NUMERICAL SIMULATION OF PLANET GROWTH: EARLY RUNAWAY GROWTH, Richard Greenberg, Planetary Science Institute, Tucson, Arizona 85719

Our earlier simulations of collisional evolution of planetesimals (1) showed that 500 km bodies (~1 per AU^2) could grow within a swarm of initially all km-size bodies very quickly (~20,000 yr). Over that stage of planet growth the size distribution of planetesimals seems to evolve in a very different way from what had been envisioned by earlier researchers, notably Safronov (2). Nevertheless, the distribution could have been interpreted as evolving towards the size distribution assumed by Safronov, so that the bulk of planetary accretion might still have followed his model, with limited runaway growth. However, continuation of our simulation forward in time indicates an alternate scenario: I find rapid runaway growth of the largest bodies up to ~4000 km diameter in ~50,000 yr.

In our earlier work the size distribution underwent evolution schematically represented in Fig. 1 through time step 2.

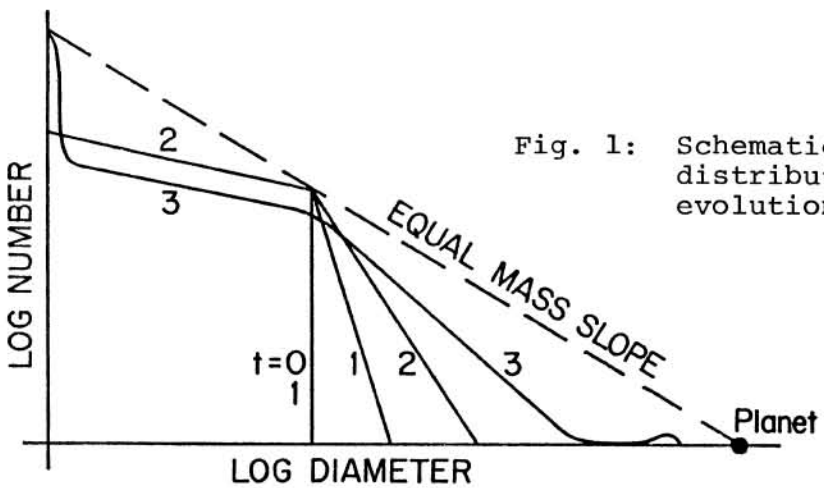

Fig. 1: Schematic of size distribution evolution.

The initial "spike" distribution at 1 km evolves into an increasingly shallow log-log slope as larger bodies form by accretion. A "small particle tail" of shallower slopes is formed of escaped crater ejecta and debris of catastrophic fragmentation. Throughout this stage of the evolution the bulk of the mass of the system resides in km-scale bodies. In agreement with Safronov's analytical theory, our numerical model yielded relative velocities of planetesimals ~V_e (the escape velocity) for dominant size bodies (1 km in this case, V_e~m/sec). These low velocities and correspondingly large gravitational cross-sections help account for the short time scale for growth.

Contrast that scenario with the growth model by Safronov, which I interpret schematically in Fig. 2. The slope of the size distribution is assumed throughout to be that characteristic of an equilibrium collisionally evolved system (such as our "small particle tail") with most of the mass in the largest bodies

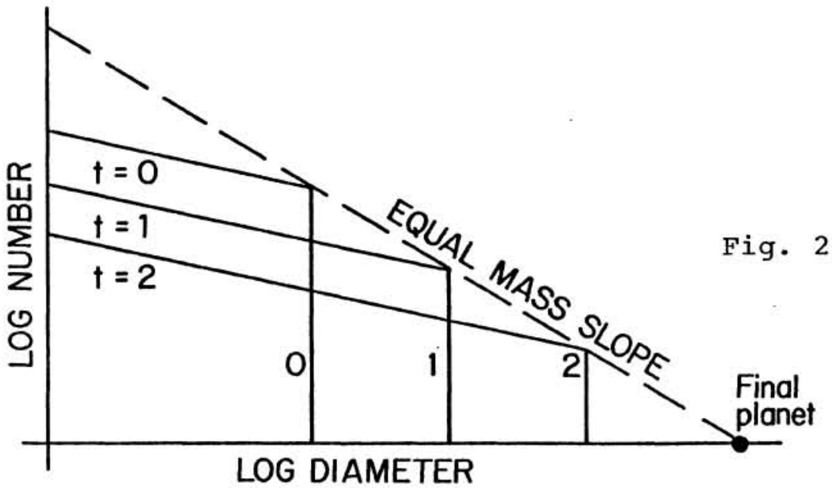

Fig. 2: Schematic of evolution described by Safronov.

existent at any given time. Hence the relative velocities increase in proportion to the size of the largest bodies. Growth is relatively slow, and runaway growth of the largest bodies is suppressed (3). At all times there is an abrupt cut-off in the distribution at the large size end.

