MODELING 2-D DUNE INTERACTIONS. Serina Diniega[1] and Karl Glasner[2], [1]Program in Applied Mathematics, The University of Arizona (Tucson, AZ 85721, serina@math.arizona.edu), [2]The University of Arizona

Introduction: This research aims to understand, through the analysis of simple equations and numerical simulations, the general trends and processes involved with transverse dune formation and evolution. Towards this end, the linearized version of the two-dimensional dune evolution equations derived by [1-3] have been analyzed and numerically simulated. Relationships between different environmental and dune parameters have been derived and/or calculated.

Additionally, this research seeks to understand the different types of dune interaction and the parameter regimes in which these interactions occur. This will aid in developing a macroscale model of dune field evolution, as models based on the movement of sand onto and over individual dunes are unfeasible.

The Model: To evolve a given sand (over rock) topography into a dune, the model first considers the interactions between the wind and sand by calculating the shear stress exerted by the wind onto the surface, via the Jackson-Hunt equation [4]. Corrections are applied to account for the separation of airflow and formation of shadow zones [5] to the lee of sharp topography (like dune crests).

This shear stress is then used to calculate the local maximal sand flux. The actual sand flux attains this amount after a spatial delay: the saturation distance. Additionally, the type of substrate the sand flux is moving over (non-erodible bedrock vs. sand) is taken into consideration.

The sand flux is then related back to changes in the dune profile, via the Exner equation. Additionally, a diffusion term is included in the dune sand evolution equation to mimic avalanching effects.

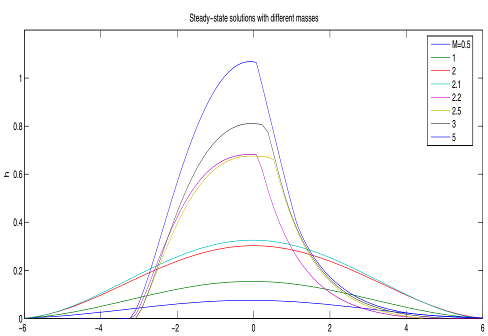

Figure 1: Plot of steady-state dune profiles for different initial system masses (height axis exaggerated). Notice the abrupt transition in shape: below the threshold mass dome dunes with smooth profiles form; above this mass crests form and the dune's aspect ratio increases.

Isolated Dunes: Steady-state dune profiles were found for a range of initial dune masses. Two types of dunes were observed to form: dome dunes when the initial mass was small and crested dunes above a mass threshold (figure 1). The crested dunes, corresponding to cross-sections of transverse or barchan dunes, were also observed to exhibit an inverse relationship between dune velocity and height, as expected (figure 2).

Additional analysis was done using a reduced dimensional model: the dune was approximated as a simple piecewise linear function. Thus, the profile of the dune could be expressed as a function of three variables: total volume of the dune, its windward slope, and the position of the dune's crest. Assuming that volume is conserved, the time derivative of the two remaining dynamic variables (windward slope and position) can be found by projecting the time derivative of the dune profile (calculated via the Exner equation in the full model) onto the space spanned by the partial derivatives of the function with respect to the windward slope and position.

The phase portrait of the windward slope (figure 3) shows that there is a stable equilibrium windward slope value (dependent on volume) that the dune profile naturally moves towards. Additionally, when the dune has this slope, the velocity of the simplified dune is inversely proportional to its height (figure 4).

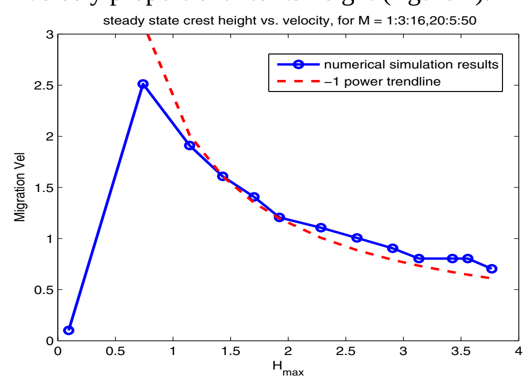

Figure 2: Plot of crest height vs. migration velocity for steady-state dune profiles with different masses. The trendline illustrates the expected inverse relationship.

