GRAVITATIONAL FOCUSSING DURING PLANETARY ENCOUNTERS

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Our recent work on gravitational encounters between planetary bodies (1) shows that failure of the two-body approximation, although it may appear to be related to low encounter velocities, is often the result of distant perturbations long before encounter. Such perturbations can modify the geometry of an encounter so that the outcome is quite different from the predictions of the Opik two-body formulation, even though the actual encounter follows two-body behavior quite well. The importance of our result is that statistical mechanical treatments of collisional, keplerian swarms may still be able to invoke two-body formulae even given the very low relative velocities that may pertain, for example during planet formation.

Monte Carlo numerical experiments by Wetherill and Cox (2,3), on the other hand, suggest that the key criterion for success of the two-body approximation is that the encounter velocity V must be sufficiently large compared with the escape velocity V_e. It generally breaks down if $V/V_e <$ 0.35, in that set of experiments.

However, we show here that some of those examples of break-down in determining "gravitational focussing" (2) are due to distant perturbations that were enhanced in the low V/V_e experiments compared with the high V/V_e ones because of systematic differences in the choice of initial separation distances. In fact, success or failure of the two-body approximation correlates better with the choice of initial integration distance than with the ratio V/V_e, further indicating that distant perturbations are the dominant factor, rather than encounter speed.

Consider Figs. 9 and 11 from (1), shown here as Figs. (a) and (b). Both show results of encounters of small particles with a hypothetical planet on a circular orbit at 1 AU. Particles have orbital eccentricities e = 0.00447 and inclinations $i = 10^{-5}$ in both figures, so the encounter velocities are equal. But the planetary mass is 6×10^{22} gm in (a) and 6×10^{25} gm in (b). Thus the parameter V/V_e is 0.67 in (a) and only 0.067 in (b).

In (a) with the larger V/V_e, the b vs. d relation agrees perfectly with the prediction of the two-body formula. Even the bend for smaller values of b fits the prediction. In (b) with the small V/V_e, there is little agreement. We have reproduced this experiment and obtain similar results (Figs. c and d).

In both cases the initial separation distance was 55 times the radius of the Tisserand sphere of influence of the planet. With the difference in the planet's mass, this initial separation was much greater for (b) than for (a) (0.054 vs. 0.0034 AU). This difference in initial separation distance is such that for (b) the test particles were exposed to strong pre-encounter perturbations, which tend to occur during critical intervals that appear as loops in the rotating, planetocentric reference frame. The much closer starting distances of Fig. (a) prevented such effects.

To test the importance of initial distance, we repeated the case with the smaller mass (as in (a) and (c)), but with the greater initial separation (as in (b) and (d)). If V/V_e is the critical parameter, this new case should also give good agreement with the two-body formula. However, as shown in (e), the agreement breaks down due to pre-encounter distant perturbations.

We also tested the converse case, with the same mass and V/V_e as in (b) and (d), but with the smaller initial separation. The V/V_e criterion predicted poor agreement with the two-body formula, but we find (Fig. f) just the opposite, because the close start minimizes pre-encounter perturbations.

These results suggest that two-body dynamics may be applied to planetary accretion, even in the intermediate stage of planet growth when values of V/V_e were very small for encounters of planetesimals with planetary embryos. Of course, the two-body approximation must be used with care under these conditions, but at least we do have the prospect of making the simulation of intermediate stage growth a tractable problem by this approach.

__References__: (1) Greenberg et al. 1988, Icarus in press. (2) Wetherill and Cox, 1984, Icarus **60**, 40. (3) Wetherill and Cox, 1985, Icarus **63**, 290.