-5} \text{ cm}$$

---

*see attachment for derivation.

However, for an O-ring seal the orifice of the leak will most likely not be circular and the characteristic diameter will be on the order of 1/5 d or less, hence

$$d_{char} \approx 1.14 \times 10^{-5} \text{ cm}$$

Next, assuming simple binary molecular collision (a valid assumption at pressures less than 10 atmospheres), the mean free path length can be estimated:

$$\lambda = \text{mean free path} = \frac{1}{\sqrt{2} \, \pi \, s^2 \, n_o}$$

where s is the molecular diameter ($3.15 \times 10^{-8}$ cm for $N_2$) and $n_o$ is the molecular number density:

$$n_o = \frac{P_o}{kT_o} = 2.45 \times 10^{19} \, \frac{\text{molecules}}{\text{cm}^3}$$

Therefore, at launch the $N_2$ molecular number density would be $2.45 \times 10^{19}$ molecules per cubic centimeter and the mean free path is:

$$\lambda = .927 \times 10^{-5} \text{ cm}$$

Clearly $\lambda \approx d$ and the continuum assumption inherent in the compressible flow analysis is not valid.

Unfortunately, there are no adequate flow theories, either continuum or kinetic which describe the flow through an orifice in the range where

$$\frac{1}{10} \, d_{char} < \lambda < 10 \, d_{char}$$

However, if we treat the mass flow from a gas-kinetic point-of-view, the molecular effusion rate into a vacuum can be expressed:

$$\dot{m}_e = a^2 \cdot \frac{8}{3} \cdot \frac{a}{L} \cdot (1/2 \, \pi \, RT_o)^{1/2} \cdot p_o *$$

where $p_o$ is the pressure and $T_o$ is the temperature of the gas in the container, and a is the orifice radius and L is the orifice length.

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*Kennard, Kinetic Theory of Gases, Para. 172.

The mass flow rate expression given above holds for gas leaks when the mean free path is much greater than the characteristic diameter of the orifice, i.e., $\lambda \gg d_{char}$  At pressures where $\lambda \sim d$, the actual flow may be up to 50% less than that predicted with this relationship.

The mass flow expression can be rewritten in a form that is readily integrable, if it is assumed that the gas temperature in the container remains fixed as the gas leak progresses and further that the orifice dimensions are fixed. With these assumptions, observe that:

$$\overset{\bullet}{m}_e = \frac{dm}{dt} \, , \quad \text{where } m = \rho V = n_o m_{N_2} V$$

and,

$$P_o = n_o k T_o \quad \text{(assuming ideal gas, which is valid)}$$

therefore the mass flow expression becomes:

$$\overset{\bullet}{m}_e = \frac{d}{dt} (n_o m_{N_2} V) = m_{N_2} V \cdot \frac{dn_o}{dt}$$

$$= -a^2 \cdot \frac{8}{3} \cdot \frac{a}{L} \cdot (1/2 \, \pi R)^{1/2} \cdot (T_o)^{1/2} \cdot n_o k T_o$$

$$\equiv -\beta \cdot k(T_o)^{3/2} \cdot n_o$$

or rearranging:

$$\frac{dn_o}{n_o} = \frac{\beta \, k}{m_{N_2} V} (T_o)^{3/2} \quad dt = \alpha \cdot T_o^{3/2} \quad dt$$

which can be integrated to yield:

$$\int \frac{dn_o}{n_o} = - \int_0^t \alpha T_o^{3/2} \, d\tau$$

or

$$\frac{n_o(t)}{n_o(o)} = e^{-\alpha T_o^{3/2} t}$$

This number density ratio can also be expressed directly as a pressure ratio by observing that:

$$\frac{n_o(t)}{n_o(o)} = \frac{p_o(t)/kT_o}{p_o(o)/kT_o} = \frac{p_o(t)}{p_o(o)}$$