Interacting Dunes: Due to the inverse relation between dune size and velocity, if a small dune is upwind of a large dune then an interaction will occur: the smaller dune's shadow zone will catch up to the larger dune and begin to steal sand. However, as the sand is stolen from the leading (initially larger) dune, the downwind dune shrinks in size/moves faster and the upwind dune grows/moves slower. Depending on the interplay between the changes in size (and speed) of the two dunes, several different types of interactions can occur: (1) Coalescence: the upwind dune catches up to the downwind dune faster than this dune can lose sand/migrate faster, and the two dunes merge into one dune; (2) Run Away: the downwind dune, by shrinking and thus migrating faster, manages to escape from the (now larger) upwind dune, resulting in two dunes with different speeds and sizes; (3) Disappear (or Attempted Run Away): the downwind dune manages to escape, but is too small to remain stable. The result is one dune with most of the sand, as the downwind dune separates, flattens, and then disappears.

It was hypothesized that the different interactions could be predicted based on the model dunes' relative and absolute sizes: for small dunes, the actual dune sizes matter as the saturation length is comparable with the dune sizes and will influence the interactions; once the two dunes are large enough, the dune mass ratio should be the deciding factor . This was found to be the case (figure 5); these results are similar to the three dimensional model results found by [6].

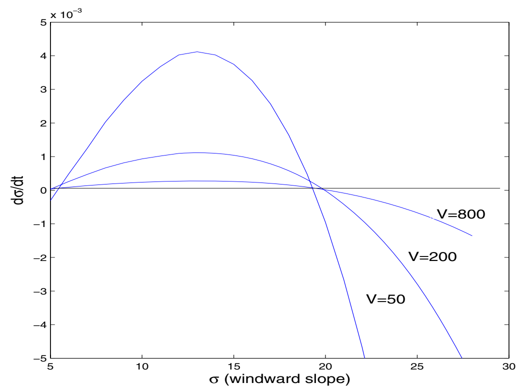

Figure 3: Phase portrait of the windward slope (in degrees), computed with the reduced dimension dune model. Note that there are two equilibrium slopes for each volume: the lower value (close to 0) is unstable and the upper value (around 20) is stable.

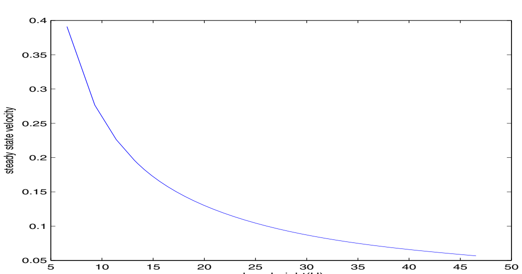

Figure 4: Plot of dune height vs. calculated dune velocity once the equilibrium slope is achieved. Note that it exhibits the expected inverse relationship.

Summary: This model appears to capture the main dynamics of dune formation and evolution: sand transport, avalanching, and separation of air flow/shadow zone, yielding physically realistic dune profiles and relations in the study of isolated dune structures. By applying the model to two dune structures, different possible interactions have been categorized and were shown to depend on the two dunes' initial masses (and only on the mass ratio for sufficiently large dunes).

Work is ongoing to understand analytically why certain interactions occur for a particular mass ratio between the dunes. Additionally, studies concerning interactions between more than two dunes will be completed to better understand and allow simulation of dune dynamics in large dune fields.

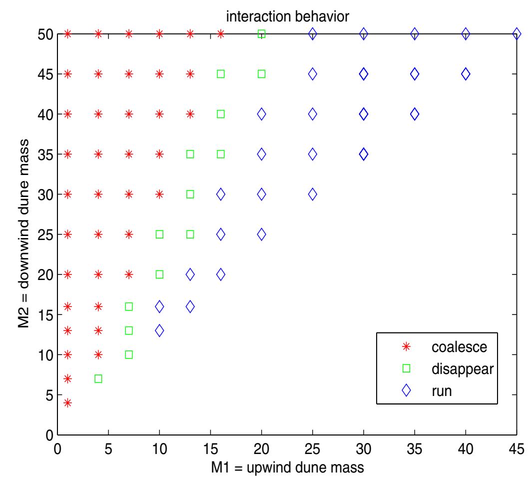

Figure 5: Plot showing the interactions observed in the simulations of two dunes, as a function of their initial masses. The axes are of the upwind dune's initial mass vs. the downwind dune's initial mass (larger). Notice that the regimes of interaction behavior (red stars = coalescence, green squares = disappear, blue diamonds = run away) depend only on the mass ratio (slope of regime boundaries) for larger dunes.

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