THREE-BODY RESONANCE TRAPPING AND THE ASTEROID BELT

Chris W. Patterson, Los Alamos National Laboratory, Los Alamos, NM 87545

A strong correlation has been shown by Dermott and Murray[1] between the width of the Kirkwood gaps and the asteroid eccentricities at two-body p/(p+q) resonances with Jupiter where

$$pn_A = (p+q)n_J - q\dot{\omega}_A.$$

The gap width is related to the square root of the resonance strength given by

$$S_2 = f(\alpha)\mu_J n_A^2 e_A^q,$$

where α is the ratio of the semimajor axes $\alpha=a_A/a_J$, μ is the mass, n is the mean motion and e is the eccentricity (J=Jupiter and A=asteroid).

Asteroids in decaying orbits in the primordial solar nebula cannot be captured by such two-body resonances but they can be trapped by the three-body resonances

$$pn_A = (p+q)n_J \pm n_{op} - (q\pm1)\dot{\omega}_A$$

which involve the mean motion of an outer planet. The strengths of the three-body resonances of an outer planet are given by

$$S_3 = g(\alpha)\mu_J n_A^2(\mu_{op}n_J/n_{op})e_A^{q\pm1}$$

and have been evaluated for resonances with $e_A \approx 0.3$ and $q \lesssim 5$ for the different outer planets. Because Saturn is very close to a 2/5 resonance with Jupiter, the three body resonances of Saturn tend to overlap gaps at the stronger ($S_2 > S_3$) two-body resonances of Jupiter. This is not the case with Uranus. It is found that trapping can readily occur at the three-body resonances of Uranus for asteroids decaying at a rate $\dot{n}_A = S_3$ corresponding to 10^{-7}-10^{-9} AU/yr. Because such three-body resonances are spaced ~0.1 AU apart, it takes only 10^6-10^8 years to capture all such asteroids between these three-body resonances of Uranus.

Strong evidence for such three-body resonances is shown in the asteroid mass spectra of Figs. 1-2. In both figures, the dotted verticle lines show the positions of the p/(p+q) two-body gaps and the dashed verticle lines show the positions of the associated three-body resonance peaks which are displaced from the gaps by $\pm n_U/p$. The correlation between the mass peaks and the three-body resonances is evident. Note that trapping occurs preferentially 'downstream' of the gaps where the asteroids could have acquired their large eccentricities making capture more favorable.

The three highest peaks in Fig. 1 occur downstream from the three strongest resonances. A least squares fit of n_U to the peaks in Fig. 1 results in $n_U=(1/3)n_S$ to within one standard deviation (0.001) which is 1/20 the peak width. Thus, it appears the asteroids were trapped while Uranus was in a 1/3 resonance with Saturn. After Uranus was collisionally displaced from this resonance, the asteroids were no longer trapped and could then collisionally evolve.

The classical asteroid families in Fig. 2 can be interpreted as a result of resonance trapping rather than collisional disruption of a parent body. The multipeaked character of the Flora family[2,3] and the large dispersion of the Koronis family become understandable in terms of resonance trapping.

1. Dermott, S. F. and Murray, C. D., Nature 301, 201-205 (1983).
2. Kozai, Y., in Asteroids (T. Gehrels, Ed.) 334-358, Univ. of Arizona Press, Tucson, 1979.
3. Tedesco, E. F., Icarus 40, 375-382 (1979)

