STATISTICAL CHARACTERIZATION OF SPATIAL DISTRIBUTION OF IMPACT CRATERS: IMPLICATIONS TO PRESENT-DAY CRATERING RATE ON MARS. ***M. A. Kreslavsky***, Geological Sciences, Brown University, Providence, RI, 02912, USA, kreslavsky@brown.edu

Introduction: Counting craters has long been widely used in planetary science to get information about age of materials and events in geological history of planetary bodies. It also has long been understood that the uniform surface age and accumulation crater population requires "random" spatial distribution of craters. Such tests for randomness were applied to martian craters, e.g., in [1]. Here I report on my work on development of practical and statistically robust methods for analysis of spatial distribution of craters. I apply these methods to the recently emplaced craters discovered by M. Malin and co-authors [2] and infer a correction factor for the present-day impact cratering rate.

Background: Estimation of ages with crater counts is based on the assumption that the cratering process is well described with a mathematical model of Poisson process [e.g., 3]. This means that each impact occurs purely randomly, "not knowing about" the site and time of previous impacts. The validity of this assumption is a special question. Double asteroids, freshly disrupted comets, secondary impacts violate this assumption. The most important of these factors, secondary cratering, plays no role for the very young surfaces on Mars, because they are younger than any large crater able to produce distant secondary craters. Here I assume that the impacts *are* random. Obliteration of craters is not random (unless they are removed by other impacts).

Under the Poisson process assumption, the observed density of craters in an area gives an unbiased estimate of the *average crater retention age*. The crater retention age depends on crater size. (Note that the crater retention age can vary over the studied area, and the estimate gives the arithmetic average of it.) What is more important for geological applications, the number of craters gives a confidence interval for the average age:

$$F_\Gamma^{-1}(1-p;N) \quad < \quad \tilde{N} \quad < \quad F_\Gamma^{-1}(p;N+1)\,, \qquad (1)$$

where $\tilde{N}$ = age × rate × area is the expected number of craters, N is the actual number of craters, p is the confidence level, for example, 0.9 or 0.95 or 0.99, and $F_\Gamma^{-1}(\cdot\,;\cdot)$ is the inverse cumulative gamma distribution. For a large number of craters, practically, for $N > 10$, this interval is well approximated by traditionally used $\sqrt{N}$ error bars:

$$N - F_n^{-1}(p)\sqrt{N} \quad < \quad \tilde{N} \quad < \quad N + F_n^{-1}(p)\sqrt{N}\,, \qquad (2)$$

where $F_n^{-1}(\cdot)$ is the inverse cumulative standard normal distribution.

An important special case is an accumulation population of craters. If (from geological considerations) we suppose that in some area some geological resurfacing event removed all craters in some area simultaneously, and no obliteration of craters occurred since that time, the total number of craters gives boundaries for the age of the resurfacing event. In particular, if the resurfacing event emplaced new material, the average crater retention age can be interpreted as the true age of this material.

In a sparse accumulation population (where craters do not overlap), craters have a purely random spatial distribution. If the spatial distribution of craters within an area is distinctive from random, the inferred average age should not be considered as an age of some material or event. That is why statistical tests for spatial randomness should accompany crater counts, if the counts are interpreted in term of particular (rather than average) ages.

Statistical test for spatial randomness: Such tests are constructed in the following way. From the actual distribution of the craters we calculate some quantity (a statistic), for example, the mean nearest neighbor distance. Then we calculate the probability distribution for this statistic for purely random spatial distributions. If the actual value of the statistic is less than, for example, 0.05-quantile of the distribution (that is this value is among 5% of the lowest values for random populations), we say that the randomness hypothesis is rejected at a confidence level of 0.95. The same, if it is greater than 0.95-quantile. Several different statistics can be applied to the same population.

