

NEPTUNE'S 2:1 ORBITAL RESONANCE IN THE KUIPER BELT. R. Malhotra, *Lunar and Planetary Institute, Houston, TX.*

Resonant orbits, i.e., of orbital period in a small integer ratio, $p : q$, with that of a planet, make a closed loop with q -fold symmetry in a reference frame co-rotating with the planet's mean motion. Long term stable resonant orbits may librate with a finite amplitude about the exact resonant orbit, tracing a pattern in the rotating frame which retains the q -fold symmetry and which usually exhibits a mirror symmetry about the Sun-planet axis. A well-known example is Pluto's orbit which librates about the 3:2 resonance with Neptune (see, for example, Malhotra & Williams, in *Pluto and Charon*, Univ. of Arizona Press, Tucson (1997)). However, eccentric orbits librating in Neptune's 2:1 resonance in the Kuiper Belt do not exhibit the usual mirror symmetry with respect to the Sun-Neptune line, as noted in Malhotra (*AJ* 111:504-516 (1996)). This is illustrated in Figure 1.

Figure 2 shows the location of the libration center and the range of stable librations of the resonance angle, $\phi = 2\lambda - \lambda_N - \varpi$. (Here λ and λ_N are the mean longitudes of a KBO and Neptune, respectively, and ϖ is the longitude of perihelion of the KBO.) Note that when a KBO is at perihelion, $\lambda = \varpi$ so that $\phi = \varpi - \lambda_N$ measures the KBO's longitude of perihelion relative to Neptune's mean longitude. For small eccentricities, the libration zone is centered 180 degrees away from Neptune (and has mirror symmetry about the Sun-Neptune axis); but for eccentricities exceeding ~ 0.03 , a bifurcation occurs and two separate libration zones exist whose centers are increasingly closer to Neptune (at $\phi = 0$) for increasing values of the eccentricity. Note that the librations are not symmetric (i.e., not sinusoidal) about the centers. For eccentricities greater than ~ 0.1 (perihelion distance $q < 40$ AU), the perihelion librates in a range of ecliptic longitudes that overlaps those of the 3:2 and the 5:3 resonances (cf., Malhotra, 1996). (We also note that this range of ecliptic longitudes encompasses the L4/L5 Lagrange points of Neptune as well.)

Although the stable libration zones of different resonances are distinct and well separated in orbital element space or phase space, they overlap greatly in observational parameter space.

Consider the observational accuracy needed to determine the association with a specific orbital resonance. Under the most favorable circumstances, observations obtained at opposition on two consecutive nights may have positional

accuracy ~ 1 arc-sec, and yield apparent motion of a KBO at a resolution of 0.04 arc-sec/hr. This is insufficient to distinguish between different resonances. To illustrate, consider the apparent motion $\dot{\theta}$ of a KBO located at a heliocentric distance r AU and observed from Earth at opposition. Assuming that the KBO is at its perihelion,

$$\dot{\theta} = 148 \text{ arc-sec/hr} \frac{1 - \sqrt{2/r - 1/a}}{r - 1}$$

where a is the semimajor axis (in AU). The apparent motions at perihelion at 30 AU of objects in the 3:2, 5:3 and 2:1 resonances are 4.07, 4.04, and 4.01 arc-sec/hr, respectively. These differences amongst these apparent motions are below the resolution of single-opposition observations, but they lead to positional differences of several minutes of arc over the 1 year time interval between successive oppositions. However, allowing for the possibility that a KBO may be detected not exactly at perihelion and not exactly at opposition, the apparent motion and positional differences will, in general, be even smaller than the numbers quoted above. Thus, recovery of a KBO at least 2, but usually several, successive oppositions is necessary to allow refinement of the orbit at a level sufficiently well to identify associations with Neptune's resonances.

The near-degeneracy of ecliptic-longitude-near-perihelion and of the apparent-motion-near-perihelion of eccentric orbits at the different Neptune resonances explains in part the recent revisions of the orbits of two KBOs, 1997 SZ10 and 1996 TR66, whose current best-fit orbits have them as candidates for 2:1 librators (Marsden, 1998, *IAU Circular no. 7073*; URL <http://cfa-www.harvard.edu/iau/cbat.html>), but were previously identified as 3:2 and 5:3 librators, respectively

Finally, we note that long term orbital stability is very sensitive to initial conditions in the vicinity of orbital resonances. Secular effects due to the cumulative perturbations of the planets may "punch holes" in the stable libration zone shown in Fig. 2. This is abundantly illustrated in an accompanying paper (Malhotra, 1999) which shows that small differences, well below the threshold of observational errors, in the orbital elements of 1997 SZ10 and 1996 TR66 may place these KBOs in either very stable or very unstable orbits.

Neptune's 2:1 resonance: Malhotra

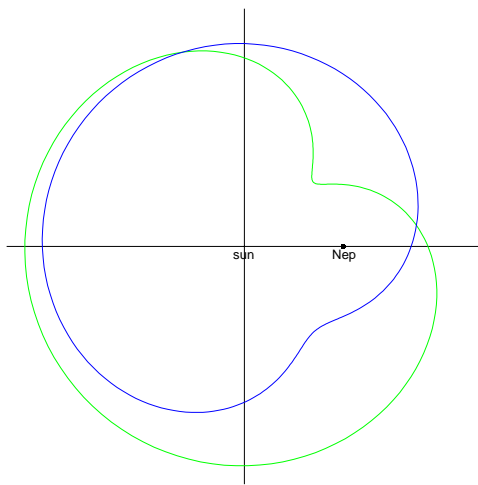


Figure 1: Examples of 2:1 resonant orbits (of zero libration amplitude) in a reference frame co-rotating with the mean motion of Neptune. Green indicates an orbit of eccentricity $e = 0.4$, blue an orbit of eccentricity $e = 0.3$, in the leading and trailing libration zones, respectively.

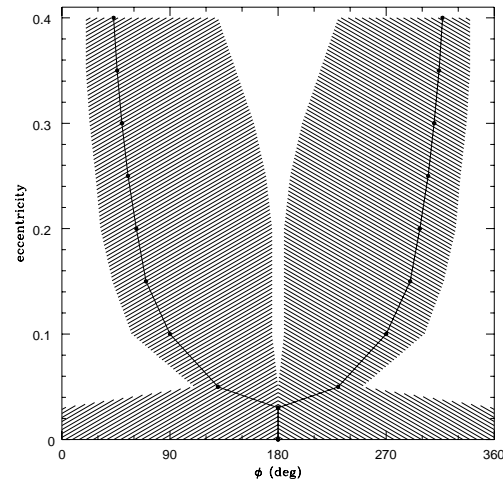


Figure 2: The dark line marks the exact resonant orbit, and the hatched region shows the zone of stable librations in the eccentricity-resonance angle parameter space, as determined from a planar, circular restricted three body model of the orbital perturbations due to Neptune on a Kuiper Belt object in the vicinity of the 2:1 orbital resonance. The resonance angle $\phi = 2\lambda - \lambda_N - \varpi$ measures the longitude of perihelion relative to Neptune's mean longitude. Note that, for long term stability, we must also consider additional effects, such as Neptune's non-zero eccentricity, coupling with a third degree of freedom (inclination), and perturbations from the other giant planets; in general, these shrink and 'punch holes' in the shaded zone.