The evolution in our scenario (Fig. 1 through step 2) seemed to be evolving towards an equilibrium size distribution slope such as that assumed by Safronov. However, we conservatively stopped the evolution beyond formation of ~500 km bodies, because for these large bodies the gravitational capture radii became comparable to the scale of departure from circular orbits. Thus amongst the largest bodies, collision probabilities were no longer governed by the particle-in-a-box statistics (with orbital e's and i's giving relative velocities) that are assumed in our algorithm. However, Kaula (4) has pointed out that in our work collisions amongst the largest bodies are rare and of negligible importance compared with collisions amongst small bodies and collisions involving accretion of smaller bodies by the largest ones. Since we found smaller bodies tended to have somewhat higher e's and i's than the largest ones, the collisions of interest are still governed by our algorithm.

With this rationale we have extended the simulation further in time. A typical case is shown in Fig. 3 for "intermediate material" and initial conditions described in (1). When the first 500 km body is produced, e's and i's for most other objects are a few 10^{-4}. As the largest body grows to about 4000 km, e's and i's are $\sim 2 \times 10^{-3}$, due to the shift of the mass peak up to 8 km from 1 km. Towards the end of the period studied, the largest body begins to contain a substantial fraction of the total system mass, as it would in Safronov's model. But it has already begun runaway growth so it cannot influence relative velocities much. (This final state in Fig. 3 is represented schematically by time step 3 in Fig. 1). There seems to be no prospect for Safronov's shallow-slope collisional equilibrium condition to be reached.

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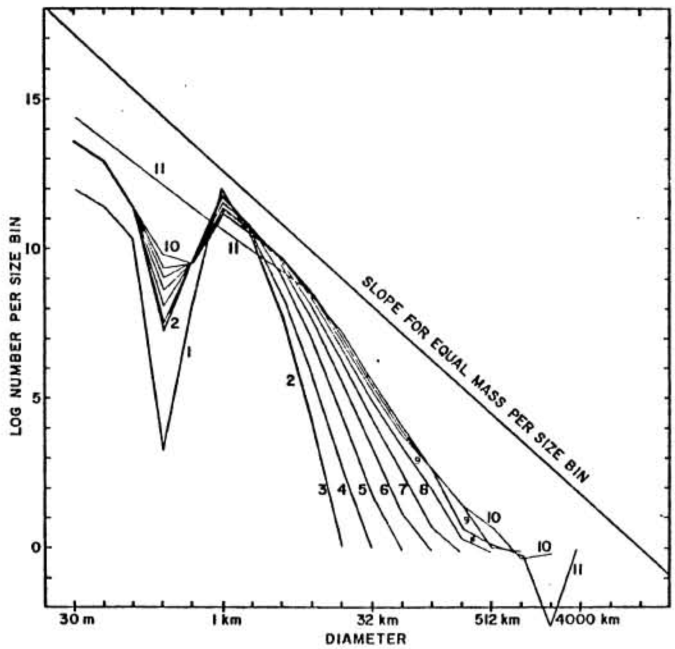
Fig. 3: Evolution of size distribution from initially all km-size bodies.

Curve	
1→	153 yr.
2	11140
3	11160
4	11810
5	13730
6	16720
7	19930
8	22860
9	25780
10	29690
11	53630



Planetesimals destined to grow into nearly full-sized planets seem to be selected by this runaway process within a few 10^4yr of our initial state. Possibly, however, a subsequent period of collisional and gravitational interactions amongst large bodies (5) may interrupt continuity of identity from the first runaway products through the final planets.

We are in the process of exploring the range of parameters and physical processes that permit the early runaway growth. Numerical experiments with a wide range of material parameters give results similar to Fig. 3. We have also incorporated a model of the effects of a gaseous medium, which was likely to have been present given the short time scale for accretion. The model includes effects of damping of random motion (e's and i's) and also enhancement of relative velocities by differential radial drift towards the sun (6). To the level of detail discussed here, the results are unchanged by the presence of a reasonable gas ($\sim 10^{-9}$gm/cm^3). We have also been experimenting with an improved "mass transfer" algorithm (7). This algorithm computes the rate at which bodies are transferred from one size bin to the next due to gradual erosion or accretion. Preliminary results suggest that if the initial size distribution is a sufficiently strict "spike," it may be possible to generate an evolution such as that shown in Fig. 2. However, more plausible initial conditions seem to yield early runaway growth.

References: (1) Greenberg, R., Wacker, J.F., Hartmann, W.K., Chapman, C.R. Icarus 35, 1 (1978; (2) Safronov, V.S., NASA TT F-677 (1972); (3) Safronov, V.S., preprint (1979); (4) Kaula, W.M., Icarus 40, 262 (1979); (5) Cox, L.P., M.I.T. dissertation (1978); (6) Adachi, I., Hayashi, C., Nakazawa, K. Prog. Theor. Phys. 56, 1756 (1976); (7) Developed by S.J. Weidenschilling.