

PHENOMENOLOGICAL THEORY OF ATTENUATION AND PROPAGATION VELOCITY OF ELASTIC WAVES IN ROCKS\*, J. M. Richardson, B. R. Tittmann, and M. Abdel-Gawad, Rockwell International Science Center, Thousand Oaks, CA 91360.

The purpose of this short communication is to present a phenomenological theory of the attenuation and propagation velocity of elastic waves in rocks in the linear regime. It pursues the objective of deriving a partial description of macroscopic behavior based on certain general assumptions concerning the nature of the microstructure. The most important assumption is that there is a broad distribution of activation energies. This latter property is almost inevitable in permanently disordered systems if the disorder directly affects the activation energies. We also allow the possibility of additional sets of relaxation processes involving very short relaxation times.

Although the theory, with the inclusion of certain additional assumptions, is capable of dealing with the dependence of velocity and attenuation on both signal frequency and water partial pressure, the experimental data considered here contains only the latter dependence. Future experimental work will emphasize frequency dependence.

It has been proposed (1) that the unusually high mechanical strength of lunar rocks arises from the absence of water. As will be shown later, there is a good inverse correlation between mechanical strength and a certain parameter related (according to the phenomenological theory) to attenuation. Thus, it is clear that any insights gained into the relation between attenuation and the presence of water will have relevance to lunar rocks.

It is assumed that the complex modulus (relevant to the process considered) can be represented in terms of superposition of elementary relaxation processes, i.e.

$$\lambda = \lambda^0 - \gamma \int_0^{\infty} d\tau P(\tau) \frac{1}{1 + i\omega\tau} \quad (1)$$

where  $\lambda^0$  is a reference modulus existing in the absence of dissipative processes,  $\tau$  is a relaxation time, and  $P(\tau)$  is a probability density representing the distribution of relaxation times. The constant coefficient  $\gamma$  reflects the density of elementary relaxation processes and their strengths of interaction with the gross elastic field.

It is assumed that the relaxation processes are thermally activated with a broad distribution of activation energies. Specifically we assume

$$\tau = \tau^0 \exp(E/kT) \quad (2)$$

where the activation energy  $E$  is a random variable uniformly distributed over an interval whose limits depend linearly on the relative partial pressure of water. With certain additional assumptions, obtain the complex modulus in the form

$$\lambda = \lambda^0 - \Gamma \phi(\omega\tau_{\min}) \quad (3)$$

where  $\phi(\omega\tau_{\min})$  a universal complex function of  $\omega\tau_{\min}$  where  $\omega$  is the signal

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frequency and  $\tau_{\min}$  is the relaxation time associated with the minimum activation energy. A detailed discussion is given in a paper by Richardson and Tittmann (2).

The theory has been fitted to velocity and attenuation data for a number of sandstones. The results for Coconino sandstone are given in Fig. 1 in which  $Q^{-1}$  and the relative velocity decreases  $(v^0 - v)/v^0$  are plotted as functions of  $p/p^0$ , the relative partial pressure of water. In Table I the dimensionless parameter combination  $\Gamma/\lambda_0^0$  is compared with mechanical properties of a number of rocks. The parameter  $\Gamma$  entering Eq. (3) is proportional to  $\gamma$  and measures the total "importance" of the collection of dissipative processes and  $\lambda_0^0$  is the dry zero-frequency value of the modulus. The correlation between  $\Gamma/\lambda_0^0$  and mechanical properties is striking.

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## References

- (1) Tittmann B. R., Clark V. A., and Spencer T. W. (1981), Proc. Lunar Sci. Conf. 11th, in press.
- (2) Richardson J. M., and Tittmann B. R. (1981), Proc. Lunar Sci. Conf. 11th, in press.

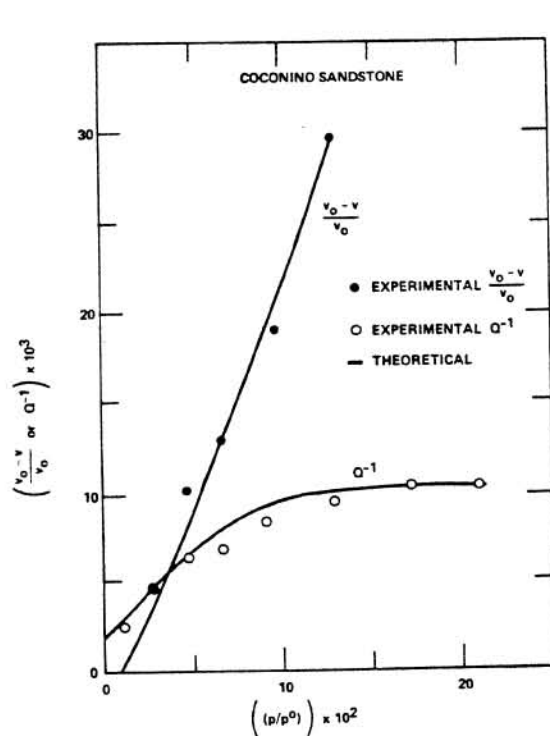


TABLE I  
Uniaxial Compressive Data for Sandstones

Sandstone	$\Gamma/\lambda_0^0$	Compressive Strength ( $10^{10}$ ) kg/cm <sup>2</sup>	Young's Modulus Dyne/cm <sup>2</sup>
Austin Chalk	0.00957	400 (est)	3.0 (est)
Coconino	0.00631	477	3.15
Tennessee	0.00594	>1758	4.28
Sioux Qtz	0.00456	>1844	4.75