Dunes on Mars, 'Venus', Earth, and subaqueous ripples: a scaling law for their elementary size

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Dunes and bedforms are observed in considerably diverse environments: aeolian dunes of sand as well as snow, dunes under water, but also dunes on Mars or Titan. Summarising our work published in [1], we would like to show in this paper that, although the fluid, the grains and the mode of sediment transport can be different in these various situations, the understanding of the size at which a flat bed destabilises – denoted below as the 'elementary' size – can be thought of in a common manner, i.e. independently of the above geological objects. Reasoning in terms of instability mechanism, we have then been able to evidence a scaling law for this elementary size, valid from centimetric subaqueous ripples to 600 m long Martian dunes.

A flat sand bed submitted to a turbulent flow spontaneously generates undulations due to the combined action of sediment transport and hydrodynamics. Because of the large separation between the hydrodynamical and dune growing time scales, these two aspects can be treated independently of each other. The amount of sediment that can be transported by a steady flow per unit time is finite above a certain threshold $u_{\rm th}$ and is a function of the shear velocity u_*. We call $q_{\rm sat}$ the corresponding 'saturated' sediment flux. Be the flux out of its saturated value, it relaxes toward equilibrium within some time or distance, depending on the nature of the perturbation. We call $\ell_{\rm sat}$ this saturation length. Consider now a wavy bottom of wavelength λ. The basal shear stress is also modulated at this wavelength, but not in phase with the bottom: the shear maximum is reached before the bottom crests. Because turbulence does not provide any intrinsic length scale, this up-flow shift is some (small) fraction of λ. As for a given shear, transport equilibrium is only reached after some lag $\ell_{\rm sat}$, the position at which the sediment flux is maximum can then be located either on the stoss side of the bottom bumps when λ is large, or on their lee side when λ is small. This position is crucial for the dune formation issue as it separates the erosion from the accretion zone. When erosion occurs at the crest, the bumps shrink, meaning that the bed is stable with respect to perturbations at this wavelength. Conversely, bumps can grow if their crest is in the deposition region. The mathematical linear stability analysis of these processes leads to a growth rate which is positive for all large wavelengths down to a cut-off value λ_c, with a maximum at λ_m. As expected, both λ_m and λ_c are proportional to $\ell_{\rm sat}$, which is the only length scale of the problem. Because it corresponds to the most unstable mode, we call λ_m the elementary size of this instability.

So far, we have not specified how $\ell_{\rm sat}$ is related to the grain, fluid and flow properties. In fact, there is not a single mechanism limiting the time and length of saturation transient but several, e.g., (i) the ejection of the grains from the bed, (ii) the grain inertia that controls the length needed for one grain to reach its asymptotic trajectory, (iii) the fluid inertia that controls the length needed for the wind to readapt to a change of the flux. One should consider for $\ell_{\rm sat}$, the *slowest* process i.e. the largest relaxation length amongst the modes of relaxation, which can be different depending on the value of $u_*/u_{\rm th}$ as well as that of the grain to fluid density ratio ρ_s/ρ_f. Initially motivated by the aeolian case at wind speeds reasonably above the threshold, we have focused on the grain inertia mechanism (ii), for which the typical length scale is the drag length $\ell_{\rm drag} = \frac{\rho_s}{\rho_f} d$, where d is the grain diameter. In order to test this analysis, the idea is then to plot the elementary size *vs* $\frac{\rho_s}{\rho_f} d$ for situations as various as possible.

