MIGRATION OF FLUID-FILLED CRACKS: APPLICATIONS TO TERRESTRIAL AND ICY BODIES. D.J. Stevenson, Div. Geol. Planet. Sci, Caltech, Pasadena, CA 91125.

Recent work (1,2) on the near-surface migration of magma has favored the notion of fluid-filled cracks which migrate by unzipping the rock ahead and zipping up the rock behind. This model is based primarily on the work of Weertman (3,4) together with experimental and theoretical expectations for the "fracture toughness" (stress intensity factor) for the medium. However, no complete theory for the migration velocity exists! Weertman was unable to quantify this crucial aspect of the model and subsequent workers have addressed only parts of the problem (either the fluid dynamic aspect or the fracture aspect but never a synthesis). I report here an attempt to construct a complete theory, and conclude that: (a) dynamic fluid cracks cannot close off at the base, unlike their fictitious static counterparts. All cracks have umbilical cords; "tadpoles" do not exist. (b) The irreversible work done at the upper tip is small compared to the viscous dissipation in the fluid. The presence of a volatile (gas) phase is not essential. In fact, the tail wags the dog. (c) Contrary to conventional wisdom, cracks can be arbitrarily long. Finite volume cracks have an asymptotic velocity of the upper tip $\propto t^{-2/3}$ where t is the elapsed time. (d) Crack "healing" at the base by magma freezing can provide approximately shape-preserving, propagating solutions at velocities appropriate for explaining terrestrial volcanism (i.e. $1\text{-}10^2$ cm/s). This work also has applicability to basaltic volcanism on other terrestrial planets and the migration of volatile-rich fluids such as NH_3-H_2O in icy satellites.

Formulation. The model is constructed in a form essentially equivalent to Weertman (4) with three important differences: the fluid velocity is allowed to vary along the crack (to allow for possible stagnation); this variability of the flow is coupled to the time variability of the crack profile by a continuity equation; the contribution to the normal stress σ on the crack walls arising from the shear stress applied by the fluid is neglected (this can be rigorously justified, notwithstanding the important role this term played in Weertman's work). The surface tension and non-Newtonian properties of the magma are neglected; this is reasonable for most applications of interest.

Crack Closure is Impossible. Consider two parallel planes with a viscous fluid in between. It is a simple fluid dynamical calculation to show that it requires infinite work to squeeze the fluid out from between these planes, even assuming incompressible flow (i.e. providing a reservoir into which the fluid can escape). Non-wetted crack tips, as in Fig. 31 of Barenblatt (5) are possible for injection from somewhere other than the tips, but irrelevant to the problem addressed here of a crack which nucleates at the roof of a magma chamber and extends upward. The common cartoon of a tadpole-shaped crack, as in Fig. 1 of Weertman (4) cannot be achieved. Instead, the dynamic contribution to σ (essentially the Pouseille flow contribution to the pressure gradient) tends to cancel the static contribution $-\Delta\rho gy$, where $\Delta\rho$ is the density difference between rock and magma, g is the gravitational acceleration and y is the vertical coordinate. The non-existence of pinch instabilities can be explicitly demonstrated. The dynamic balance occurs notwithstanding the tendency for fluid stagnation near the base of a finite volume crack.

The Top is where the Action Isn't. Whilst it is important to understand the processes near the crack tip (6), these processes do not appear to determine the rate at which magma can migrate. If the crack tip moves at velocity v, then the total work rate by gravity is almost $\Delta\rho gVv$, where V is the volume (the amount of stagnant fluid is small). The dissipation rate by irreversible

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processes (such as plastic deformation) at the tip is $2\Gamma v$ where $\Gamma \equiv K_{ic}^2/D(1-\nu^2)$ is the fracture surface energy, K_{ic} is the 'critical' stress intensity factor, E is Young's modulus and ν is Poisson's ratio. For the particular case of a crack where the base is approaching a tadpole shape, this dissipation is one eighth of the total power available. Similar factors apply for other configurations. This is actually an upper bound; it follows that the viscous dissipation of the fluid (especially in the lowermost, narrow part of the crack) is most important.

Extending Cracks. Although the base of a crack is pinned by its inability to pinch off, the top can move upward. It can be shown that such a solution is possible provided the fluid velocity $v(y) \simeq \Delta\rho g D^2(y)/12\eta$, everywhere except near the top, where D is the crack width and η is the magma dynamic viscosity. Together with the continuity equation, $\partial D/\partial t = -\partial(vD)/\partial y$, this implies an asymptotic solution of the form $v \propto (x/t)$, $D \propto (x/t)^{1/2}$ and crack length $\propto t^{1/3}$ where x is the distance from the crack base. (Notice the tendency towards crack closure for x finite and $t \to \infty$.) For plausible parameter choices, such cracks could "migrate" from 100 km depth to the earth's surface without magma freezing. Conduits of unlimited extent are possible.

Crack Healing. More realistically, the rind of frozen magma, which develops on the walls of the crack, can plug the crack base. It can be shown that the viscous dissipation at the base is then finite and an approximately shape-preserving, near-tadpole-like crack can migrate up leaving behind a frozen "umbilical cord" (actually a sheet). The velocity is dictated primarily by viscous dissipation near the base, at least for laminar flow elsewhere, and is given to order of magnitude by

$$v \simeq 0.1\, U_0^{6/7}\, U_1^{1/7}$$

$$U_0 \equiv g\Delta\rho D_0^2/12\eta$$

$$U_1 \equiv (KL/D_0^2)$$

where D_0 is an average crack width, K is the thermal diffusivity of the wall rock and L is the length of the fluid-filled portion of the crack.

Applications. The above formula for v is found to be compatible with estimates based on purely thermal considerations (7). It also provides a means for the quantification of magma eruption temperatures of other planets and the eruption volumes. In particular, the theory predicts larger, less frequent igneous events on bodies with smaller g. This has important implications for the interpretation of icy bodies such as Enceladus.

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