It is not practical and often is not appropriate to use a theoretical probability distribution of the statistic chosen because of boundary effects. Unless the number of the craters reaches millions, the boundary effects are significant, and the distribution for a given area deviates from the theoretical one for an infinite plane or a sphere or a simple geometrical shape. (Recently this was discussed in [4]). Instead of cumbersome theoretical calculations, it is practical to estimate the probability distribution through numerical Monte-Carlo calculations as a frequency distribution of the statistic calculated for a large enough number of pseudo-randomly generated crater populations in the same area. Such an approach was widely applied to venusian craters [5-7]. It is important to use advanced pseudo-random number generators for such calculations, because many standard generators may produce bad side effects.

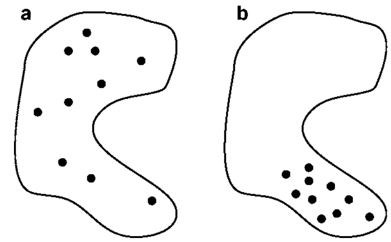

Fig. 1. Example of two spatial distributions of 10 craters in the same area. In the frame of purely random spatial distribution model, all spatial distributions have the same "probability", including those in **a** and **b**. For a geologist, however, distribution **a** represents a rather typical random spatial distribution, while distribution **b** argues for age difference between the northern and southern parts of the area. A "good" statistic would reject randomness on **b** and would not reject randomness of **a**.

The proper choice of statistics for the randomness tests is not a trivial problem. Surprisingly, it is not a purely mathematical problem and demands some "geological" input. A "good" statistic should be sensitive to "geologically" plausible deviations from pure randomness (**Fig. 1**.) For example, the most often used mean nearest neighbor distance is a rather good statistic; at least it does reject randomness on Fig. 1b.

To find "good" statistics, I run Monte-Carlo simulations of crater populations with parameterized deviation from randomness in some "geologically plausible way" and search for statistics most sensitive to such deviation. For example, I model a square area split in two rectangular halves with certain "age" difference, (Fig. 2). etc. This search for "good" statistics is in progress. It is not clear, what is the best set of statistics, or what set is good enough for practical use. I, however, came to some useful recipes.

So far I systematically analyzed statistics which are some characteristics of frequency distributions of the nearest neighbor distance, the adjacent area (a sub-area of the whole area, each point of which is closer to a given crater than to any other crater), and the local density (the inverse adjacent area). The characteristics we analyzed were: mean, mean square root, standard deviation, skewness, kurtosis, maximum, minimum, and a set of quantiles. I found that for a square-shaped area, the following statistics are better than others: (1) the traditionally used *mean nearest neighbor distance* is sensitive to the presence of small (2-3 craters) tight clusters (decreases, when clustered); (2) the *standard deviation of nearest neighbor distance* is sensitive to larger clusters (increases, when clustered); (3) the *standard deviation of adjacent area* is sensitive to large-scale "age" difference (increases). I recommend these statistics for areas of more or less isometric simple shapes and large number of craters (> ~ 20).

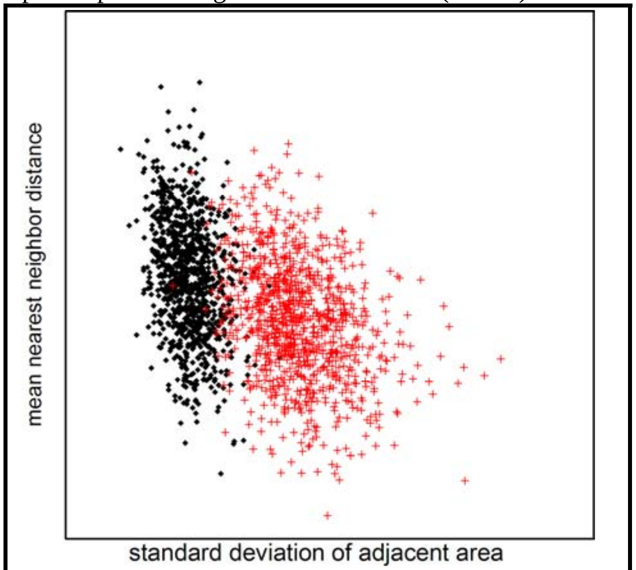

Fig. 2. Black dots, the standard deviation of adjacent area and the mean nearest neighbor distance (both in arbitrary units) for 1000 pseudo-random realizations of a random population of 128 craters in a square area. Red crosses, the same statistics for 1000 pseudo-random realizations of a model with a factor of 3 age difference between halves of the square area. The sets of realizations are well separated along the horizontal axis and widely overlapped along the vertical axis. In 96% of the cases with the age difference, the standard deviation of the adjacent area rejects pure spatial randomness at a confidence level of 0.96.

