FRAC TAL PLANETS: A GENERALIZED SURFACE ROUGHNESS MODEL FOR REMOTE SENSING. M. K. Shepard and B. A. Campbell, 1Department of Geography and Earth Science, 400 E. Second St., Bloomsburg University, Bloomsburg, PA 17815, mshepard@bloomu.edu, 2Center for Earth and Planetary Studies, National Air and Space Museum, Smithsonian Institution, Washington D.C. 20560, campbell@ceps.nasm.edu

Remote sensing plays an enormous role in our understanding of the geology of solid bodies in the solar system. Inferences based on remote sensing of planetary surfaces require the use of two types of models: one for the scattering and emission of electromagnetic radiation from matter; and one which quantitatively describes the surface composition and roughness of geologic surfaces. Here, we use the term roughness to indicate any deviations, statistical or otherwise, from a planar surface. In order for our geologic inferences to be soundly based, each of these models must be realistic, i.e., based upon physical principles and/or empirically observed behavior, and general enough to encompass most all types of surfaces one is likely to encounter. While the interaction of electromagnetic waves and matter has been reasonably well characterized for over a century, realistic quantitative models of geologic surface roughness have only recently been developed [1,2,3]. In this abstract, we advocate the use of fractals as a simple, quantitative, and general model for geologic surface roughness. Fractals implicitly assume that the surface is “noise” and therefore do not model deterministic structures well. However, in areas of homogeneous surface structure, i.e., areas where a single geologic “theme” is randomized to form the landscape, they provide a robust estimator of the scale-dependent surface roughness. We briefly describe fractal statistics, and then illustrate their utility for remote sensing by computing a shadowing function for fractal surfaces observed at nadir. To demonstrate the generality of fractal surface models, we compare the shadowing behavior of fractal surfaces to the shadowing behavior of three published surface roughness models, including the well known Hapke [4] model.

The term “fractal” is often taken to be synonymous with “scale-invariance”, the well known property that many geologic phenomenon look the same at all scales. However, for topography and surface roughness, scale-invariance is rare. More commonly, the vertical and horizontal axes of a surface scale at different rates. Quantitatively, this behavior, called “self-affinity”, can be described by two relations:

\[ \sigma^2 = aL^{2H} \]  
\[ \nu^2 = b(\Delta x)^{2H} \]

(1)

where \( \sigma^2 \) is the variance of surface heights about the mean on a profile of length, \( L \); \( \nu^2 \) is the mean square deviation of heights between two points separated by a distance \( \Delta x \) (also referred to as the Allan variance, variogram, or structure function); \( a \) and \( b \) are constants of proportionality; and \( H \) is a parameter variously referred to as the Hurst exponent or Hausdorff measure [3,5]. For natural surfaces, \( H \) is observed to be restricted to 0<H<1. The behavior described in Eq. (1) is an empirical description and it is not clear why these relationships are valid. However, numerous studies have found that these (or related derivative formulations) hold over scales ranging from micrometers to kilometers [6,7,8]. Note that a fractal surface model requires two roughness parameters instead of the single parameter found in most (if not all) surface models used for remote sensing. Two parameters allow one to specify the roughness at a single scale (the constant of proportionality) and characterize how the roughness changes with scale (the parameter \( H \)).

We examine the shadowing properties of fractal surfaces for two reasons. First, surface self-shadowing is an important component of photometric and radar scattering models. Second, comparing the shadowing behavior of a range of fractal surfaces with published models is a relatively simple way to compare surface “appearances” and provides a measure of the generality of a fractal model for planetary surfaces. To date, no rigorous analytical model of self-shadowing on a fractal surface exists. However, fractal surfaces are relatively easy to synthesize and ray-trace, thus providing a method for determining their shadowing properties.

Using published spectral methods [2,3], we generated ten each of fractal surfaces with Hurst exponents of 0.1, 0.3, 0.5, 0.7, and 0.9, each with root-mean-square (RMS) slopes at the smallest model scale of 10, 20, 30, 40, and 50 degrees. Each surface was viewed at nadir (emission angle = 0) and illuminated at 10 degree intervals from 10 to 80 degrees incidence. The mean fraction of surface illuminated (not in shadow) from the 10 similar surfaces (same \( H \), RMS slope, and incidence angle) was taken as the best estimate of the shadowing behavior. Based on the observed shadowing behavior, we have empirically derived the following shadowing function for a fractal surface, given its Hurst exponent, \( H \), RMS slope at the smallest scale, \( \theta_0 \), and incidence angle, \( i \), (nadir observations only):

\[ S = 1 - \text{SUM} \{ \frac{2}{3.16} \left[ 0.5*\text{erfc}(1/\sqrt{2}\tan(i)\tan(\theta_0)\ln^{1.5}) \right] \} \]

(2)

where \( S \) is the fraction of surface illuminated, SUM indicates a summation over integer “n” from 1 to infinity, and \( \text{erfc} \) is the error function complement. (In what follows, we will utilize the parameter \( s_0 = \tan(\theta_0) \). In practice, the summation only needs to be carried out to \( n=6 \) to be within \(-1\% \) of the actual value. We have found Eq.(2) to be within \(-5\% \) or better of the synthetic ray-traced behavior. Figure 2...
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We compared the shadowing function of the synthetic fractal surfaces to three published analytic shadowing functions for randomly rough surfaces; Hapke [4], Smith [9], and Wagner [10]. Hapke’s shadowing function is embedded in a much larger photometric roughness correction function for any combination of incidence, emission, and phase angles, while the other two models assume nadir observation only. Each model assumes similar random surface morphology characterized by Gaussian statistics. Hapke assumes a surface characterized by a mean surface slope parameter, commonly referred to as theta-bar [4], symbolized here by $s_H = \tan(\theta)$, Smith utilizes a single RMS slope parameter, here symbolized by $s_S$, while Wagner utilizes a composite slope parameter, here symbolized by $s_W$, consisting of the ratio of the standard deviation of heights about the mean to the autocorrelation length of the surface. Comparisons to fractal surface shadowing behavior reveal the following approximate relationships:

$$s_S \sim s_0 \quad \text{for } H = 0.7$$

(3)

$$s_W \sim 1.3s_0 \quad \text{for } H = 0.1$$

(4)

$$s_H \sim 0.7s_0 \quad \text{for } H = 0.5$$

(5).

The implication we draw from Eqs. (3), (4) and (5) is that a fractal surface model can reproduce the shadowing behavior of the surfaces assumed in these three models. Further, it appears that photometric or radar scattering models which utilize the shadowing behavior of these three surface models will only be valid for a restricted set of surface statistics.

In conclusion, we have empirically demonstrated that the shadowing characteristics of several common surface roughness models can be represented as specific cases of shadowing on a fractal surface. Because natural surfaces obey a wide range of fractal statistics, surface roughness models which do not include a scale dependence will have limited applicability. We propose that the next generation of photometric and radar scattering models incorporate fractal surface behavior in order to be as realistic and general as possible.

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References.