MEASUREMENT OF LUNAR HEAT FLOW USING ORBITAL RADIOMETRY. W.W. Mendell, Johnson Space Center, Houston, TX 77058

Global heat flow is a key piece of information for the determination of the evolutionary state of a planet. The magnitude of the heat flow can be related to the abundance of long-lived radio isotopes [1]. In turn, the calculated uranium abundance can be used to estimate the abundance of refractory elements in geochemical models of the planet [2]. Measurement of a constant subsurface thermal gradient combined with the thermal conductivity yields the heat flow.

$$\dot{Q} = -k \frac{dT}{dz}$$

The Apollo Heat Flow Experiment was performed by the implantation of temperature sensors in the lunar surface at depths great enough that no temperature excursions due to the solar heating cycle could be detected [3]. The orbital experiment will utilize microwave radiometers operating at different wavelengths to detect the subsurface gradient.

The temperature in the uppermost surface layer of the Moon is determined almost entirely by the effects of solar heating. The lunation cycle generates a harmonic series of thermal waves. With increasing depth the temperature excursions become smaller, with the higher harmonics being damped more strongly. The lunar surface layer is such a good insulating material that even the fundamental is damped out within a meter of the surface.

Boundary conditions for a thermal model of the surface layer include solar albedo, infrared emissivity, and time dependence of insolation. The latter quantity encompasses geometrical effects such as local slope, latitude, and Sun-Moon relationships. The first-order thermal model treats the Moon as a semi-infinite solid with conductive heat transfer and constant thermal properties [4]. The solution depends upon a single parameter which contains, among other things, the product of the thermal conductivity, the bulk density of the material, and the specific heat in the form $\gamma = (k\rho c)^{-1/2}$. Taking into account the periodicity of the surface temperature and the heat flow gradient, we can write the solution as

$$T(z,t) = T_0 + \frac{\dot{Q} z}{k} + \sum_n T_n \exp(-\beta z \sqrt{n}) \cos(n\omega t - \beta z \sqrt{n} + \phi_n)$$

where T_n, ϕ_n are the amplitudes and phases of the surface temperature Fourier expansion, ω is the lunation frequency, and $\beta = \sqrt{\omega}/(\gamma k \sqrt{2})$ is the thermal damping coefficient.

Consideration of the microwave emission will assume observation along the vertical with a thin beam antenna for simplification. The observed antenna temperature $\Theta(\nu)$ at frequency ν can be found from [5]

$$\Theta(\nu,t) = (1-R_\nu)\int_0^\infty T(z,t) \exp(-\chi_\nu z)\, \chi_\nu \, dz$$

$$= (1-R_\nu)\left\{ T_0 + \frac{\dot{Q}}{k\chi_\nu} + \sum_n \frac{T_n}{a_{n\nu}} \cos(n\omega t + \phi_n - \psi_{n\nu}) \right\}$$

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where $R(\nu)$ is the microwave reflection coefficient and $\chi(\nu)$ is the electromagnetic absorption coefficient. Thus the microwave thermal emission from a lunar region is a harmonic series related to the surface temperature variation through the microwave emissivity, attenuation coefficients $a(\nu)$, and additional phase shifts $\psi(\nu)$. Both a and ψ are functions of the ratio β/χ.

$$a_n(\nu) = \left(1 + 2\beta\sqrt{n}/\chi(\nu) + 2\beta^2 n/\chi^2(\nu)\right)^{1/2}$$

$$\psi_n(\nu) = \operatorname{arccot}\left[1 + \left(\beta\sqrt{n}/\chi(\nu)\right)^{-1}\right]$$

The magnitude of χ increases directly with the wavelength. In practice, two harmonics can be detected; but the attenuation coefficient increases with wavelength such that no variation is seen for wavelengths $\gtrsim$20 cm [6].

Solution for Heat Flow. The heat flow can be calculated in two ways. Both assume observations at two frequencies and that $R(\nu_1) = R(\nu_2) = R$. This assumption is valid for Fresnel reflection because the dielectric constant is essentially frequency independent in the appropriate range.

Using the observed constant components of the radio temperature,

$$\dot{Q} = \frac{\Theta_0(\nu_2) - \Theta_0(\nu_1)}{1 - R} \cdot \gamma\sqrt{2/\omega} \cdot \left(\frac{\beta}{\chi(\nu_2)} - \frac{\beta}{\chi(\nu_1)}\right)$$

Here R must be calculated from some previous knowledge of the dielectric constant, and the ratios β/χ can be obtained from

$$\frac{\Theta_1(\nu_1)}{\Theta_1(\nu_2)} = \frac{a_1(\nu_1)}{a_1(\nu_2)} \quad ; \quad \frac{\Theta_2(\nu_1)}{\Theta_2(\nu_2)} = \frac{a_2(\nu_1)}{a_2(\nu_2)}$$

Note that measurement of a second harmonic at both wavelengths must be obtained for this approach. In any case, the expression also requires a value for γ, which requires a set of infrared observations of the surface temperature for the region.

If infrared observations exist, then one can use T_0, T_1, and ϕ_1 to avoid calculation of R and the requirement for observing a second harmonic in the radio temperature. We then can write

$$\dot{Q} = \frac{T_0\left(\Theta_0(\nu_1) - \Theta_0(\nu_2)\right)\sqrt{\omega/2}}{\gamma\left(\frac{\beta}{\chi(\nu_1)}\Theta_0(\nu_2) - \frac{\beta}{\chi(\nu_2)}\Theta_0(\nu_1)\right)}$$

The ratios β/χ can now be obtained from the measurement of the phase shift $\psi(\nu)$ or

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$$a_n(\nu) = \frac{\Theta_0(\nu)}{T_0} \cdot \frac{T_n}{\Theta_n(\nu)}$$

Complications. The development presented so far has been simplified as much as possible to focus on the basic principles. The most serious omission from this model is the radiative component of heat transfer in the surface. Since radiative transfer is highly temperature dependent, solar heat is much more readily conducted into the surface during the day than it can be conducted out during the cold lunar night. Steady state considerations require that the heat flux average to zero over a lunation. Thus a net heat flow outwards must exist in the solar heated layer due to the temperature dependence of the conductivity and having nothing to do with the heat flow from the interior. The temperature gradient associated with this pseudo heat flow was detected by the Apollo Heat Flow Experiment [3].

Microwave observations will also detect the extra gradient although its effect is reduced in longer wavelength measurements. Observations at three frequencies give two determinations of the sum of the two gradients. In principle, the two gradients can be separated. However, the pseudo heat flow is much larger than the true one, and the situation must be carefully modelled for an accurate determination.

References:

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