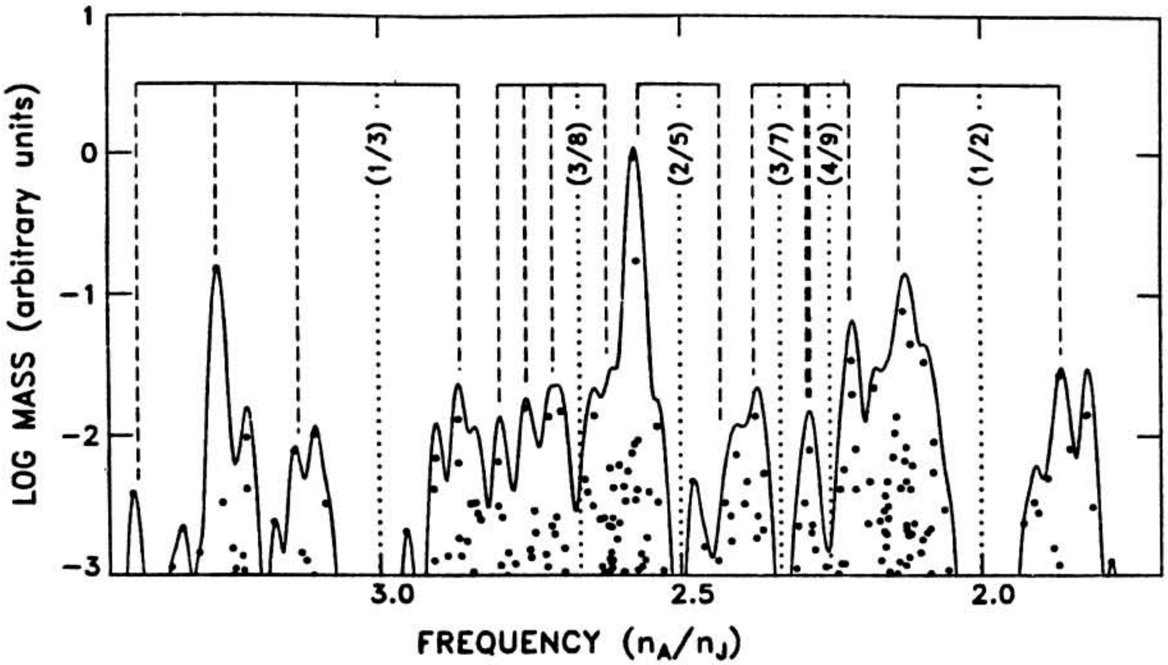

Fig. 1 **Asteroid Mass Spectrum.** Plot of all asteroids with diameters greater than 100 km. The solid curve is the sum of the asteroid masses in $\Delta n_A=(0.02)n_J$ size bins corresponding to a tangential velocity dispersion of 50 m/sec.

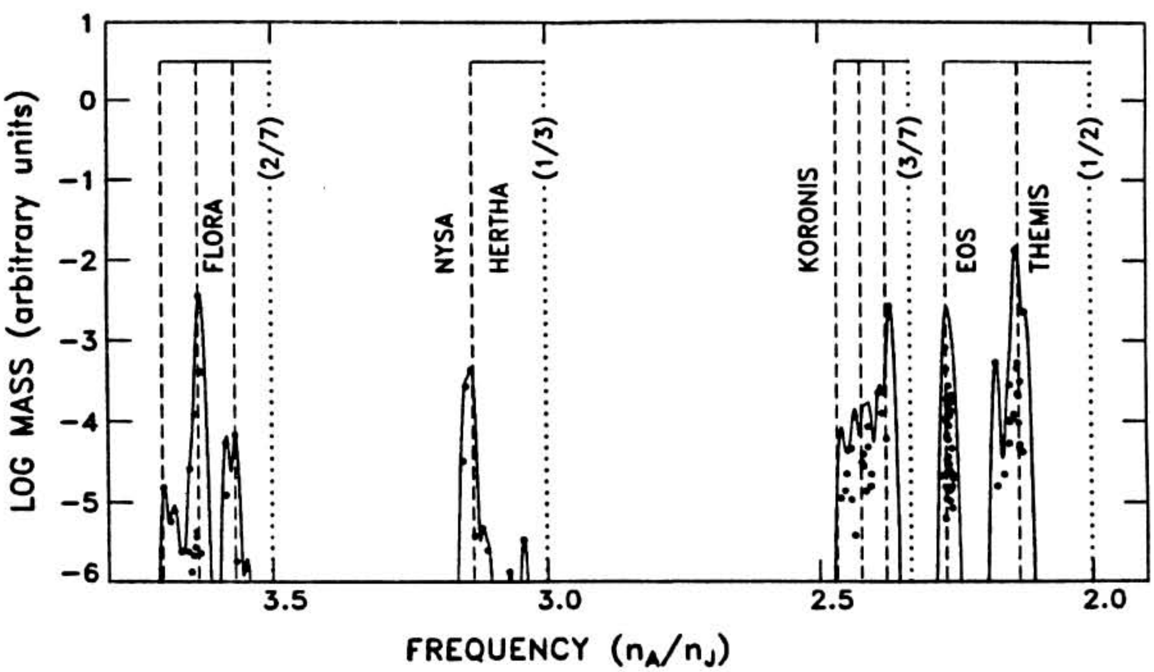

Fig. 2 **Mass Spectrum of Asteroid Families.** Plot of the classical asteroid family members (including Nysa-Hertha) with diameters greater than 10 km and a $\Delta n_A=(0.01)n_J$ bin size corresponding to 25 m/sec.