

**WHAT IS THE YOUNG'S MODULUS OF ICE?.** F. Nimmo, *Dept. Earth and Space Sciences, University of California Los Angeles, (nimmo@ess.ucla.edu).*

The Young's modulus  $E$  of ice is an important parameter in models of tidal deformation [1, e.g.] and in converting flexural rigidities to ice shell thicknesses [2, e.g.]. There is a disagreement of an order of magnitude between measurements of  $E$  in the laboratory (9 GPa) and from field observations ( $\approx 1$  GPa). Here I use a simple yielding model to address this discrepancy, and conclude that  $E = 9$  GPa is consistent with the field observations. I also show that flexurally-derived shell thicknesses for icy satellites are insensitive to uncertainties in  $E$ .

**Lab Measurements** Because ice may creep or fracture under an applied stress, it behaves elastically only if the loading frequency is high and stresses are small. Lower temperatures expand the parameter space in which elastic behaviour is expected. The most reliable way of determining  $E$  in the laboratory is to measure the sound velocity in ice and thus derive the elastic constants. The values of  $E$  found are consistently about 9 GPa [3; 4].

**Field Measurements** Field techniques rely on observing the response of ice shelves to tidal deformation [5]. In this case, loading frequencies are much lower ( $\sim 10^{-5}$  Hz) and stresses much higher ( $\sim 1$  MPa). Fractures are commonly observed, and creep is also likely to occur [6]; thus, not all of the shelf may respond in an elastic fashion.

The length-scale of the response of an ice shelf to tidal deformation is determined by the parameter  $\beta$  [7], where:

$$\beta^4 = \frac{3\rho g (1 - \nu^2)}{ET_e^3}. \quad (1)$$

Here  $\rho$  is the density of the sea,  $g$  is the acceleration due to gravity,  $\nu$  is Poisson's ratio and  $T_e$  is the effective elastic thickness of the ice shelf. The effective elastic thickness is defined as the thickness of a purely elastic plate which would produce the same response to loading as the actual ice shelf. Note that  $T_e$  will be less than the total ice shelf thickness  $h$  if ductile creep or fracture are important [8].

Field measurements of ice shelf deformation allow  $\beta$  to be determined [7]. Inspection of equation 1 shows that in order to derive  $E$ , a value for  $T_e$  must be assumed. One approach is to assume that the elastic thickness  $T_e$  is the same as the total ice shelf thickness  $h$ , which may be measured directly. Doing so results in values for  $E$  which are significantly smaller than the lab values. For instance, Vaughan [7] concluded that the effective Young's modulus from a variety of tidal deformation studies was  $0.88 \pm 0.35$  GPa. [9] used radar observations of tidal flexure to conclude that  $E$  varied in both time and space, from 0.8 to 3.5 GPa. They ascribed the variation to viscoplastic effects. Similarly, [10] observed a time-delay between tidal forcing and ice-shelf response, which is also probably due to viscous effects.

Tension fractures are inferred to form at both the top and bottom of ice shelves due to tidal flexure, and may be tens of metres deep [11, p. 204]. The presence of these fractures will reduce the effective elastic thickness  $T_e$  of the ice shelf.

Furthermore, for typical curvatures of  $10^{-6} m^{-1}$  and shelf thicknesses of 1 km (see Table 1), the maximum elastic stresses will be  $O(1)$  MPa. These stresses are comparable to or exceed the likely elastic limit of ice [12] and suggest that much of the ice shelf will deform in a ductile rather than elastic fashion (see also [9]). It would therefore be surprising if the simple assumption that  $T_e = h$  were correct. An alternative [5; 13] is to assume that the *effective* elastic thickness of the ice shelf is some fraction of the total shelf thickness. Doing so results in a larger value of  $E$ ; equation 1 shows that reducing  $T_e$  to 50% of the ice shelf thickness results in an eight-fold increase in  $E$ . This increase is sufficient to bring the field results into agreement with the laboratory measurements.

**Yielding Model** Below I develop a simple model to show the effect of ice yielding on  $T_e$ . It will be assumed that the ice is an elastic-perfectly plastic material [14] where the stress increases linearly with strain up to a particular yield stress,  $\sigma_y$ , and remains constant thereafter. The perfectly plastic regime represents the area in which either fracturing or ductile flow occurs.

The first moment of the stress-depth relationship for this rheology may be used to infer the effective elastic thickness  $T_e$  of the material [8]. Assuming that stress profile is symmetrical, it can be shown that

$$Te^3 = \frac{\sigma_y(1 - \nu^2)}{EK} \left( 3h^2 - 4 \left[ \frac{\sigma_y(1 - \nu^2)}{EK} \right]^2 \right) \quad (2)$$

where  $K$  is the curvature and it is assumed that  $2\sigma_y(1 - \nu^2) < EK h$ . If this inequality is not satisfied, the whole shelf behaves elastically and  $T_e = h$ .