Therefore, the pressure at any time t is expressed:

$$p_o(t) = p_o(o) \, e^{-\alpha T_o^{3/2} t}$$

The value of $\alpha$ depends on the quantity $\beta$, the volume of the container and molecular mass of the gas in the container. $\beta$ can be evaluated from the leak rate measured during acceptance testing of the timer control module:

$$\beta = a^2 \cdot \frac{8}{3} \cdot \frac{a}{L} \cdot (1/2 \, \pi \, R)^{1/2}$$

$$= \left( \frac{\dot{m}_e}{p_o(T_o)^{1/2}} \right) \quad \text{measured}$$

Assuming a leak rate of $1 \times 10^{-6}$ STD cc/sec ($2.556 \times 10^{-12}$ lbm/sec) measured with $T_o = 70°F$ and $p_o = 16$ psia:

$$\beta = \frac{2.556 \times 10^{-12}}{16 \times 144 \times (530°R)^{1/2}}$$

$$= .4819 \times 10^{-16}$$

The exponential coefficient $\alpha$ is then evaluated:

$$\alpha = \frac{\beta k}{m_{N_2} V} = 4.276 \times 10^{-11} \, sec^{-1} \, °R^{2/3}$$

where $m_{N_2}$ is the mass of a single $N_2$ molecule, k is boltzman's constant and V is the volume of the sealed container.

The pressure ratio at time t can be computed assuming that the pressure at launch is 14.7 psia and the temperature is +80°F; it is further assumed that immediately after launch the temperature is increased to the maximum for a vacuum environment, +185°F. This increase in temperature raises the initial pressure of the nitrogen gas inside the sealed container:

$$p_o(o)^t = p_o(o)_{launch} \left( \frac{T_{o \text{ vacuum at } t = o}}{T_{o \text{ at launch}}} \right)$$

$$= 14.7 \left( \frac{645}{540} \right)$$

$$= 17.558 \text{ psia}$$

$$= 907.8 \text{ torr}$$

The time required for the pressure to drop to 1 torr is then found from:

$$\frac{p_o(t)}{p_o(o)} = \frac{1}{907.8} = e^{-\alpha T_o^{3/2} t} = e^{-7.005 \times 10^{-7} t}$$

or

$$-7.005 \times 10^{-7} t = 6.867$$

and the time elapsed is:

$$t = 9.803 \times 10^6 \text{ sec}$$

or

$$t = 1.135 \times 10^2 \text{ days}$$

Therefore the seals are adequate!

## ATTACHMENT

Contained herein is the derivation of the relationship for the variation of pressure with time in a sealed container having a small leak.

The pressure inside a constant volume container that is maintained at a constant temperature is directly related to the gas mass remaining inside the container. Based on the principle of conservation of mass, the flow rate through an orifice (leak) can be expressed as follows:

$$\frac{dm}{dt} = -\rho_e \, u_e \, A_e .$$

This rate is controlled by the pressure drop across the orifice, and for a compressible fluid the ratio of these pressures is:

$$\frac{P_e}{P_o} = \left(1 + \frac{\gamma - 1}{2} \, M^2\right)^{\frac{-\gamma}{\gamma - 1}}$$

and the flow rate is maximum when the flow is choked (i.e., M = 1):

$$\frac{P_e}{P_o} = \left(\frac{\gamma + 1}{2}\right)^{\frac{-\gamma}{\gamma - 1}} \qquad = .5283 \text{ (for } \gamma = 1.4\text{)}$$

For pressure ratios less than .5283, the flow remains choked and is independent of the external pressure at the orifice. For leakage into a vacuum this condition is always satisfied.

A similar expression exists for the density ratio across the orifice:

$$\frac{\rho_e}{\rho_o} = \left(1 + \frac{\gamma - 1}{2} \, M^2\right)^{-\frac{1}{(\gamma - 1)}}$$

and for choked flow:

$$\frac{\rho_e}{\rho_o} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}} \qquad = .6339 \text{ (if } \gamma = 1.4\text{)}$$

The flow velocity in the orifice can be obtained from the conservation of energy relationship

$$\frac{1}{2} u_o^2 + \frac{\gamma}{\gamma-1} \cdot \frac{P_o}{\rho_o} = \frac{1}{2} u_e^2 + \frac{\gamma}{\gamma-1} \cdot \frac{P_e}{\rho_e}$$

and if ( )$_o$ indicates reservoir or sealed volume conditions, $u_o \approx o$ and the velocity at the exit is expressed:

$$u_e^2 = \frac{2\gamma}{\gamma-1} \left( \frac{P_o}{\rho_o} - \frac{P_e}{\rho_e} \right)$$

$$= \frac{2\gamma}{\gamma-1} \left( \frac{P_o}{\rho_o} - \frac{.5283 \, P_o}{.6339 \, \rho_o} \right)$$

$$= \frac{2\gamma}{\gamma-1} \left( \frac{P_o}{\rho_o} \right) \left( 1 - .8334 \right)$$

