GRAIN TEMPERATURE IN PROTOPLANETARY DUST CLOUDS

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A protoplanetary dust cloud represents an open thermodynamic system[1], continually losing heat irreversibly to the surrounding space. This permits disequilibrium quasi-steady states to prevail in the interior of the cloud. For those dust grains in the cloud that can radiate directly into the surrounding space (i.e. the shortest path between the grains and the cloud surface is transparent to the infrared grain radiation), the steady state relationship between the gas temperature T and the grain temperature T' has been studied in the past[2,3]. Here we study the effect of a non-negligible infrared opacity on the T-T' relationship. This is a problem of nonequilibrium thermodynamics, and an analytical solution to the problem does not exist. We have therefore developed a numerical method to study the problem.

The method consists in an iterative scheme where the first approximation is that the cloud is entirely transparent. The effect of opacity is then gradually introduced in such a way that a convergence of the iterations gives a self-consistent solution to the problem. For the sake of specificity of discussion, let us refer to a conceptual solar nebula (Fig. 1) which is invoked in the context of condensation or capture of grains leading eventually to formation of planets and satellites. We find that the i-th iteration T', T'_i, for a given value of h is the solution of

$$\sigma T_i'^4 = \sigma T_o''^4 + 2CNVk(T-T_i') + \frac{1}{2}\pi\sigma a^2 \int_0^R \int_0^\pi n(\underline{r})\,[T'_{i-1}(\underline{r})]^4 \sin\theta \, d\theta \, dr \quad (1)$$

Here N,n = gas, grain number densities, k, σ = Boltzmann, Stefan-Boltzmann constants, C = thermal accomodation coefficient of hydrogen, a = grain radius, V = impingement velocity of gas molecules onto the grain surface, R = photon mean free path, and T_o'' = equivalent radiation temperature[3] in the surrounding space (i.e. the temperature that a blackbody placed in the region in question would attain in steady state due to the luminosity of nearby stars). Similarly, an equivalent radiation temperature T"(h) for the extant radiation field in the interior of the nebula may be defined. The bivariate integral in Eq.(1) represents the contribution of opacity to the grain temperature. This contribution to a given grain is estimated by assuming that this grain receives radiation directly from all other grains within a sphere of radius R, and does not receive any radiation from the grains outside this sphere. Without the integral, Eq.(1) simply gives us the grain temperature in a transparent nebula[3]. We have assumed above that the infrared opacity is due to the grains alone. The gas contributes little to this opacity for the example that we shall discuss. We have further assumed the grains to behave as blackbodies, and their

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absorption cross-section to be their geometric cross-section. Sophistications of these simplifying assumptions can be easily introduced in our scheme.

We have made some preliminary calculations of T' and T" as functions of the height h above the midplane and the optical depth τ. We have considered roughly the asteroidal region with $T''_o = 150$ °K. Arbitrary assumptions have been made regarding the variation of N and T with h ($N = 10^{15} \exp(-h/H)\, cm^{-3}$, $T = 2000 \exp(-h/H)$ °K) – just to take into account in some fashion the fact that these quantities decrease with h. Iron, with an abundance ratio of 10^{-5} is assumed to be fully condensed in the form of 1μ grains, and at the instant of calculation, distributed homogeneously throughout the nebula($n \propto \exp(-h/H)$). This is also an ad hoc assumption, invoked for the purpose of visualizing the effect of a distributed opacity on the temperature relationship. The results (Fig. 2) show that contrary to what we have believed in the past[2], even large optical depths do not bring about an equality of T and T'. On the other hand, a near-equality of T' and T" is established already at small optical depths. The essential physical consideration here is that the gas as a thermodynamic entity does not exchange heat directly with the infrared radiation field. As reservoirs of heat, this radiation field and the gas are coupled only via the agency of the grains.

For the sake of completeness we should also discuss the dependence of the T-T' relationship on the gas density. This dependence is shown in Fig. 3 (calculated from Eq.(1) without the opacity term). The densities at which a a near-equality of T and T' obtains are rather high for known or physically realistic cosmic gas-dust environments – certainly for the peripheral regions of the nebula. The other situations in which equilibration may be approached are: (i) larger H, and (ii) large gaseous infrared opacity. In the former case equilibration will be achieved near the midplane, but disequilibrium will prevail elsewhere. In the second case – in the present example for instance – the gaseous opacity would have to far exceed grain opacity for equilibration to occur near the midplane. This appears unlikely (see e.g.[4]).

Condensation: If the grains have come to exist as a result of in situ condensation in the nebula, then at the onset of condensation the grain temperature would be represented more realistically by the curve marked T'(transparent) – calculated for a transparent nebula – than the curve marked T', so that disequilibrium would be even more pronounced during early condensation. Furthermore, if condensation occurs as a result of radiative cooling of the gas, then the peripheral regions of the nebula – where disequilibrium is most pronounced – would be a major locale of condensation since cooling would take place more efficiently here than near the midplane[2,5]. Thus, as far as the nebular type of condensation scenarios are concerned, we may make the general conclusion that condensation favors disequilibrium.

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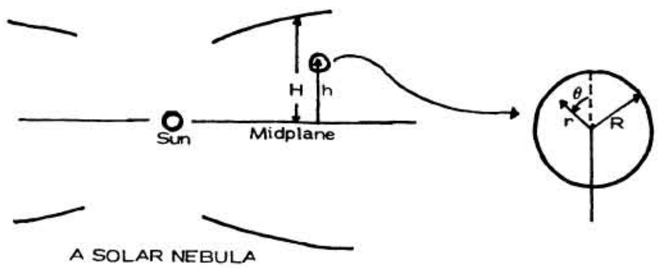

Figure 1

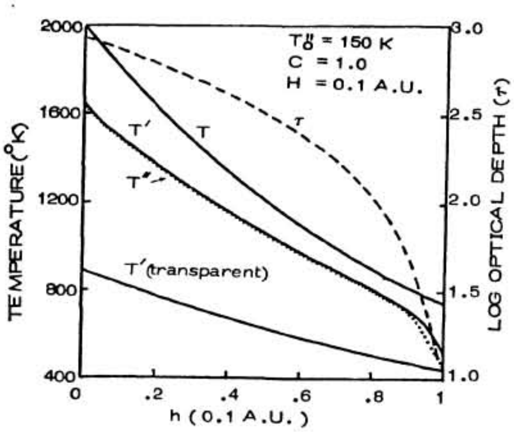

Figure 2

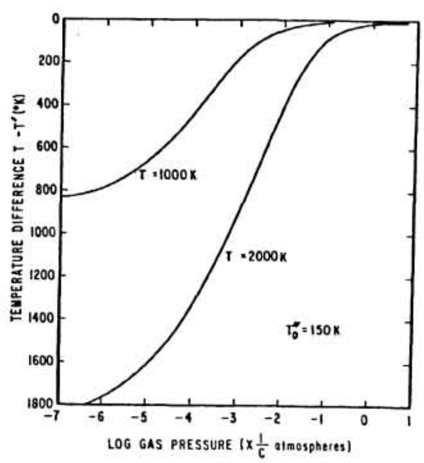

Figure 3