STATISTICAL TEST OF THE $\sqrt{2}$ SPACING RULE FOR BASIN RINGS, Gary D. Clow and Richard J. Pike, U.S. Geological Survey, Menlo Park, CA 94025

The regularity in the radial separation of adjacent rings in impact basins has been noted since the first study of lunar basins by Hartmann and Kuiper [1]. By appealing to the sagging plate model of Lance and Onat [2], Fielder [3] may have been the first to suggest that the ratio of adjacent basin ring radii should ideally be $\sqrt{2}$. In their extensive study, Hartmann and Wood [4] agree that $\sqrt{2}$ and its multiples have a special role in the spacing of lunar basin rings, although the mechanism of their formation remains elusive. The purpose of this work is to test the hypothesis that some process involved in basin formation is so highly dominant that the ratio of the radii of adjacent rings is $\sqrt{2}$ throughout the inner solar system, although small deviations from this value can be expected due to the complexity of the impactor-target interaction.

As a first step in the testing procedure, we assign numerical values, termed ranks, to the relative position of rings within each basin according to the method of Pike [5]. The rationale and procedure for ranking rings described in [5] is summarized and slightly changed here: Based on the known regularity of lunar basin ring spacing, we assume that ring radii form a geometric sequence and that the common ratio (incremental spacing factor) for each basin has a value between, say, 1.2 and 1.6. The most prominent ring is arbitrarily assigned to the fourth member of the sequence. A sequence of radius intervals $\{(L_n,U_n)\}$ is constructed using the radius r_4 of the most prominent ring with $L_n=1.6^{n-4}r_4, U_n=1.2^{n-4}r_4$, if $1 \leq n \leq 4$ and $L_n=1.2^{n-4}r_4$, $U_n=1.6^{n-4}r_4$ if $n>4$. A ring is associated with the n^{th} member of the geometric sequence if its radii is within the interval (L_n,U_n). Such a ring is defined to have rank n by Pike [5]. With this scheme, all 90 rings for the 24 basins in this study (11 lunar, 3 mercurian, 5 martian, and 5 terrestrial) can be unambiguously ranked. Observed rank values range from 1 to 7.

Using a geometric sequence, the radius r_n of a ring with rank n can be expressed by $r_n=r_4c^{n-4}$ or equivalently,

$$\log r_n = \log r_4 + (n-4)\log c \tag{1}$$

where c is the incremental spacing factor. A least-squares fit to equation (1) using rank (n) and radius (r_n) data for each basin yields a statistical estimate of the incremental spacing factor c* (table 1) and ring radius r_4*. Sample correlation coefficients exceed .98 in each case, demonstrating that ring radii actually do follow geometric sequences as has been assumed and that the incremental spacing factor is the same inside and outside of the main basin ring. The parameter c* appears to be independent of r_4 for each planet and thus of lithospheric thickness and also of the ratio (r_4/planet radius), which is a measure of the planetary surface curvature experienced during basin formation.

A two-sided t-test used to test the hypothesis that $c=\sqrt{2}$ for each basin reveals that this hypothesis can be accepted for 21 of the 24 basins, when the probability of accepting the hypothesis, given that it is true, is

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set at 90% [6]. The observed frequency of acceptance (88% with 4% resolution) compares well with what is expected if the proposed hypothesis is actually true. A test of the hypothesis that the mean value of c for each planet is equal to $\sqrt{2}$ shows that this hypothesis must be accepted when the decision procedure is based on the t-distribution and the probability of accepting the hypothesis, given that it is true, is set at 90%. Thus c is independent of the value of surface gravity. Since c appears to be independent of lithospheric thickness, relative surface curvature, and surface gravity, the hypothesis that $c=\sqrt{2}$ for the entire set of 24 basins can also be tested with a two-sided t-test. The hypothesis is accepted with this test when the probability of accepting the hypothesis, given that c is actually equal to $\sqrt{2}$, is 90%. Figure 1 shows the probability that the hypothesis would be accepted as a function of the true value of c for the decision procedure used here. Note that the probability of accepting the hypothesis $c=\sqrt{2}$ is only 10% if c is actually 1.397 or 1.432.

In summary, the dominant expression of basin ring spacing is a geometric sequence with an incremental spacing factor equal to $\sqrt{2}$. The incremental spacing factor is the same inside and outside of the main basin ring and appears to be independent of lithospheric thickness, relative basin size, and surface gravity. These results hold on bodies throughout the inner solar system with the possible exception of Venus, which remains untested.

References: [1] Hartmann, W.K. and Kuiper, G.P. (1962) Comm. Lunar Planet. Lab. Univ. Ariz., v. 1, p. 51-66; [2] Lance, R. and Onat, E. (1962) J. Mech. Phys. Solids, v. 10, p. 301; [3] Fielder, G. (1963) Nature, v. 198, p. 1256-1260; [4] Hartmann, W.K. and Wood, C.A. (1971) The Moon, v. 3, p. 3-78; [5] Pike, R. J. (1981), NASA Tech. Memo. 84211, p. 123-125; [6] Bowker, A.H. and Lieberman, G.J. (1972) Engineering Statistics, p. 134-346.

Table 1

Basin	(r_4/r_{planet})	C*	$H \mid c= 2$
Moon			
Moscoviense	0.128	1.451	reject
Australe	0.158	1.377	accept
Humorum	0.161	1.353	reject
Hertzsprung	0.164	1.467	accept
Mendel-Rydberg	0.181	1.468	"
Crisium	0.196	1.402	"
Smythii	0.242	1.423	"
Nectaris	0.250	1.421	"
Orientale	0.268	1.414	"
Imbrium	0.388	1.432	"
Procellarum	0.921	1.372	"
Mercury			
Tolstoi	0.119	1.401	accept
Sobkou	0.165	1.440	"
Haydn-Raphael	0.183	1.408	"
Mars			
Schiaparelli	0.068	1.414	accept
Mars-So. Pole	0.125	1.412	"
Argyre	0.162	1.382	"
Isidis	0.221	1.409	"
Hellas	0.324	1.384	reject
Earth			
Haughton	0.001	1.434	accept
Ries	0.002	1.396	"
Clearwater W.	0.002	1.403	"
Manicouagan	0.006	1.407	"
Popigai	0.007	1.393	"

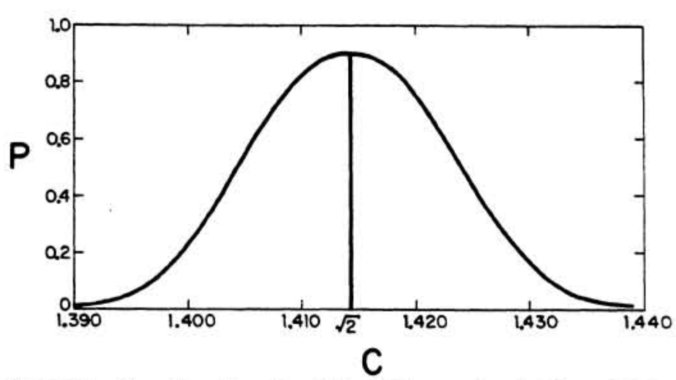

Figure 1. Operating-characteristic (OC) curve for testing validity of the $\sqrt{2}$ hypothesis for spacing of adjacent basin rings on silicate planets. P, probability; c, incremental spacing factor.