MARGINAL STABILITY IN VARIABLE VISCOSITY FLUIDS. A. Warford and J.-Cl. De Bremaecker, Department of Geology, Rice University, P. O. Box 1892, Houston, Texas 77001.

The thermal history of the terrestrial planets is greatly influenced by the possibility of convection (so called "solid-state convection"). The viscosity essentially decreases exponentially with temperature and increases with pressure. It is thus of interest to determine the condition of marginal stability for convection in such a fluid. For Rayleigh-Bénard convection this condition corresponds to the minimum eigenvalue of R as a function of a in the following equation:

$$\eta(D^2-a^2)^3w+Ra^2w = -4\eta'(D^2-a^2)^2w'+\eta^{IV}(D^2-a^2)w -6\eta''(D^2-a^2)w''-2\eta^{IV}w''-4\eta^{III}w^{III}$$

(McFadden, 1969) where η is viscosity, $D=d/dz$, a is the wavenumber, w is the vertical velocity, R is the Rayleigh number defined using η_{max}, and primes or roman numerals indicate derivatives with respect to z, taken as positive upwards. The depth of the fluid layer is taken as unity, and so is the maximum value of the viscosity. An equivalent formulation in terms of temperature is given i.a. by Booker and Stengel (1978).

This equation is solved numerically by transforming it into a system of six first order equations which may be solved e.g. by the Runge-Kutta method. For any a we can find the value of R; a is then varied until R_{min} is found. This is the critical Rayleigh number and the corresponding critical wavenumber.

It is physically evident that fixed (i.e. no-slip) boundary conditions will not contribute to decrease a, i.e. to increase the aspect ratio of the convection cell. Consequently we have concentrated our attention to free (i.e. free-slip) conditions. The main results are summarized in the figures.

Figure 1 shows the variation in R (really the critical Rayleigh number) vs. the aspect ratio of the convection cell in 4 different cases. Curves A, B and C correspond to a variation of viscosity with depth symmetrical with respect to $z=\frac{1}{2}$. For Curve A, $\eta=1-(1-\eta_{min})\sin^5\pi z$. The numbers on the curves are the values of $1/\eta_{min}$ i.e. the viscosity contrast between $z=\frac{1}{2}$ and z=0 or 1. Curve B shows the same information for $\eta=1-(1-\eta_{min})\sin\pi z$. Except for a smaller R, these curves are the same. Curve C is for $\eta=(e^{-az}+e^{-a(1-z)})(1+e^{-a})^{-1}$. In all cases the aspect ratio increases when the viscosity minimum becomes more pronounced, but this increase is not large; the corresponding decrease in R shows that convection becomes increasingly easy to start. Curve D is for an exponential decrease in viscosity downward: one would expect the convection to concentrate in the low viscosity zone, thus reducing the aspect ratio and R. The figure shows, however, that the aspect ratio first increases slightly; it becomes less than the one for constant viscosity only when the viscosity contrast exceeds 1000; R, on the other hand decreases uniformly.

Figure 2 shows the variation of R with a for the variation of η shown in the insert and the values of $1/\eta_{min}$ shown on the curves. This variation of η approximates that expected in the planets. It is seen that the sharpness of the minimum (corresponding to R critical) greatly decreases as the viscosity contrast increases (Curve 1 is for constant η).

Richter (1978) reports a similar phenomenon for layered fluids and suggests that it facilitates convection at a variety of aspect ratios.

Figure 3 shows the same information for the variation of η given by curve B Figure 1, and shows the same phenomenon.

The variation of velocity with depth in these various cases (not shown) give an insight into the "reason" for the phenomena described.

REFERENCES

Booker, J.R. and Stengel, K.C., 1978, J. Fluid Mech., 86, 289-291.

McFadden, C.P., 1969, The effect of a region of low viscosity on thermal convection in the Earth's Mantle, Unpubl. Thesis, Univ. of Western Ontario, 64 pp.

Richter, F.M. and Dalen, S.F., 1978, J. Geophys. Res., 83, 4951-4956.

MARGINAL STABILITY

Warford, A., et al.

