PLANETARY COLLISION CALCULATIONS: ORIGIN OF THE MOON

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We previously [1] described a three-dimensional smoothed particle hydrodynamics (SPH) code and its use for investigations of the single-impact hypothesis for the origin of the Moon. These simulations showed that substantial amounts of mass could be injected into orbit about the Earth.

We have now made these simulations more realistic by introducing iron cores into the planetary bodies. The iron cores play a distinct and important role in the mechanics of mass injection into orbit around the Earth.

Stevenson ([2] and private communication) has pointed out that our use of the Tillotson equation of state does not represent adequately a two-phase medium, since for any given internal energy one can have an enormous range of pressures, depending on the liquid mass fraction. Therefore pressure gradients computed from the Tillotson equation of state in any two-phase medium may be inaccurate. Since we find that a great deal of material can be put into orbit with this possible underestimation of pressure-gradient acceleration, we believe that our results can only be enhanced by use of a better equation of state.

The present series of simulations involves collisions between the protoearth and impactors of various masses. The mass ratio between the two planets was obtained by using different numbers of particles for both the protoearth and impactor. All the simulations were started with a total angular momentum just slightly higher than the present angular momentum of the Earth-Moon system (see the Table for exact values). This ensures that the final system has about the right angular momentum after allowing for the few particles that escape. The total mass of the system was chosen to be equal to the mass of the Earth-Moon system. The impact parameter is thus automatically determined. For the three higher velocity cases we chose a somewhat larger value to account for the larger mass-loss occurring during those collisions.

The simulation is initiated by setting both planets on a collision course. The initial positions correspond to a separation of 10 protoearth radii along the x-axis and the appropriate impact parameter along the y-axis. The relative velocity was computed from energy conservation so as to match the desired relative velocity at infinity.

A total of 9 new simulations were run for various combinations of parameters; the initial characteristics of these simulations can be found in the Table.

The results of the low-velocity simulations are very sensitive to the mass of the impactor. However, the first stages are roughly the same. As the impactor approaches the protoearth it becomes deformed by the tidal field. At the time the impactor hits, it slows down and there is a corresponding dissipation of kinetic energy into heat in the strong shock at the interface. Material is then vaporised and ejected in two high speed jets, one directed forward and one smaller directed backwards. The intensity of this jetting is of course determined by the intensity of the shock which is in turn determined in part by the impact parameter. In our simulations, the impact parameter is fixed by the mass of the impactor since all our computations are done at (almost) fixed angular momentum. This means that we get more prominent jetting in the case of a large mass impactor than a small one. Once this jetting begins and once the impactor starts to be destroyed the various simulations differ quite strongly from each other.

Low mass impactors ($M_{\rm impactor}/M_{\rm protoearth} \leq 0.12$). The collision is almost a grazing one (owing to the angular momentum constraint); therefore the shock is weak, and the impactor is thrown onto a very eccentric orbit that makes it collide again with the protoearth. The second collision destroys the impactor completely and a great part of it is flung into orbit. Most of the iron core of the impactor ends in orbit as well, leading to an iron-rich accretion disk. We computed the orbits of all the particles to see whether they will collide with the protoearth, reenter the Roche lobe, or even escape the system. The escaping fraction is quite large. The shock is not strong enough to cause this ejection; this mass was thrown away by the gravitational torques due to the various clumps in orbit.

The Table gives the result of these calculations. We also give in the Table the mass fraction with equivalent circular orbits inside and outside the Roche limit. We determined the iron content in each category as well as the amount of material which originated from the protoearth. Note that in none of our simulations does iron from the core of the protoearth get into orbit; any iron in orbit always comes from the core of the impactor.

Large Mass Impactors ($M_{\rm impactor}/M_{\rm protoearth} \geq 0.17$). Large mass impactors collide with a relatively small impact parameter (due to the angular momentum constraint). The shock is very strong and large jets are created. A significant amount of mass can escape the system despite the collision occurring just at the escape velocity and take a good fraction of the energy and angular momentum with it; most of the material that is left is quickly accreted by the protoearth. The fraction left in orbit is iron poor but its mass is less than a lunar mass.

Intermediate Mass Impactors ($0.12 < M_{\rm impactor}/M_{\rm protoearth} < 0.17$). This is the only mass range that, together with our angular momentum constraints, leads to a suitable prelunar accretion disk or even, in one case, to a Moon-sized object orbiting the protoearth outside the Roche lobe. During the collision the iron core is more resistent to the shock than was the granite mantle, and as a result it is not completely destroyed. When it gets sufficiently far away from the protoearth its self-gravity is able to pull it back together. We have now formed primarily a three-body system: the protoearth, the impactor's core, and the most of the remaining parts of the impactor's mantle. The latter body is by far the smallest of the three. The iron core is on a close, eccentric orbit about the protoearth.

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The rocky clump is on a trajectory that takes it further out and therefore it travels at a somewhat smaller velocity, lagging behind the core. This results in a tremendous gravitational torque acting on the outer material arising from the protoearth and the iron core. Enough angular momentum is transferred in the outer regions to make sure that the rocky material will not collide with the protoearth again. Having thus given away a large part of its angular momentum, the iron core hits the protoearth at the end of its first revolution, giving rise to a gigantic splash. For a mass ratio of 0.136 we are left with a pure granite clump of one lunar mass orbiting safely outside the Roche lobe, together with some smaller pieces, and with slightly more than half a lunar mass orbiting inside the Roche limit. That system has a total angular momentum equal to the present Earth-Moon system to within half a percent. For other mass ratios in this range the granite material is spread out and sometimes comes within the Roche lobe.

