

**THERMAL-MECHANICAL PROPERTIES OF POSSIBLE ICE DIAPIRS ON EUROPA.** J. A. Rathbun, *Department of Astronomy and Space Sciences, Cornell University, Ithaca, NY 14853, USA, rathbun@astrosun.tn.cornell.edu*, G. S. Musser, Jr., *Scientific American, New York, NY, USA*, S. W. Squyres, *Center for Radiophysics and Space Research, Cornell University, Ithaca, NY 14853, USA*.

Early Galileo results suggest evidence for ice diapirism on Europa [1]. An analytic thermal-mechanical model for diapirs has already been developed to address the problem of corona formation on Venus [2]. Here, we adapt this model to consider the conditions that would be required for ice diapirism to occur on Europa.

We model the evolution of an initially spherical “droplet” of warm ice as it buoyantly rises through a surrounding medium of cooler ice. The model includes the conductive cooling of the droplet, the mechanical deformation of the droplet as it approaches the surface and the thermal deformation of the droplet as it cools. This model is represented by three equations. In these equations, a caret denotes a non-dimensional quantity, lengths are scaled by the initial radius, velocities by the initial terminal speed and times by the time required for the droplet to move a distance equal to its own initial radius. The equations are

*Rise :*

$$\frac{d\hat{z}}{d\hat{t}} = \hat{v} = -\Gamma \frac{1}{\hat{r}f(\hat{z}/\hat{r}, E)} \quad (1)$$

*Cooling :*

$$\frac{d\Delta\hat{\rho}}{d\hat{t}} = -\frac{3}{2} \frac{1}{Pe_0} \frac{1+E^{-2}}{\hat{r}^2} \Delta\hat{\rho} \left\{ 1 + \left[ 1 + \left( \frac{4}{3\pi} Pe_0 |\hat{v}| E \hat{r} \right)^{2/3} \right]^{3/4} \right\} \quad (2)$$

*Deformation :*

$$\frac{d\hat{r}}{d\hat{t}} = \frac{1}{4E} \frac{\Gamma}{\hat{r}} \left\{ f \left( \frac{\hat{z} + \hat{h}}{\hat{r}}, E \right)^{-1} - f \left( \frac{\hat{z} - \hat{h}}{\hat{r}}, E \right)^{-1} \right\} - \frac{1}{3} \frac{\hat{r}}{\Delta\hat{\rho}} \frac{d\Delta\hat{\rho}}{d\hat{t}} \quad (3)$$

where  $\hat{r}$  is the radius of the droplet,  $\hat{z}$  is the depth of the droplet,  $\hat{t}$  is the time since the droplet formed,  $\hat{v}$  is the velocity of the droplet,  $\Delta\hat{\rho}$  is the density contrast between the droplet and the surrounding medium,  $E = h/r$  is the aspect ratio of the droplet and  $h$  is the semiminor axis of the now oblate spheroidal droplet. The Péclet number ( $Pe$ ) is the ratio of the thermal diffusion timescale ( $L^2/\kappa$ ) to the dynamical timescale ( $L/v$ ):

$$Pe = \frac{2vh}{\kappa} \quad (4)$$

where  $\kappa$  is the thermal diffusivity of the medium. Two modifications to Stoke’s flow are  $f$ , a drag amplification factor, and  $\Gamma$ , which arises because the motion of the droplet is controlled by the buoyancy of the boundary layer, whose temperature is lower than that of the central part of the diapir. Numerical

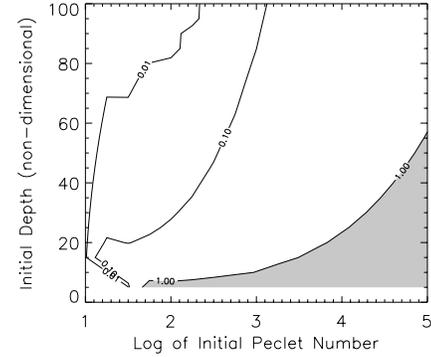


Figure 1: Cooling time divided by rise time. Only values greater than one yield physical diapirs.

simulations for corona formation have found  $\Gamma \approx 0.7$  [2]. The drag amplification arises because the presence of the lid distorts the velocity field from ideal Stokesian flow

$$f(x, E) \approx 1 + \frac{9}{8x} + \frac{323}{192x^2} + \frac{1}{4(x-E)^3}. \quad (5)$$

These equations depend on only two physical parameters, the non-dimensional initial depth of the droplet ( $\hat{z}_0$ ) and the initial Péclet number ( $Pe_0$ ). These equations have no analytical solution except in the limit of small Péclet numbers, which corresponds to pure conduction. In this limit, the droplet does not rise, it merely expands thermally and there is no diapirism. In general, these equations require numerical integration.

We carry out this numerical integration for a range of initial depths and initial Péclet numbers. The maximum non-dimensional initial depth is the maximum thickness of the  $H_2O$  layer on Europa, which we take to be 200 km, divided by the smallest droplet radius, which we take to be about 1 km. Our calculations include Péclet numbers from less than unity to  $10^5$ . For each set of Péclet number and non-dimensional depth, the rising time, cooling time, and radius at the cooling time, are calculated. The droplet rising time  $\hat{t}_{rise}$  is reached when the instantaneous rise timescale exceeds the elapsed time; *i.e.*, when

$$\frac{h(t)}{|v(t)|} > t \Rightarrow \hat{h} > \hat{t}|\hat{v}|. \quad (6)$$

Equivalently, the rising time is achieved when the time derivative of the instantaneous rise timescale exceeds unity. The cooling time  $\hat{t}_{cool}$  is reached when the thermal contrast has fallen to a fraction of its original value such that it is in the pure conduction limit. This criterion implies  $\Delta\hat{\rho} = 3^{-\frac{3}{2}}$ .

