A PROBABILISTIC APPROACH TO BALLISTIC DIFFERENTIATION OF SURFACIAL SOILS ON MOON-LIKE PLANETS; Abhijit Basu, Dept. of Geology and Rabi N. Bhattacharya, Dept. of Mathematics, Indiana Univ., Bloomington, IN 47405, USA

We have examined a situation where soil particles of a smaller mass m, and a greater mass M, are ejected by the same impulsine force F, in response to micrometeoritic bombardment on the surface of a moon-like planetary body. Assume that the initial velocities in the direction of projection are governed by the equation:

$$mv_m = F = Mv_M \quad : \quad V_m = F/m, \; V_M = F/M$$

The horizontal distances (h) traveled are:

$$h_m = t_m v_m \cos\Theta = \frac{2F^2 \sin\Theta\cos\Theta}{m^2 g} = \frac{F^2 \sin 2\Theta}{m^2 g}, \quad h_M = \frac{F^2 \sin 2\Theta}{M^2 g}$$

where, t = time of flight, Θ = angle of ejection, and g = accelaration due to gravity. Assume that Θ is uniformly distributed on $[0,\frac{\pi}{2}]$. Because micrometeoritic bombardment is random, the angle ϕ between the vertical plane containing the path of the ejected particle and an arbitrary axis originating at the point of impact and on the horizontal plane will be distributed uniformly on $[0,\frac{\pi}{2}]$. Thus,

$$\text{Prob}\,(h_m \leq r) = \text{Prob}\,(F^2 \sin 2\Theta / m^2 g \leq r)$$

$$= \text{Prob}\left(\text{Sin}2\Theta \leq \frac{m^2 gr}{F^2}\right) \quad = \text{Prob}\left(2\Theta \leq \text{Sin}^{-1}\left(\frac{m^2 gr}{F^2}\right),\; 0 \leq 2\Theta \leq \frac{\Pi}{2}\right)$$

$$+ \text{Prob}\left(2\Theta \geq \Pi - \sin^{-1}\left(\frac{m^2 gr}{F^2}\right),\; \frac{\Pi}{2} < 2\Theta \leq \Pi\right)$$

$$= \text{Prob}\left(\Theta \leq \frac{1}{2}\sin^{-1}\left(\frac{m^2 gr}{F^2}\right) \quad + \text{Prob}\left(\Theta \geq \frac{\Pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{m^2 gr}{F^2}\right)\right)\right.$$

$$= \text{Prob}\left(\Theta \epsilon \left[0, \frac{1}{2}\sin^{-1}\left(\frac{m^2 gr}{F^2}\right)\right]\right) + \text{Prob}\left(\Theta \epsilon \left[\frac{\Pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{m^2 gr}{F^2}\right), \frac{\Pi}{2}\right]\right)$$

$$= 2\left(\frac{\frac{1}{2}\sin^{-1}\left(\frac{m^2 gr}{F^2}\right)}{\Pi/2}\right) \quad = \frac{2}{\pi}\sin^{-1}\left(\frac{m^2 gr}{F^2}\right) \quad \left[\begin{array}{l}\text{Since Prob } (\Theta\epsilon\ (a,b)) \\ = (b-a)/\frac{\Pi}{2} \text{ for} \\ 0 \leq a < b \leq \frac{\Pi}{2}\end{array}\right]$$

Similarly,

$$\text{Prob}\,(h_M \leq r) = \frac{2}{\Pi}\sin^{-1}\left(\frac{M^2 gr}{F^2}\right), \quad \left[0 \leq r < \frac{F^2}{M^2 g}\right].$$

In the annular region

$$A_m = \{(r,\phi): \frac{F^2}{M^2 g} \leq r < \frac{F^2}{m^2 g}, 0 \leq \phi 2\Pi\}$$

no M-particles land. The proportion of m-particles that land in A_m is

$$p_m = \text{Prob} \quad \frac{F^2}{M^2 g} \leq h_m < \frac{F^2}{m^2 g} \quad = \frac{2}{\Pi} \cos^{-1} \frac{m^2}{M^2} .$$

All the M-particles land within the circle

$$R_M = \{(r,\phi): 0 \leq r < \frac{F^2}{M^2 g}, 0 \leq \phi < 2\Pi\} .$$

The area of A_m = $a_m = \frac{\Pi F^4}{g^2} \left(\frac{1}{m^4} - \frac{1}{M^4} \right)$

and that of R_m = $b_M = \frac{\Pi F^4}{M^4 g^2}$

Note that $a_m > 0$ ($M > m$) and although there will be an overlap of R_m of one impact on to the A_m of a neighboring impact, larger proportion of m-particles are in flight for greater duration. Thus, over time there will be a concentration of lighter particles above heavier particles in the soil profile unless events more forceful than micrometeoritic bombardment disrupt the system.

This suggests that particles of lesser mass, either because of smaller size or because of lower density, are likely to form a thin veneer at the surface of moon-like bodies that may undergo gardening by micrometeoritic bombardment. In the case of the moon, we know that the finest size fraction of soils are always compositionally different from the rest of the regolith (e.g. Devine et al., 1982). Therefore, the topmost veneer of lunar soils is not likely to be representative of the regolith. Remote sensing methods that obtain signals from this topmost veneer would, therefore, obtain compositional data of the finest fraction of lunar soils and probably not of the average regolith (cf. Basu, 1985).

REFERENCES: Basu, A. (1985) 48th Met. Soc. Mtg. Abs., 7; Devine, M. et al. (1982) PLPSC 13th, JGR, 87, A260-A268.