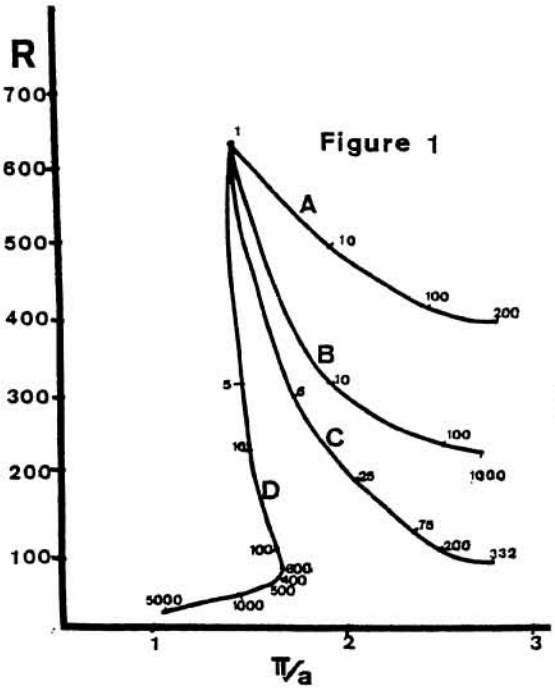

Figure 1

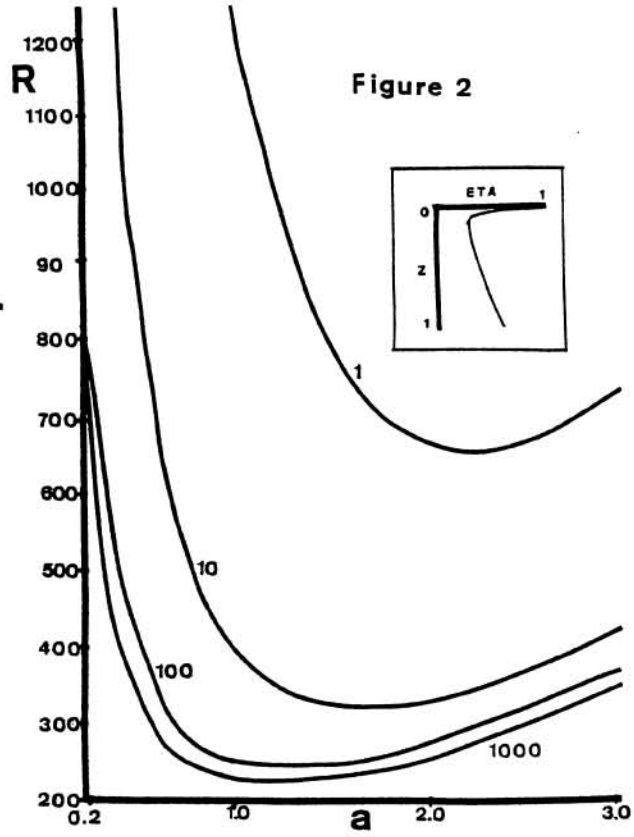

Figure 2

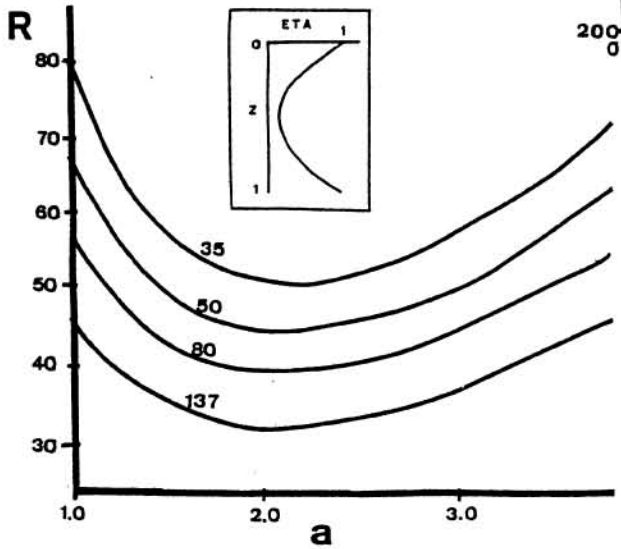

Figure 3