NEW FORMULAS FOR THE EVOLUTION OF PLANETESIMAL VELOCITIES

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We have derived a self-consistent set of equations for the velocity evolution of a planetesimal population that is induced by mutual gravitational scattering and inelastic collisions. Our central assumption was to approximate the velocity distribution in the Fokker-Planck and Boltzmann collision operators by a Schwarzschild distribution such that $\langle v_r^2\rangle = 4\langle v_\theta^2\rangle$ and $\langle v_r^2\rangle = 4\langle v_z^2\rangle$. The first equality is exact in the limit of collisionless orbits, while the second equality approximates the result of numerical simulations (1) which found mean square orbit inclinations to be about 1/4 the mean square orbit eccentricities. A similar calculation by Hornung et al. (2) leaves the ratio $\langle v_r^2\rangle:\langle v_z^2\rangle$ arbitrary and therefore obtains somewhat more complicated results.

The resulting rate of change of the kinetic energy density of non-circular motions, $\partial(n_a m_a \langle v_a^2\rangle/2)/\partial t$, of planetesimals of mass m_a due to gravitational encounters and inelastic collisions with planetesimals of mass m_b and number density n_b consists of four terms:

1. Viscous stirring due to gravitational scattering

$$\frac{3\sqrt{\pi}}{4}G^2 N_{ab}\ln\Gamma\; V_{ab}^{-3}\left\{\frac{(9L-12\sqrt{3})}{(m_a+m_b)^2}\left[3m_a m_b(m_a\langle v_a^2\rangle+m_b\langle v_b^2\rangle)+(m_a^3+m_b^3)\langle v_a^2\rangle\right] + L(m_b\langle v_b^2\rangle-m_a\langle v_a^2\rangle)\right\}$$

2. Energy exchange due to dynamical friction

$$4\sqrt{3\pi}\; G^2 N_{ab}\ln\Gamma\; V_{ab}^{-3}\;(m_b\langle v_b^2\rangle - m_a\langle v_a^2\rangle)$$

3. Viscous stirring due to inelastic collisions

$$\frac{\sqrt{\pi}}{8}(\sqrt{3} - \tfrac{11}{36}L)N_{ab}(R_a+R_b)^2 B V_{ab}\left\{m_b(\langle v_a^2\rangle-\langle v_b^2\rangle) + 2m_a\langle v_a^2\rangle\right\}(m_a+m_b)^{-2}$$

4. Energy damping due to inelastic collisions

$$-\sqrt{\pi}(\tfrac{11}{18}\sqrt{3} - \tfrac{L}{24})N_{ab}(R_a+R_b)^2 B V_{ab}\left\{m_b(\langle v_a^2\rangle-\langle v_b^2\rangle)+ 2m_a\langle v_a^2\rangle\right\}(m_a+m_b)^{-2}$$

where G is the grav. constant, $N_{ab} = n_a n_b m_a m_b$, $V_{ab}^2 = \langle v_a^2\rangle+\langle v_b^2\rangle$,

$$\Gamma = \frac{\langle v_b^2\rangle^{1/2}\Omega^{-1}}{\max\left\{\dfrac{G(m_a+m_b)}{\langle v_a^2\rangle+\langle v_b^2\rangle},\ (R_a+R_b)B^{1/2}\right\}}, \quad L = \ln\left\{\frac{\tan(5\pi/12)}{\tan(\pi/12)}\right\} \simeq 2.634,$$

Ω is the orbit frequency, and $B \gtrsim 1$ is the gravitational enhancement of the collision cross-section. A recent investigation (3) finds that the factor B is limited to a maximum value of about 3000 at 1 A.U. from the sun due to the essentially two dimensional nature of the planetesimal disk at large encounter distances. Note that the above expressions assume that the entire relative velocity is damped during an inelastic collision.

In order to determine the relative importance of the four contributions listed above, we have calculated the steady state velocities of a simple bimodal population consisting of just two different planetesimal masses where $m_a = 10^{21}$g and $m_b = 10^{20}$g. Three separate cases were considered in which the ratio of the mass surface density of m_a to that of m_b was 5, 1, and 0.1 for cases 1, 2, and 3, respectively. The surface density of the more abundant size was 10g/cm^2 in all three cases.

Dynamical friction was found to be the dominant energy loss term for the larger particles in cases 1 and 2, depressing the rms velocity of m_b by a factor of 5 in case 1 and a by factor of 2 in case 2 relative to the rms velocity of m_b in case 3, which was 1.0 km/s. This value may be compared with 1.2 km/s, the escape velocity from the surface of a body of mass 10^{20} g and density 3 g/cm^3. In all cases the rms velocity of m_a was about 0.2 km/s greater than that of m_b. Dynamical friction was a significant energy source for the smaller particles in all three cases, being roughly equal to the energy input due to viscous stirring of the smaller particles in cases 2 and 3. Inelastic collisions among the smaller particles accounted for most of the energy loss from the system in cases 1 and 2, whereas collisions amoung the larger particles was the major loss mechanism for case 3.

We believe our equations have a firmer theoretical basis than the analogous expressions obtained by relaxation-time arguements. Application of these equations to the planetary accretion problem is described in a companion abstract (4).

References:

(1) Wetherill, G.W., Geol. Soc. Canada Spec. Pap. 20, 3-24, (1980)
(2) Hornung, P., Pellat, R., and Barge, P., Icarus, in press, (1985)
(3) Wetherill, G.W. and Cox, L.P., Icarus 63, 290-303, (1985)
(4) Wetherill, G.W. and Stewart, G.R., Lunar Planet. Sci., 17, this meeting. (1986)