Section E

Increased travel distance due to craters and other obstacles on the Moon

by H. J. Moore

Introduction

Craters, blocks, scarps, and linear depressions present obstacles which must be crossed or by-passed by Lunar Roving Vehicles. Small craters and blocks with reliefs of 1/2 to 1 meter or less can be crossed. Craters and blocks with larger reliefs must be circumvented. Rilles or linear depressions can generally be crossed at some point along their length but there are little data on the frequency of occurrence of crossings. Thus, only craters and blocks will be considered below.

Method of analysis

Procedures for analyzing increased path lengths of vehicle travel required by the presence of obstacles have been discussed by Brooks (1958), Ulrich (1968), and Rozema (unpublished data). The analysis below is similar to those above but it differs in that the obstacles are described by a frequency distribution rather than by a discrete size; and the size of the vehicle is taken into account. No consideration is given to the vehicle turning radius which may have considerable importance when frequency of small obstacles is high.
Increased travel distance for one circular obstacle

The vehicle will encounter a circular obstacle over any interval \(dy\) with equal probability (see fig. E-1):

\[
dy = \sin \theta \, d\theta
\]  
\[\text{(1)}\]

Figure E-1. Circular obstacle

It will travel an additional distance \(p_1\) of:

\[
p_1 = 2 \left( \theta - \sin \theta \right),
\]
\[\text{(2)}\]

and the mean additional distance traveled \(p_2\) is:

\[
p_2 = \frac{\int_{0}^{\frac{\pi}{2}} 2(\theta - \sin \theta) \sin \theta \, d\theta}{\int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta} = \frac{2(1 - \frac{\pi}{4})}{(1 - 0)} = 2(1 - \frac{\pi}{4})
\]
\[\text{(3)}\]

The effective size of an obstacle is a function of the size of the obstacle \(D\) and the vehicle size (figure E-2) because the vehicle must clear the obstacle by half of its width \(w\). Then for obstacles with diameters \(D\) and vehicles with width \(w\), the mean additional distance traveled per obstacle \(p_3\) is:

\[
p_3 = (D + w)(1 - \frac{\pi}{4})
\]
\[\text{(4)}\]
in travel distance:

\[ dP = \frac{\frac{\pi}{4} \left( 1 - \frac{\pi}{4} \right) (D + w) dD}{(D + w)} \]  

(9)

Equation 9 may be restated:

\[ dP = \frac{\pi}{4} \left( 1 - \frac{\pi}{4} \right) nk \int_{D1}^{D2} \left( \frac{D^{\frac{n+1}{n+2}} + 2wD^{n+1} + \frac{2D^{n-1}}{n} }{D} \right) dD \]  

(10)

or

\[ P = \frac{\pi}{4} \left( 1 - \frac{\pi}{4} \right) nk \left[ \frac{D^{\frac{n+2}{n+2}}}{n^{\frac{n+2}{n+2}}} + \frac{2wD^{\frac{n+1}{n+1}}}{n^{\frac{n+1}{n+1}}} + \frac{2D^{\frac{n-2}{n-2}}}{n^{\frac{n-2}{n-2}}} \right]_{D1}^{D2} \]

where:

\[ n \neq -2, n \neq -1, \text{ and } n \neq 0. \]

Solution of equation 10 for \( n = -2 \) gives:

\[ P = \frac{\pi}{4} \left( 1 - \frac{\pi}{4} \right) (-2k) \left[ D^{\frac{n+1}{n+1}} + \frac{2wD^{\frac{1}{-1}}}{-1} + \frac{2D^{\frac{-2}{-2}}}{-2} \right]_{D1}^{D2} \]

(11)

Solution of equation 10 for \( n = -1 \) gives:

\[ P = \frac{\pi}{4} \left( 1 - \frac{\pi}{4} \right) (-k) \left[ D + 2w \frac{D^{\frac{-1}{-1}}}{-1} + \frac{2D^{\frac{-2}{-2}}}{-2} \right]_{D1}^{D2} \]

(12)

Because the path length increases, there must be corresponding increases in subsequent path lengths resulting from the initial increased path lengths. For example, the added path length \( (P') \), when taken as unity, is accompanied by an increase \( (P') \):
\[ P' = nk \left( \frac{n}{4} \right) \left( 1 - \frac{n}{4} \right) \int_{D_1}^{D_2} \left( D^{n+1} + 2WD^n + W^2 D^{n-1} \right) dD \] (13)

But \( P \) is a fraction the map length (M) so that:

\[ PP' = P^2. \] (14)

Then the traverse length (L) becomes the sum of the map length (M=1) and sum of all the increased path lengths:

\[ L = 1 + P + P^2 + P^3 + \ldots + P^{n-1}, \] (15)

or

\[ L = \frac{1}{1-P}. \] (16)

**Criterion for impassability**

In order to establish if a block field or a cluster of sharp craters is passable or impassable requires detailed examination of that field because statistical approaches do not necessarily yield such information. However, an estimate can be made from the foregoing equations. For a field of obstacles (D+W) that cover all the area, the area is clearly impassable. The vehicle might drive through if the area covered by obstacles (D+W) is less than 1.0. Thus, the integral of equation 6 may be used to estimate passability:

\[ A = \frac{n}{4} \left( \frac{n}{4} \right) \int_{D_1}^{D_2} \left( D^{n+1} + 2WD^n + W^2 D^{n-1} \right) dD, \] (17)

where:

\[ n \neq -2, \; n \neq -1, \; \text{and} \; n \neq 0. \]
Solutions for equation 17 are similar to those for equation 10. Examples of the use of equations 5, 10, 16 and the integral of equation 8 are given in tables E-I, E-II, and E-III.