Assuming an ideal gas (which is a valid assumption for gases at these pressures), the velocity can also be expressed:

$$u_e = \left( \frac{2\gamma}{\gamma-1} (R_{N_2} T) (1 - .8334) \right)^{1/2}$$

$$= \left( 1.1662 \, R_{N_2} T_o \right)^{1/2} \qquad (\text{if } \gamma = 1.4)$$

The remaining unknown relationship required for the leak rate expression is the orifice exit area, $A_e$.   If it is assumed that the orifice size remain fixed, a value for $A_e$ can be computed from the known initial leak rate (ref: is being determined during acceptance testing):

$$\dot{m}_e = \left( \rho_e u_e A_e \right)_{ref}$$

Where,

$$\rho_{e_{ref}} = \rho_{o_{ref}} (.6339) = \left(\frac{P_o}{RT_o}\right)_{ref} (.6339)$$

$$u_e = \left(1.1662 \ RT_o\right)^{1/2}_{ref}$$

$$\dot{m}_e \quad \text{is specified}$$

therefore,

$$A_e = \left(\frac{\dot{m}_e}{\rho_{e_{ref}} u_e}\right)_{ref}$$

$$= \left(\frac{\dot{m}_e}{\left(\frac{P_o}{RT_o}\right)(.6339)\left(1.1662 \ RT_o\right)^{1/2}}\right)_{ref}$$

The flow rate can now be re-expressed:

$$\frac{dm}{dt} = -\rho_e u_e A_e$$

$$= -(.6339) \rho_o \left(1.1662 \ RT_o\right)^{1/2} \left(\frac{\dot{m}_e}{\frac{P_o}{RT_o} (.6339)\left(1.1662 \ RT_o\right)^{1/2}}\right)_{ref}$$

$$= -\rho_o \left(\frac{T_o}{T_{o_{ref}}}\right)^{1/2} \cdot \left(\frac{\dot{m}_e}{P_o / RT_o}\right)_{ref}$$

and

$$\frac{dm}{dt} = \frac{d}{dt}\left(\rho_o V\right) = V \cdot \frac{d\rho_o}{dt}$$

therefore we can write:

$$\frac{d\rho_o}{dt} = -\rho_o \left( \frac{T_e}{T_o} \right)^{1/2}_{ref} \cdot \left( \frac{\dot{m}_e}{p_o V /RT_o} \right)_{ref}$$

Integrating over time, an expression for the gas density variation with time is obtained:

$$\int_{\rho_o(o)}^{\rho_o(t)} \frac{d\rho_o}{\rho_o(\tau)} = - \int_0^t \left( \frac{\dot{m}_e}{p_o V/RT_o} \right)_{ref} \cdot \left( \frac{T_o}{T_o}_{ref} \right)^{1/2} d\tau$$

$$= \left( \frac{-\dot{m}_e}{p_o V/RT_o} \right)_{ref} \cdot \left( \frac{T_o}{T_o}_{ref} \right)^{1/2} \int_0^t d\tau$$

or

$$\ln \frac{\rho_o(t)}{\rho_o(o)} = \left( \frac{-\dot{m}_e}{p_o V/RT_o} \right)_{ref} \cdot \left( \frac{T_o}{T_o}_{ref} \right)^{1/2} \cdot t$$

$$\frac{\rho_o(t)}{\rho_o(o)} = e^{\left( -\frac{\dot{m}_e}{p_o V/RT_o} \right)_{ref} \left( \frac{T_o}{T_o}_{ref} \right)^{1/2} \cdot t}$$

This density ratio can also be expressed as a pressure ratio in the sealed container:

$$\frac{\rho_o(t)}{\rho_o(o)} = \frac{p_o(t)}{p_o(o)}$$

And finally we have an expression that will yield the time required for the pressure to drop from an initial value to some minimum terminal value:

$$p_o(t) = p_o(o) e^{-\left(\frac{me}{p_o V/RT_o}\right)_{ref} \left(\frac{T_o}{T_o}\right)_{ref}^{1/2} \cdot t}$$