High Velocity Collisions (V_∞ = 10 km/s). Only three simulations were run in this velocity regime since the results indicate that there is no chance of forming a Moon or a suitable prelunar accretion disk at this velocity. A large impactor mass is needed if enough mass is to be put into orbit. A mass ratio of 0.18 is the only one of the three simulations that leads to more than a lunar mass in orbit. However, more that 1.3 lunar masses of iron is included in that mass, and a large part of the impactor's mantle escapes. There is no convincing mechanism that would allow the separation of the rocks from the iron in orbit.

None of our "successful" simulations led to the formation of a disk of material confined within the Roche limit. At least a comparable amount of material always orbited beyond the limit, either in the form of a major clump or divided into several small pieces. We computed the mean angular momentum of all material in orbit and deduced a corresponding mean radius from it. This mean radius is listed in the tenth column of the Table. All of these mean radii for the cases having more than a lunar mass in orbit are larger than the Roche limit. For high velocity collisions this mean radius is quite large.

Table: Principal results of the simulations. The columns give, in order, the number of the run, the relative velocity of the projectile at infinity, the total angular momentum in the collision (normalised to the Earth-Moon system angular momentum $J_{\rm Earth-Moon} = 3.5 \times 10^{41}$ gm cm^2 sec^{-1}), the mass ratio of the impactor to the protoplanet, the mass orbiting the protoearth inside Roche limit, the iron fraction in that mass, the mass orbiting the protoearth outside the Roche limit, the iron fraction in that mass, the total mass of material from the protoearth in orbit, the mean circular radius of the orbiting material, and finally the mass escaping the system. All masses are given in units of one lunar mass.

Run	v_∞ km/sec	$J_{\rm tot}$ $J_{\rm Earth-Moon}$	mass ratio	$M_{\rm in}$ $M_{\rm Moon}$	%Fe	$M_{\rm out}$ $M_{\rm Moon}$	%Fe	$M_{\rm Earth}$ $M_{\rm Moon}$	$\bar{R}_{\rm circ}$ $R_{\rm Earth}$	$M_{\rm escape}$ $M_{\rm Moon}$
1	0.0	1.06	0.10	1.23	60	0.67	16	0.0	2.8	1.37
2	0.0	1.08	0.12	0.78	14	0.92	6	0.08	3.0	0.86
3	0.0	1.09	0.14	0.62	0	1.32	0	0.13	3.7	0.32
4	0.0	1.08	0.16	0.73	0	0.51	0	0.16	3.1	0.54
5	0.0	1.10	0.18	0.48	0	0.24	0	0.11	2.8	0.54
6	0.0	1.15	0.25	0.12	0	0.00	0	0.05	1.6	0.69
7	10.0	1.17	0.10	0.05	0	0.08	0	0.05	3.5	6.17
8	10.0	1.18	0.14	0.32	75	0.54	70	0.05	4.1	5.50
9	10.0	1.35	0.18	0.35	62	1.85	61	0.27	5.2	5.98

Thompson and Stevenson [3] have pointed out that a hot and thick disk may extend quite a long way outside the Roche distance without necessarily becoming gravitationally unstable to breaking up into clumps. Thus the formation of a disk extending outside the Roche lobe in many of our simulations does not mean that gravitational instabilities will immediately break up the outer part of the disk. However, that event must happen sooner or later, when the outer part of the disk has cooled sufficiently. If several clumps are formed at that time, many of them will soon collect into a single body, since a circular distribution of mass points is gravitationally unstable, and a few of the clumps may be expelled from the system in the process. It thus appears that a single body will be formed beyond the Roche lobe after an indeterminate history in which the material may go through the stages of being part of a disk and as being divided among several other bodies.

The outer orbiting clump will greatly perturb the material orbiting inside the Roche limit at least down to a certain distance. It is likely that the outer part of the disk will not last long once the clump has formed. Some of the perturbations may increase the radial distance of part of the material so that it will either accrete onto the main clump or be ejected from the system. But the perturbations will certainly also raise tides throughout the disk, and these tides should be capable of transferring much of the angular momentum in the disk to the clump, with an accompanying inward flow of the material in the disk [4,5].

We therefore propose that one should consider as prime lunar formation candidates those simulations that lead to about a Moon mass of iron-free material in orbit in the general vicinity of the Roche limit. This consideration limits the impactor's mass to the relatively narrow range 6.5×10^{26} gm $\leq M_{\rm impactor} \leq 8.2 \times 10^{26}$ gm.

[1] Benz, W., W. L. Slattery, and A. G. W. Cameron 1986, *Icarus*, **66**, 515-535. [2] Stevenson, D. J. 1987, To appear in *Ann. Rev. Astron. Astrophys.* [3] Thompson, A. C., and D. J. Stevenson 1987, preprint. [4] Goldreich, P., and S. Tremaine 1980, *Astrophys. J.* **241**, 425–441. [5] Lin, D. N. C., and J. Papaloizou 1979, *Mon. Not. Roy. Astron. Soc.* **186**, 799–812.