References cited
Rosiwal, August, 1898, Uber geometrische Gesteinsanalysen: Verhandlungen Geologischen Bundesanstalt (Reichsanstalt), p. 143-175.
Table E-1. Examples of calculations for increased travel distances due to blocks.

<table>
<thead>
<tr>
<th></th>
<th>( N_A ) (^{1/} )</th>
<th>( D_2 ) &amp; ( D_1 ) (^{2/} )</th>
<th>( P )</th>
<th>( L_B ) (^{3/} )</th>
<th>( N_L ) (^{4/} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough maria</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( w = 2 ) meters</td>
<td>( 0.002 , D^{-3} )</td>
<td>( 1 - 30 )</td>
<td>( 0.0043 )</td>
<td>( 1.0044 )</td>
<td>( 2.0 )</td>
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<tr>
<td>Smooth maria</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w = 2 ) meters</td>
<td>( 0.0002 , D^{-3} )</td>
<td>( 1 - 30 )</td>
<td>( 0.0004 )</td>
<td>( 1.0004 )</td>
<td>( 0.2 )</td>
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<tr>
<td>Large fresh crater flank (crater diam. 80 km)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w = 2 ) meters</td>
<td>( 0.027 , D^{-3} )</td>
<td>( 1 - 100 )</td>
<td>( 0.059 )</td>
<td>( 1.063 )</td>
<td>( 31.7 )</td>
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<td>Block field</td>
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<td></td>
<td></td>
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<tr>
<td>( w = 2 ) meters</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Section A</td>
<td>( 0.100 , D^{-1.1} )</td>
<td>( 1 - 10 )</td>
<td>( 0.358^* )</td>
<td>( 1.558 )</td>
<td>( 178 )</td>
</tr>
<tr>
<td>Section B</td>
<td>( 0.032 , D^{-1.08} )</td>
<td>( 1 - 2.36 )</td>
<td>( 0.040 )</td>
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<td>( 22.6 )</td>
</tr>
<tr>
<td></td>
<td>( 0.336 , D^{-3.80} )</td>
<td>( 2.36 - 9.44 )</td>
<td>( 0.059 )</td>
<td></td>
<td>( 31.5 )</td>
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<tr>
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<td>( 0.012 , D^{-1.07} )</td>
<td>( 1 - 2.36 )</td>
<td>( 0.014 )</td>
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<td>( 8.0 )</td>
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<tr>
<td></td>
<td>( 0.080 , D^{-3.32} )</td>
<td>( 2.36 - 9.44 )</td>
<td>( 0.022 )</td>
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<td>( 11.7 )</td>
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<tr>
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<td></td>
<td>( 0.002 , D^{-0.585} )</td>
<td>( 2.36 - 9.44 )</td>
<td>( 0.006 )</td>
<td></td>
<td>( 2.8 )</td>
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* Each section is 47 x 47 meters.

1/ \( N_A \) = number of craters per square meter.

2/ \( D, D_1, D_2 \) are crater diameters in meters.

3/ \( L, L_B, L_C \) = ratio of distance traveled to map length.

4/ \( N_L \) = number of obstacles encountered for each kilometer of distance traveled.
Table E-II. Examples of calculations for increased travel distances due to craters.

<table>
<thead>
<tr>
<th></th>
<th>$N_A^{1/}$</th>
<th>$D_2$ &amp; $D_1^{2/}$</th>
<th>$P$</th>
<th>$L_B^{3/}$</th>
<th>$N_L^{4/}$</th>
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<tr>
<td>Rough maria</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w = 2 meters</td>
<td>0.05 $D^{-2}$</td>
<td>2 - 200</td>
<td>0.119</td>
<td>1.136</td>
<td>38.8</td>
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<tr>
<td>Smooth maria</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w = 2 meters</td>
<td>10.0 $D^{-3}$</td>
<td>200-10,000</td>
<td>0.026</td>
<td>1.026</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>0.05 $D^{-2}$</td>
<td>2 - 40</td>
<td>0.091</td>
<td>1.100</td>
<td>37.3</td>
</tr>
<tr>
<td></td>
<td>2.0 $D^{-3}$</td>
<td>40 - 60</td>
<td>0.009</td>
<td>1.009</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>0.033 $D^{-2}$</td>
<td>60 - 300</td>
<td>0.018</td>
<td>1.019</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>10.0 $D^{-3}$</td>
<td>300 - 10,000</td>
<td>0.017</td>
<td>1.017</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Large fresh crater flank (crater diam. 80 km)

w = 2 meters 0.1 $D^{-3}$ 2 - 10,000 0.059 1.063 29.4

1/ $N_A$ = number of craters per square meter.

2/ $D$, $D_1$, $D_2$ are crater diameters in meters.

3/ $L$, $L_B$, $L_C$ = ratio of distance traveled to map length

4/ $N_L$ = number of obstacles encountered for each kilometer of distance traveled.