On the Relaxation of Herschel Basin and Mimas' Tidal Bulge

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The rate at which viscous relaxation of topographic relief occurs on a planetary body is determined by the viscosity structure of the underlying medium to a depth comparable with the wavelength of the topographic feature. The features of largest topographic wavelength on the icy satellites include the large impact basins Herchel (on Mimas), Odysseus (on Tethys) and the provisionally named Tirawa basin on Rhea. These craters have diameters that are, respectively, 71, 82 and 46% of the radius of the satellites on which they lie. Odysseus and Tirawa exhibit extensive relaxation, whereas Herschel does not.

For a homogeneous model of Mimas, the e-folding time for topographic reduction of Herschel's relief (relaxation time), t_c, is given by

$$t_c = \frac{4\pi\mu f(D/R_p)}{\rho g D} \tag{1}$$

where ρ is density (930 kg m^{-3}), g gravitational acceleration (0.08 m s^{-2}), D crater diameter (140 km), R_p the radius of Mimas and μ the dynamic viscosity [1]. For analytic models of crater relaxation in a semi-infinite planar layer, $f(D/R_p) = 1$. For the case of a spherically symmetric body, however, it may take other values. Both analytic calculations by Cathles [2] and numerical calculations by Thomas and Squyres [3] predict that viscously relaxing topographic features with wavelengths comparable to the radius of the satellite have relaxation times significantly smaller than those calculated from planar considerations. For the Herschel basin ($D/R_p = 0.71$), our numerical calculations indicate that $f(D/R_p) = 0.40$ [3]. If we assume that Herschel basin was formed in the primordial bombardment of the saturnian satellites, > 4 Gya, and has not undergone significant relaxation, we find from equation (1) that the global viscosity of Mimas, μ, is required to be $> 2.63 \times 10^{23}$ Pa s. Given the viscosity law of Weertman [4], with material parameters selected for ice by Reynolds and Cassen [5] we find that this corresponds to a global mean temperature of < 146 K.

Another class of large scale surface features on the icy satellites are the tidal distortions of the satellites' shape. The extent of the distortion for Mimas has been measured to high precision by Dermott and Thomas [6]. Mimas is well represented by a triaxial ellipsoidal, with a ratio of the axes, $(b-c)/(a-c)$, of 0.27 ± 0.04. This value is very close to the value of 0.25 expected for a body in hydrostatic equilibrium. Assuming that the satellite is indeed in hydrostatic equilibrium, Dermott and Thomas use the shape to infer the degree of differentiation of Mimas' interior, and propose that Mimas contains a silicate core of radius 0.43R_p (where R_p is the radius of Mimas). An alternative set of models, not involving the assumption of hydrostatic equilibrium, have been proposed by Ross and Schubert [7].

The relaxation time of a tidal bulge of a homogeneous satellite is given by Darwin [8]

$$t_b = \frac{19\mu}{2\rho g R_p} \tag{2}$$

Dividing equation (1) with the appropriate values for D and $f(D/R_p)$ by equation (2) we find that $t_c/t_b = 0.75$. If we substitute a viscosity of $> 2.63 \times 10^{23}$ Pa s in equation (2), we obtain $t_b > 5.51 \times 10^9$ y. This excessively long time argues against the likelihood of Mimas relaxing to hydrostatic equilibrium. On the other hand, if the extraordinary agreement between the figure of Mimas and that of a triaxial ellipsoid is real, then one has to account for the rapidity of relaxation of the satellite to a figure of hydrostatic equilibrium while at the same time preserving the Herschel basin in an unchanged form.

Can the presence of a rigid core change the value of t_c/t_b to a less restrictive value? To remove the conflict we would wish $t_c/t_b \gg 1$, to allow relaxation of the tidal bulge to occur without any relaxation of the Herschel basin. In fact the trend with increasing core size is in the opposite direction. Numerical calculations using finite element simulations of viscous flow in a spherical medium containing a rigid core indicate that for a core radius 25% of the satellite radius ($R_{core}/R_p = 0.25$), $t_c/t_b = 0.65$ and for $R_{core}/R_p = 0.50$, $t_c/t_b = 0.34$. The reason for this trend is that tidal bulges produce deeper flows and thus are influenced more by the presence of a core than a crater basin. Larger silicate cores are excluded by the low average density of Mimas (1170 kg m^{-3}).

Viscosity gradients were incorporated into the model to simulate global temperature gradients. However, in no case did a significant alteration of the relaxation time behavior occur. For example, for a temperature gradient of 0.1 K km^{-1} (greatly in excess of the adiabatic thermal gradient of $\sim 10^{-4}$ K km^{-1}), t_c/t_b was calculated to be 0.79.

Finally, the effect of a strong power-law rheology for ice was investigated, in which viscosity varied with deviatoric stress σ as $\mu \propto \sigma^{-3.8}$ [9]. In this case, values of t_c/t_b were found that would allow relaxation of the tidal bulge in timescales much shorter than Herschel. For example, if $R_{core}/R_p = 0$, $t_c/t_b = 170$. The effect of a core is to suppress this effect: for $R_{core}/R_p = 0.25$, $t_c/t_b = 110$ and for $R_{core}/R_p = 0.50$, $t_c/t_b = 3.3$. The large difference between this result and those derived from Newtonian viscosities above is a result of the much larger deviatoric stresses globabally associated with a tidal bulge. In a medium with a power-law rheology, these stresses result in decreased viscosities and significantly enhanced flow.

In conclusion, there are two conditions under which the well-preserved state of Herschel basin could be reconciled with relaxation of Mimas to a hydrostatic equilibrium shape: (a) Mimas accreted at high temperatures, and Herschel formed after the satellite cooled. Since the temperature rise due to accretional energy for Mimas is very small [10, 11], this possibility requires that Mimas formed in a gaseous circum-saturnian nebula that substantially elevated ambient temperatures during the satellite's formation. Herschel would then have formed, probably as a result of cometary impact, after the nebula dissipated and the satellite cooled. Cooling times for Mimas are short ($\sim$ 10 My) [10], so Herschel could still be an ancient feature in this scenario. Alternatively, (b) the deformation of Mimas could follow a strong power-law rheology like that found for ice in laboratory investigations [9]. In this scenario no pronounced early heating is required, relaxation of the tidal bulge can take place over an extended period, and no restrictions are placed on the timing of the Herschel impact.

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