The measure of the elementary size is easy in the lab, for a bed of grains submitted to a water flow. In this case, the emergence of the pattern can be well controlled, and its λ_m deduced from a correlation or a Fourier transform technique. The data plotted in figure 1 correspond to different grain diameters [2,3,4]. For dunes in the field, the measure is less strict. In particular, isolated small barchans are not necessarily at the elementary size: λ_m is a non-local measure of the typical initial wavelength of the dune pattern. Fortunately, large barchans often present corrugated flanks with superimposed undulations, especially after few days of storm or unusual winds. In fact, a large dune is not far from a flat sand bed, and these rows of undulations are precisely emergent pattern at the elementary size [5]. Although less accurately, these superimposed undulations can also be seen on photos of Martian dunes [6]. From the analysis of 'microscopic' photos taken by the rovers at the

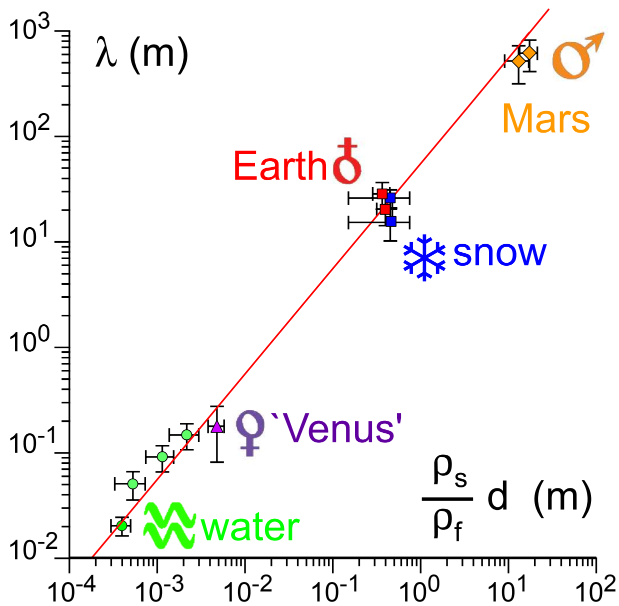

Figure 1: Scaling law for the elementary size of dunes. Symbols: aeolian sand dunes in Atlantic Sahara (♁); aeolian snow dunes in Antartica (∗); Martian dunes (♂); bedforms in the Venus wind tunnel (♀); subaqueous ripples (♒).

surface of the planet, as well as theoretical calculations on the threshold for transport in the Martian conditions, one can deduce that the aeolian grains on Mars are rather small, with a diameter slightly smaller than 100μm. The plot of figure 1 is completed by two less documented situations: snow dunes in Antarctica [7,8], as well as bedforms obtained in a high pressure CO_2 wind tunnel reproducing conditions close to the Venus atmosphere [9]. Interestingly, the two aeolian Earth data point (sand and snow dunes) coincide on the plot although the very different values of d and ρ_s, but which compensate. Overall, the scaling law $\lambda \propto \frac{\rho_s}{\rho_f} d$ is well verified over almost five decades.

Given the few information available on Titan [10] and following this scaling law, one expects the elementary size there to be of the order of one or few meters. This size is much too small to be detectable on the Cassini radar images showing the dunes [11], which are presumably of 'giant' type, like terrestrial star dunes. Aeolian sand ripples are another pattern that should not be included in this law, as their formation mechanism is of different nature – a screening rather than hydrodynamical instability. Finally, we shall emphasize again that we do not claim to capture all the dependences on this plot, but that $\frac{\rho_s}{\rho_f} d$ is the dominant scaling factor. As mentioned above, we expect subdominant dependences on the wind speed, on finite size effects, etc, related to other relaxation mechanisms.

This scaling law at hand, one can finally address the more speculative issue of the time scales of Martian dunes. The linear stability analysis predicts that the dune growth rate scales with $\ell_{\text{sat}}^2/\bar{Q}$, where $\bar{Q}$ is the mean sand flux. Its value is difficult to estimate because we do not know the fraction of the time for which Martian winds are above the transport threshold. Assuming that this is the case few days per year, we end up with a factor of the order of 10^5 between terrestrial (a week) and Martian (tens of centuries) dunes. Similarly, one gets a factor 10^{-4} between the propagation velocities. As some satellite high resolution pictures definitively show some evidence of aeolian activity – e.g. avalanche outlines – it may then well be that the martian dunes are fully active but not significantly at the the human scale.

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