I found with such models, that for areas of elongated or digitate shapes these statistics should be replaced with their rank analogs: (1) the *median nearest neighbor distance*, (2) the *interquartile amplitude of the nearest neighbor distance*, and (3) the *interquartile amplitude of the adjacent area*. Certainly, when the areas of analysis are large, all distances should be calculated on a sphere rather than on a flat map.

Statistical tests have the ability to reject statistical randomness at a given level of confidence, but they cannot confirm randomness. The greater the number of craters, the higher the sensitivity of tests to deflections from randomness. If "good" tests do not reject randomness, this may mean either that the distribution is similar to random or that we have too few craters. The fact that a test does not reject randomness is meaningless unless we quantify somehow the sensitivity of this test to some deflections from randomness for a given number of craters. This can be done with Monte-Carlo models in the same way as we analyzed the sensitivity of statistics. The results strongly depend on the shape of the area, and the models should be run for each area under consideration. Fig. 3 shows an example of such a sensitivity analysis. This kind of analysis can bring some interpretable sense to the fact that an observed distribution is not distinguishable from random.

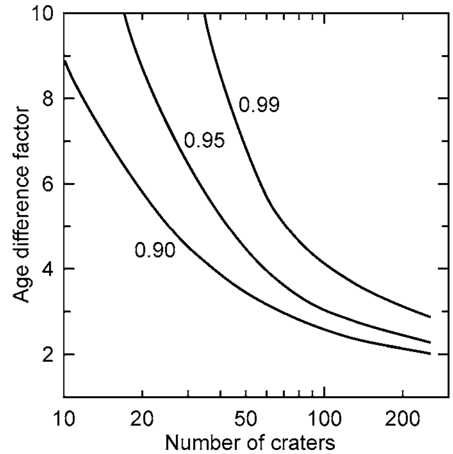

Fig. 3. Sensitivity analysis of the standard deviation of an adjacent area for the model of square area with an age difference between two halves. The model age difference necessary for confident rejection of randomness is plotted against the number of craters for three confidence levels. Although this statistic is the best among those analyzed for this particular model, it still cannot reliably detect a factor of two age difference with 250 craters.

Application to new craters from Malin et al. (2006): Malin et al. [2] found 20 new small craters formed between May 1999 and March 2006 in two wide dusty regions of total area of $21.5 \cdot 10^6$ km^2. These craters were found due to new large dark halos formed due to the impacts. Two systematic wide-angle surveys made it possible to identify these features.

I ran my preferred spatial randomness tests for the set of 20 impact events from [2] within the area used for the survey. The median nearest neighbor distance test rejects spatial randomness with great confidence (Fig. 4). Thus the observed new craters do not form an accumulation population within the survey area.

The reason for this is rather natural. It is quite possible that the wide dark halos form only in some parts within the survey area, and the actual area where the craters are potentially detectable is smaller. The craters form an accumulation population is some sub-area of the survey area. Malin at al. mentioned this possibility and stated that "undersampling was substantially less than a factor of two".

I ran a series of Monte-Carlo simulations to figure out what part of the survey area should be removed to provide spatial randomness. I manually cut out some parts of survey area without actual craters and other new dark spots listed in [2], and ran the set of statistical tests for the remaining area and the actual set of craters. I tried to maximize the remaining area and simultaneously to keep the set of three statistics off 95% confidence rejection limits. I did not develop any algorithm searching rigorously for the maximum. However, from first tries I figured out how to "please" the statistics, and I believe, my result is close to the mathematical maximum. I found that the largest remaining area where the 20 craters may form an accumulation population is 1/2.9 of the survey area, that is $4.3 \cdot 10^6$ km^2.