If the rise time exceeds the cooling time, the droplet is in the pure conduction limit, and diapirism will not occur. For much of the parameter space, this is the case (figure 1). We therefore have a constraint on the maximum initial depth of the droplets. Also, there is a minimum value of the initial Péclet number (about 30), lower than which the droplet is always in the pure conduction limit.

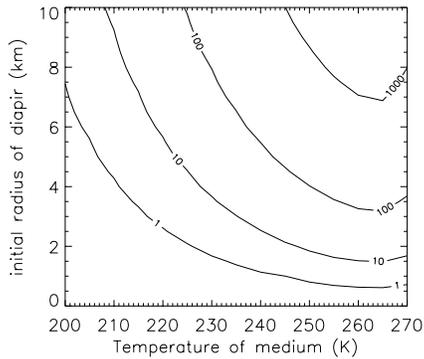


Figure 2: Péclet number

Péclet number is a function of the size of the droplet and the material properties of the medium. We calculate values of the Péclet number for conditions on Europa. The initial Péclet number is

$$Pe_0 = \frac{2U_0 r_0}{\kappa} \quad U_0 = \frac{6}{5} \frac{V_0 g \Delta \rho_0}{6\pi\eta r_0} \quad (7)$$

where  $V_0$  is the initial volume,  $U_0$  is the initial velocity,  $\Delta\rho_0$  is the initial density contrast, and  $\eta$  is the viscosity. Putting these two formulas together

$$Pe_0 = \frac{8}{15} \frac{g r_0^3 \Delta \rho_0}{\eta \kappa}. \quad (8)$$

The thermal diffusivity is given by  $k/\rho c_p$ , where  $k$  is the thermal conductivity and  $c_p$  is the specific heat capacity. The density contrast is due to a temperature difference between the surrounding medium and the droplet, since there is likely only one phase of ice (Ice I) present on Europa,

$$\Delta\rho = \alpha\rho\Delta T \quad (9)$$

where  $\alpha$  is the volumetric thermal expansivity and  $\Delta T$  is the temperature contrast between the surrounding medium and the droplet. Putting these in equation 8 yields

$$Pe_0 = \frac{8}{15} \frac{g r_0^3 c_p \alpha \rho^2 \Delta T}{\eta k} \quad (10)$$

All of the parameters in this equation, with the exception of  $r_0$ ,  $g$  and  $\Delta T$  are properties of the medium, and for ice they all vary predictably with temperature. For a Newtonian rheology, we take the viscosity to be

$$\eta(\text{Pa s}) = 10^{14} \exp\left[25.2 \left(\frac{273}{T(\text{K})} - 1\right)\right] \quad (11)$$

[3]. Tabular data were used to describe the volumetric thermal expansivity [4], the thermal conductivity [5], and the specific heat capacity [5]. We assume the temperature of the droplet to be the melting temperature of ice. The Péclet number therefore reduces to a function of only two parameters, the initial radius of the droplet and the temperature of the medium. Contours of Péclet number as a function of these parameters are shown in figure 2.

From figure 1 we see that each initial Péclet number corresponds to a maximum non-dimensional initial depth of the droplet. We can incorporate this information with the data in figure 2 to show maximum initial depth (in km) as a function of initial radius of the droplet and temperature of the medium (figure 3). With Galileo data that will be released soon, the final radii of the diapirs can be approximated from the dimensions of the surface deformation they cause, yielding constraints on the depth of formation of the diapirs and the temperature of the subsurface ice on Europa.

If ice diapirism is occurring on Europa, the diapirs must be originating from a position no deeper than the base of the ice layer, whether it is underlain by liquid or by rock. The initial depths of the diapirs in our calculations are on order a few tens of kilometers (figure 3). If the total thickness of  $\text{H}_2\text{O}$  on Europa is 100-200 km, then the presence of small diapirs could be interpreted as evidence for a liquid region beneath the ice.

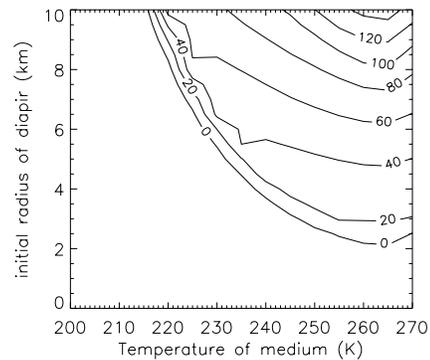


Figure 3: Initial Depth (km)

**References:** [1] Head, J.W. et al. (1997), *BAAS* vol. **29** no. **3**, 983. [2] Musser, G.S. and Squyres, S.W. (1997), *J. Geophys. Res.* **102**, 6581. [3] Hillier, J. and Squyres, S.W. (1991), *J. Geophys. Res.* **96**, 15,665. [4] Eisenberg, D. and Kauzmann, W. (1969) *The Structure and Properties of Water*, Oxford University Press. [5] Dorsey, N.E. (1940) *Properties of Ordinary Water-Substance*, Reinhold, New York.