The yield stress of ice is not well known, and probably varies with both temperature and strain rate. [12] argues that a yield stress of 0.1 MPa is appropriate for glacier ice. [15] show that the yield stress is independent of pressure, but depends on temperature and strain rate. Extrapolating their results to a strain rate of  $10^{-8} s^{-1}$  suggests yield stresses of 0.6 MPa at  $-5^\circ C$ . A fracture depth of 10-100 m implies an effective yield stress of 0.1-1 MPa. I assume that  $\nu=0.3$ ,  $\rho = 1000 kg m^{-3}$ ,  $g = 9.8 m s^{-2}$  and generally use  $E=9$  GPa.

Table 1 lists the observational data from [7]. Rather than assuming that  $T_e = h$ , column 4 lists the implied value of  $T_e/h$  assuming that  $E=9$  GPa and using equation 1. Column 5 lists the value of  $T_e/h$  obtained using the yielding model (equation 2). The agreement is generally quite good (except for Jakobshavn) and shows that yielding is a valid way of explaining the observations, and is consistent with the laboratory-derived value of  $E$  (9 GPa).

In summary, the observed flexure at ice shelves can be reconciled with the laboratory-determined values of  $E$ , if some fraction of the shelf experiences yielding (either fracture or creep). Yielding is expected to occur based on the likely behaviour of ice, and a simple elastic-plastic model shows that the amounts of yielding required are reasonable.

## REFERENCES

location	$\beta \times 10^{-4}$ $m^{-1}$	$h$ m	$T_e/h$ (obs.)	$T_e/h$ (theor.)
Rutford	$2.4 \pm 0.4$	2000	$0.48 \pm 0.09$	0.47
Ronne	$5.4 \pm_{1.5}^{2.7}$	700	$0.47 \pm_{0.27}^{0.15}$	0.46
Doake	$3.4 \pm 0.7$	1000	$0.61 \pm 0.15$	0.52
Bach	$11.0 \pm_{1.7}^{2.5}$	200?	$0.63 \pm_{0.16}^{0.11}$	0.54
Ekstrom	$11.0 \pm 0.7$	150-200	$0.72 \pm 0.05$	0.51-0.46
Jakobs- havn	$17.0 \pm_{2.0}^{4.0}$	450-800	$0.16 \pm_{0.04}^{0.02}$	0.47-0.38

Table 1: The values of  $\beta$  and  $h$  are obtained from [7].  $T_e/h$  (obs.) is the value inferred from equation 1 using the observed  $\beta$  and  $h$  and assuming that  $E=9$  GPa.  $T_e/h$  (theor.) is the value inferred from equation 2 assuming  $\sigma_y=0.3$  MPa.

**Icy satellites** The ice shells of outer solar system satellites differ in several respects from terrestrial ice shelves. Firstly, strain rates are lower: around  $10^{-10} s^{-1}$  on Europa, and less elsewhere. Secondly, surface temperatures are very much lower (typically 100-120 K), indicating that the ice may deform in a brittle fashion. Thirdly, the ice shells are probably 10's-100's km thick, implying that creep, rather than fracture, will occur towards the base of the ice shell. It is therefore more appropriate to use the yield-strength envelope (YSE) approach of [16; 8]. In this model, the near-surface ice deforms in a brittle manner, that at the base deforms in a ductile fashion, and that near the mid-plane elastically. The YSE approach allows us to convert a measurement of rigidity into a shell thickness, given a value for  $E$ . The conversion depends on the strain rate and curvature of the feature.

As an example, we will use an apparently flexural feature on Europa studied by [2]. We assume a conductive ice shell in which the thermal conductivity varies as  $567/T$  and the top and bottom temperatures are 105 K and 260 K, respectively. The values of  $\rho, g$  and the coefficient of friction are  $900 kg m^{-3}$ ,  $1.3 m s^{-2}$  and 0.6, respectively. We will use the grain-boundary sliding ( $n=2.4$ ) rheology of [17] which is grain-size independent. The strain rate is assumed to be  $10^{-15} s^{-1}$ .

[2] obtained a flexural parameter  $\beta$  of  $6.25 \times 10^{-5} m^{-1}$  and a maximum curvature of  $7.5 \times 10^{-7} m^{-1}$ . Assuming a Young's modulus of 9 GPa, equation 1 gives  $T_e=2.8$  km and the YSE approach gives a conductive ice shell thickness of 11 km. Taking  $E=1$  GPa yields a  $T_e$  of 6 km and a shell thickness of 12 km. Increasing strain rate by two orders of magnitude decreases the inferred shell thickness by 1 km. Reducing the friction coefficient by 30% increases the shell thickness by 1 km. Using the grain-boundary sliding ( $n = 1.8$ ) rheology of [17] with a grain-size of 1 mm reduces the shell thickness by 1 km. Thus, uncertainties in most parameters do not significantly affect the final result.

The shell thickness is insensitive to variations in  $E$  because the elastic portion of the shell is small. Hence, the YSE approach is a robust way of converting estimates of rigidity into ice shell thicknesses.

## References

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