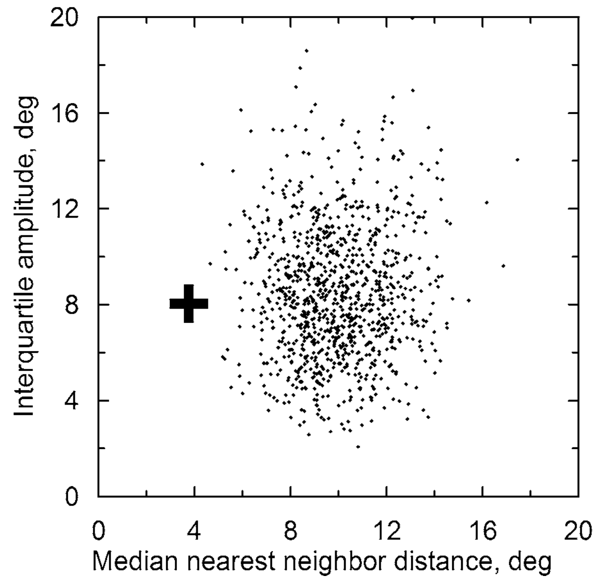

Fig. 4. The mean and the interquartile amplitude of nearest neighbor distance for 20 new craters from [2] (bold cross) and for 1000 pseudo-random simulated populations of 20 craters within the dusty areas surveyed in [2] (dots).

This estimate is an upper boundary. Formally, statistical inferences do not constrain the lower boundary. However, on the basis of assessment from [2] cited above, I believe that this upper boundary is a reasonable estimate for actual accumulation area.

Implications for the cratering rate: This area estimate gives the formal cratering rate of $6.7 \cdot 10^{-7}$ km^{-2}yr^{-1} for craters larger than 10 m in diameter.

Fig. 5 compares the size-frequency distribution of the new craters (for the corrected area) with G. Neukum's model for crater production function [8]. Here I excluded the largest crater: it falls well off the trend and is suspected to form much before 1999 (as noted in the catalog by M. Malin et al. accessible from http://www.msss.com). With this crater excluded, the larger two thirds of the population beautifully follows the 6.9 yr isochron, corresponding to the actual population age. This shows that the production function from [8] can be immediately applied for dating the youngest terrains on Mars with small craters larger than 20 m. W. Hartmann's recent iteration of the martian crater dating technique [9] coincides with [8] for the domain of small craters and young ages and is also fully applicable.

The smaller one third of the population shows a distinctive rollover, which is obviously due to observational bias, or atmospheric shielding, or both.

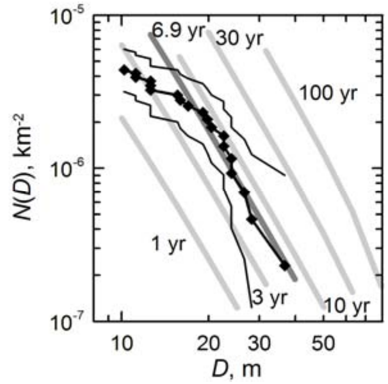

Fig. 5. Cumulative size-frequency distribution of 19 new craters from [2], calculated with the area correction factor of 1/3. The largest crater is excluded. Thin lines show formal 90% confidence interval calculated with (1). Thick gray lines represent Neukum isochrons calculated according to [8] with craters smaller than 100m excluded.

To illustrate the measured cratering rates, I list here the following numbers. For a typical HiRISE image, one crater larger than 20 m is formed on average each ~30,000 years; for a typical MOC image (672 x 8064 pixels, 4.5 m/pixel), each ~35,000 years, for a typical CTX image, each ~800 years. The total absence of small craters at a whole CTX image indicates that the crater retention age is younger than 1,900 years (90% confidence level).

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