ORBITAL RESONANCES IN THE SOLAR NEBULA: TIMESCALES AND RESONANCE WIDTHS; Stuart J. Weidenschilling and Donald R. Davis, Planetary Science Institute, Tucson, Arizona 85719

An important property of the solar nebula is non-Keplerian rotation of the gaseous component due to a radial pressure gradient (1,2). Solid bodies on Keplerian orbits experience a headwind that causes decay of their orbits due to drag. Weidenschilling and Davis (3) examined the combined effects of gas drag and gravitational perturbations by a planetary embryo on the orbital evolution of a planetesimal. They showed that perturbations at commensurability resonances (ratio of periods (j+1)/j, where j is an integer) tend to push the planetesimal away from the embryo's orbit; the opposing effects allow trapping in stable resonant orbits. Resonant perturbations pump up eccentricities, leading to disruptive collisions and comminution of planetesimals. Weidenschilling and Davis suggested a scenario in which this phenomenon allows an early-formed embryo to inhibit the growth of potential rivals, while it grows rapidly by capture of small fragments that are brought through the resonances by drag. We report here some additional properties of such resonances that may modify this scenario.

In order for a planetesimal to be trapped, the drag force must not exceed the effects of resonant perturbations, i.e., the area/mass ratio must not be too large. The minimum size for trapping corresponds to a planetesimal radius s_{min}. The expression for s_{min} given in (3) and Davis is in error; a corrected derivation yields

$$s_{min} \simeq (\rho/\rho_s)(\Delta V/V_k)a_1/(3\mu C_j\ j^{2/3}(j+1)^{1/3})$$

where ρ and ρ_s are densities of the gas and planetesimal, $\Delta V/V_k$ is the fractional deviation of the gas from Keplerian motion (typically a few times 10^{-3}), a_1 is the embryo's orbital radius and μ its mass in solar units. C_j is a coefficient that depends on j; in general, $C_j \simeq 2j$. The maximum value of j that corresponds to a stable resonance is given by Wisdom's (4) criterion for resonance overlap at $j \gtrsim 0.5\ \mu^{-2/7}$.

A trapped body has its eccentricity pumped up to an equilibrium value

$$e_{eq} \simeq 1.14\ [(\Delta V/V_k)/(j+1)]^{1/2},$$

independent of the planetesimal's size or the embryo's mass; this value is in excellent agreement with the results of numerical integrations of test orbits. At a resonance, the rate of eccentricity pumping is given by

$$de/dt \simeq (e_{eq} - e)/\tau,$$

where the time constant τ is given by

$$\tau \simeq (s/a_1)(\rho_s/\rho)(a_1/1AU)^{3/2}/(\Delta V/V_k)\ \text{yr}.$$

An important consideration for evaluating the effects of resonant trapping is the width of a resonance, i.e., the range of semimajor axis over which a planetesimal responds to resonant perturbations. As with any damped oscillator, lower damping (a larger planetesimal or lower gas density) produces a narrower resonance near exact commensurability; higher damping yields a broader range of response with a lower peak. Using

ORBITAL RESONANCES
Weidenschilling, S.J. and Davis, D.R.

analytic theory (5), it can be shown (6) that the width of a resonance is approximately

$$\delta a/a_1 \simeq 0.5[((j+1)/j^3)C_j\mu\ (\rho/\rho_s)(a_1/s)]^{1/2}$$

Some numerical values of s_{min}, δa, and τ are given in Table I, assuming $\Delta V/V_k = 5 \times 10^{-3}$. In principle, an embryo of 0.1 $M_\oplus$ at 1 AU can trap km-sized planetesimals. However, the timescale for eccentricity pumping is ~10^3-10^5y, depending on j. Resonance widths are typically a few times 10^{-5} AU for this embryo mass; a velocity perturbation $\gtrsim$ 100 cm/s may move a planetesimal through a resonance. Perturbations of this magnitude due to collisions or close encounters with other planetesimals occur on a timescale of the same order as τ. Both timescales increase directly with the planetesimal size, but resonance width, and the velocity increment needed to cross resonance, decreases. Thus, resonant trapping is marginally possible for $s \sim s_{min}$, but becomes less likely for larger bodies. The situation is similar for early-stage accretion in Jupiter's zone; the lower gas density yields larger τ for a given planeteismal size, but smaller values of s_{min}.

These results suggest a modification to the accretion scenario of (3). Trapping of planetesimals in stable resonances for times sufficient to reach e_{eq} may be rare, or confined to a limited size range $\gtrsim s_{min}$. The dominant effect of resonances may be "non-equilibrium" eccentricity pumping of larger and smaller bodies as they pass through resonant zones, by diffusion or gas drag, respectively. Resonances are "transparent" to sufficiently large bodies. The mass arriving at the embryo's orbit, and accreted by it, may consist of a bimodal distribution of small fragments and large impactors; the latter may be needed to explain planetary obliquities. These phenomena must be included in any realistic model of planetary accretion in the presence of gas.

TABLE I

a_1=1 AU	ρ_s=3g cm^{-3}		$\rho=10^{-9}$g cm^{-3}			
		μ=3E-7			μ=3E-6	
j	s_{min}, cm	δa, AU	$\tau(s_{min})$, yr	s_{min}, cm	δa, AU	$\tau(s_{min})$, yr
1	2.5E7	5.1E-6	1E6	2.5E6	5.1E-5	1E5
2	2.4E6	1.7E-5	1E5	2.4E5	1.7E-4	1E4
3	1.3E6	1.7E-5	5E4	1.3E5	1.7E-4	5E3
5	5.0E5	1.9E-5	2E4	5.0E4	1.9E-4	2E3
9	1.5E5	2.4E-5	6E3	1.5E4	2.4E-4	6E2
18	5.0E4	2.7E-5	2E3	5.0E3	2.7E-4	2E2
33	1.5E4	3.6E-5	6E2	—	—	—
	a_1=5.2AU		ρ_s=2g cm^{-3}	$\rho=10^{-11}$g cm^{-3}		
		μ=3E-5			μ=1E-3	
j						
1	2.0E4	2.6E-3	1.2E5	6.0E2	8.7E-2	3.6E3
2	1.9E3	8.8E-3	1.2E4	5.7E1	2.9E-1	3.5E2
3	1.0E3	8.8E-3	6.1E3	3.0E1	2.9E-1	1.8E2
5	4 2E2	9.4E-3	2.6E3	—	—	—
9	1.4E2	1.1E-2	8.5E2	—	—	—

1. Adachi, I., C. Hayashi, and K. Nakazawa (1976). Prog. Theor. Phys. 56, 1756-1771. 2. Weidenschilling, S.J. (1977). MNRAS 180, 57-70. 3. Weidenschilling, S.J. and D.R. Davis (1985). Icarus 62, 16-29. 4. Wisdom, J. (1980). Astron. J. 85, 1122-1133. 5. Greenberg, R. (1978). Icarus 33, 62-73. 6. Weidenschilling, S.J. (1987). Gerlands Beiträge zur Geophysik, submitted.