DERIVATION OF AN AVERAGE SINGLE PARTICLE PHASE FUNCTION FOR THE LUNAR REGOLITH

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We have developed a scattering model for a particulate medium bounded by a stochastic interface. The model includes effects of surface shadowing, multiple scattering inside a surface element and scattering by a single particle which is the smallest element in our treatment. At least in principle, all the free parameters can be measured in laboratory samples and, therefore, the model is verifiable for any tests.

BACKGROUND. In a number of papers Lumme and Bowell (1,2), Lumme and Irvine (3), and Hapke (4) have considered models to interpret both the disk-integrated and disk-resolved data of atmosphereless bodies. Although these models differ considerably in details and produce different values of the individual parameters, they can fit the data very well. This seems to indicate that the solution is not unique. However, the major difficulties in these models are in the manner in which they treat both the surface roughness and the single particle phase function. Neither in the models can the surface roughness be qualitatively related to real laboratory samples and the single particle phase functions have been parametrized on a mathematical rather than on a physical basis. Also a verification of both models with strignent laboratory tests has been in its infancy. Voyager spacecraft have in recent years greately increased the disk-resolved data base for atmosphereless bodies. For these reasons it has turned out to be appropriate to reopen the question of a scattering model. It also seems natural that the enormous amount of the lunar photometric data will be among the first ones to be analyzed.

LPI-MODEL. Lumme et al. (5) have recently developed a fairly general scattering model for a particular medium bounded by a rough interface. It appeared to be both convenient and realistic to describe the interface of the surface by a stochastic process, which was chosen to be multivariate normal statistics. With this choise the surface roughness ρ becomes a well defined quantity σ/l where σ is the standard deviation of the height distribution and l is the correlation length. This quantity can also easily and uniquely be measured for a laboratory sample. It must be noted, however, that in real surfaces there is a whole spectrum of superimposed roughnesses.

In the LPI-model it is further assumed that an inclined surface element is composed of particles which both shadow each other and scatter multiply. Single scattering of an element is assumed to follow the Lommel-Seeliger's law with a proper mutual shadowing correction while the functional form of the multiple scattring is approximated by the Lambert's law. The absolute size of an element is immaterial but can be thought to be on the order of ten times the mean particle size. Due to surface roughness any element can either be shadowed, obstructed from a viewer or both.

The fairly complicated expressions for a single and multiple scattering surface brightness components, I_1 and I_m, are greatly simplified in a special case which is considered here. Along the photometric equator with angle of incidence ι and that of reflection ϵ, being equal and the phase angle $\alpha = \iota/2 = \epsilon/2$ (azimuth $= \pi$) the surface brightness components in terms of the incident flux πF read

$$\begin{aligned}
\frac{I_1}{F} &= \frac{\varpi_0}{4} P(\alpha)\Phi_S(\alpha, D) W(\alpha, \rho) \left\{ (1 - \frac{1}{\xi^2})[2N(\xi) - 1] + \frac{2}{\sqrt{2\pi}\xi} e^{-\frac{1}{2}\xi^2} \right\} \\
\frac{I_m}{F} &= a_M W(\alpha, \rho) \frac{\cos\alpha/2}{2\pi\rho} e^{\frac{1}{4\rho^2}} \int_{-\xi}^{\xi} (1 - \frac{x^2}{\xi^2}) e^{-\frac{1}{4}x^2} K_0(\frac{1}{4\rho} + \frac{1}{4}x^2) dx \qquad (1) \\
W &= \left[2N(\xi) - 1 + \frac{2}{\sqrt{2\pi}\xi} e^{-\frac{1}{2}\xi^2} \right]^{-1} \\
\xi &= \frac{\cot\alpha/2}{\rho}
\end{aligned}$$

where ϖ_0 is the single scattering albedo, $a_M \approx 0.82(1 - \sqrt{1 - \varpi_0})^{2.5}$ is an approximation to "multiple scattering albedo" for any classical radiative transfer theory for which the asymmetry factor is close to zero (this does not require isotropic scattering), P is the single particle phase function, Φ_S is the mutual shadowing function (6), (we assume here for the volume density $D = 0.4$), W is the probability that a surface element is both illuminated and visible, N is the Gaussian probability distribution and K_0 is the modified Bessel function of zeroth order.

ANALYSIS OF THE LUNAR DATA. We have used the disk-integrated phase function of the Moon as given by van Diggelen (7), which is based on Rougier's original data, and Shorthill et al. data (8) for the disk-resolved brightness. Shorthill et al. data were fitted for smoothing purposes by a combination of Lommel-Seeliger and Lambert functions. From these fits the proper values for I/F, accordant with Eq.(1), were chosen. Provided with two known functions of α we disk-integrated our model and used Eq.(1) to solve the unknowns ϖ_0, ρ and $P(\alpha)$ in least square sense. The resulting parameter domain and the constraining quantities are given in Table I. The resulting P must naturally have the normalization property that the integral of it over the whole space must equal to 4π. Our maximum phase angle was 150°. This allowed the use of four point Gaussian quadrature to calculate the integral. We assigned a ten percent accuracy to this integral. Quite independently of our present procedure we also require that there is no limb darkening at $\alpha = 0°$. This constraints ρ to an interval 0.6 – 0.9 for then also I_m/F will be roughly constant. Finally the overall rms error is constrained to less than ten percent.

Table 1: Derived parameters and their constraints for the lunar regolith.

Quantity	Constrained by
min $\varpi_0 = 0.4$	Normalization of P
max $\varpi_0 = 0.5$	Normalization of P , 10% scatter
min $\rho = 0.6$	Normalization of P , limbdarkening
max $\rho = 0.9$	Normalization of P , limbdarkening

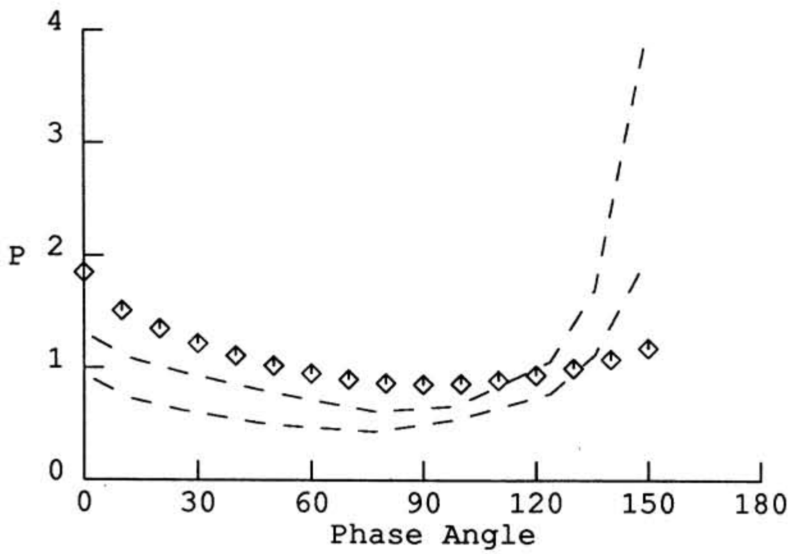

Figure 1: Comparison of the derived single particle phase function for the lunar regolith (domain bounded by two dashed lines) and that of interplanetary dust (squares),(9).

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