FOURTH ORDER EQUILIBRIUM FIGURE FOR THE ATMOSPHERES OF JUPITER AND SATURN, J. D. Anderson, Jet Propulsion Laboratory, Pasadena, California, 91103

The potential function $V(r,\phi')$ in the atmosphere of a planet can be represented by the sum of the external gravitational potential plus the rotational potential at distance r from the center of mass. The rotational term is characterized by the small dimensionless parameter $q = \omega^2 a^3/GM$, where a is the equatorial radius and ω is the angular velocity of rotation. The planet centered latitude is given by ϕ'. The equipotential surfaces are derived by expanding $V(r, \phi')$ in powers of q and by recognizing that $J_{2i} \sim q^i$ (1), (2), (3), (4). The radius r has been developed to the fourth order in q by requiring that $V(r, \phi') - V(a, 0) = 0(q^5)$. Expressions have also been derived for the acceleration of gravity and for the deflection of the local vertical $(\phi-\phi')$ in the atmosphere. The flattening of $f = (a-b)/a$ of the equipotential surface has been derived from the expression for the radius. The result follows for the reference surface (1.0 Bar).

$$
\begin{aligned}
f = {} & \frac{3}{2} J_2 + \frac{1}{2} q + \frac{9}{4} J_2^{\,2} - \frac{1}{4} q^2 + \frac{5}{8} J_4 + \frac{81}{8} J_2^{\,3} + \frac{27}{8} J_2^{\,2} q + \frac{3}{8} J_2 q^2 \\
& + \frac{1}{8} q^3 + \frac{15}{2} J_2 J_4 + \frac{15}{8} J_4 q + \frac{21}{16} J_6 + \frac{783}{16} J_2^{\,4} + \frac{81}{4} J_2^{\,3} q + \frac{9}{8} J_2^{\,2} q^2 \\
& - \frac{1}{4} J_2 q^3 - \frac{1}{16} q^4 + \frac{175}{64} J_4^{\,2} + \frac{1575}{32} J_4 J_2^{\,2} + \frac{255}{16} J_4 J_2 q + \frac{15}{32} J_4 q^2 \\
& + \frac{35}{16} J_6 q + \frac{21}{2} J_6 J_2 + \frac{93}{128} J_8 + 0(q^5)
\end{aligned}
$$

The various fourth order quantities in the atmospheres of Jupiter and Saturn have been evaluated numerically by combining data from the Pioneer flybys of Jupiter (5) with experimental and theoretical data given by Slattery (6).

TABLE I
Numerical Data for Jupiter and Saturn

Parameter	Jupiter	Saturn
a km	71,400	60,400
$a\omega$ km/sec	12.56	10.14
$q \times 10^6$	88,850	165,000
$J_2 \times 10^6$	14,733	16,670
$J_4 \times 10^6$	-587	-1,020
$J_6 \times 10^6$	34	81
$J_8 \times 10^6$	-3	-10

For example, the dynamical flattening for Jupiter and Saturn is 0.064777 and 0.101213 respectively. However, in the atmosphere, the rotation rate varies with latitude (7) and hence the value of q is uncertain. This can be

important.

The linear relationship between the error in q and the error in the equilibrium quantities can be established by differentiating each quantity of interest with respect to q. The resulting expression for the error in the local vertical depends on the latitude, and for Jupiter is given by

$$\Delta(\phi-\phi') = (27\overset{\circ}{.}9 \sin 2\phi' + 3\overset{\circ}{.}1 \sin 4\phi' + 0\overset{\circ}{.}3 \sin 6\phi')\Delta q$$

while for Saturn the corresponding expression is

$$\Delta(\phi-\phi') = (26\overset{\circ}{.}6 \sin 2\phi' + 5\overset{\circ}{.}4 \sin 4\phi' + 1\overset{\circ}{.}0 \sin 6\phi')\Delta q$$

The uncertainty in q will depend on the uncertainty in the zonal wind velocity at the latitude of interest. At the equator, an uncertainty of 100 m/sec in the zonal wind will result in an uncertainty in q of 0.0014 for Jupiter and 0.0033 for Saturn. The error in the local vertical is greatest at temperate latitudes. The maximum error occurs at $38\overset{\circ}{.}8$ on Jupiter and is equal to $28\overset{\circ}{.}3\ \Delta q$. On Saturn the maximum error occurs at $33\overset{\circ}{.}5$ where it is $27\overset{\circ}{.}9\ \Delta q$. The error in the local vertical can easily amount to more than $0\overset{\circ}{.}01$ in these latitudes if the zonal winds are not known to fairly high accuracy.

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