THERMAL MODELS OF INSOLATION-DRIVEN NITROGEN GEYSERS ON TRITON;

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Introduction A highlight of the Voyager 2 encounter with Triton was the discovery of geyser-like plumes in the atmosphere, along with clouds and surface deposits (streaks) that may be related to the plumes in origin [1]. At least two active plumes were observed at multiple emission angles, rising vertically to an approximate altitude of 8 km and flowing westward for some hundreds of kilometers. These plumes were at 50° and 57°S latitude respectively, well within the south polar cap and close to the current subsolar latitude of 45°S. The observed limb and terminator clouds were also close to the subsolar latitude, suggesting that the plumes are powered (or at least in some way triggered) by insolation, though the presence of plume-like clouds on the morning terminator means that the eruption process is able to continue through the night. The roughly ten-to-one ratio of the number of surface streaks to the number of plumes and clouds suggests a typical active lifetime for the plumes of roughly 5 years.

Photometric analysis [Soderblom, *et al.*, work in progress] of the plumes and their shadows indicates that the plumes contained entrained dust of low single-scattering albedo and of an optical depth on the order of 0.05. If (1) the dust grain size is $\sim 1\,\mu$m (constrained from below by the need for significant optical absorbtion and from above by the lack of significant settling of the plume cloud along its length), and (2) the horizontal velocity is $\sim 10\,\mathrm{m\,s^{-1}}$ [Ingersoll, submitted to *Nature*], the dust-entrainment rate is $\sim 1\,\mathrm{kg\,s^{-1}}$. If the gas eruption rate is ~ 20 times greater, a power of $\sim 5 \times 10^6$ W is required to sustain the eruption. The energy expended over 5 years is then $\sim 8 \times 10^{14}$ J. Modeling of the geyser-like ascent of the plumes [Soderblom, *et al.*, work in progress] indicates that venting of nitrogen with an initial temperature of 41 K (4 K above the ambient near-surface temperature) from a region 100 m in diameter can roughly account for the observed plume height of 8 km. This 100 m is the diameter of the base of the plume and probably corresponds to the size of a region of small, closely spaced vents rather than a single large orifice. The estimate is relatively "soft" in that the plume height is proportional to the square root of the geyser radius; furthermore, the actual height of eruption may be limited by atmospheric stratification rather than by the source radius and velocity of escape from the vents.

Smith *et al.* [1] sketched an insolation-driven model for the plumes in which heat is trapped by a greenhouse layer of clear nitrogen ice, which increases the subsurface temperature and creates a reservoir of high-pressure gas to feed the geyser in a subjacent porous layer. (Use of the term "reservoir" must not be misinterpreted to imply that the gas supply is necessarily "charged" and "discharged" in temporally distinct episodes. The term "conduit" would perhaps be more appropriate.) Such a model is not ruled out at least by very simplistic estimates of the available energy. Deposition of the solar flux (1.5 W m^2 peak value, with a diurnal average roughly half this in the polar regions) at the base of a 2-m nitrogen layer with a conductivity of $\sim 0.24\,\mathrm{W\,m^{-1}\,K^{-1}}$ [2] could result in a subsurface temperature as high as 6 K above the ambient 37 K. A circular region as small as 1.3 km in radius would suffice to collect the required power, provided the energy could be channeled to the geyser with perfect efficiency. My purpose outlined here is to investigate, using scaling arguments supported by numerical calculations, the mechanisms by which energy can be transferred from an extended greenhouse absorber to a central geyser. Only when this energy-transport process—in particular, its efficiency—is understood can we evaluate the plausibility of the insolation-driven geyser model and estimate the dimensions of the "machinery" responsible for the plumes.

Energy-Transport Processes Two processes of energy transport will operate in a porous subsurface layer on Triton. The first is ordinary thermal conduction. The second results from the strong variation of the equilibrium vapor pressure of nitrogen with temperature. Localized injection of heat will result in increased pressure and hence gas flow through the pores toward cooler regions, where the gas will partly condense to maintain local equilibrium; because of the latent heat of vaporization, energy as well as mass will be transported. (I will assume for simplicity that the amount of mass transport is insufficient either to change the porosity significantly or to exhaust the local supply of nitrogen, which may be a thin coating on a water-ice substrate rather than a pure nitrogen layer. These effects may in reality be important.) This second process is analogous to the operation of the vapor-filled "heat pipes" used, among other things, for thermal control in spacecraft. The importance of energy transport by the gas depends on the temperature (which determines the vapor pressure and density) and the D'Arcy permeability k_D of the region. If one ignores all but the exponential effects of temperature on the gas-flow rate when taking derivatives, one can conveniently define a "potential temperature" $\Theta = T + LR^*T\rho(T)^2k_D/2\eta k$ that appears in place of T on the right side of the thermal-diffusion equation and, in particular, obeys Poisson's equation in steady state. (There is also a negligible correction to the heat capacity that multiplies $\partial T/\partial t$.) In this equation L is the latent heat of sublimation, R^* the gas constant divided by molar mass, ρ the gas density, η the viscosity, and k the ordinary thermal conductivity. One can also calculate an "effective thermal conductivity" $k_{eff} = k\partial\Theta/\partial T$. Figure 1 shows k_{eff} as a function of temperature for nitrogen with porosity $\phi = 0.1$ and several grain sizes, calculated using a simple porosity-permeability model [3], along with the ordinary conductivity of N_2 [2] and of H_2O[4].

It is clear that we must appeal to highly efficient energy transport by the gas phase to make the insolation-driven geyser work. The reason is that the drops in temperature, both from the sub-greenhouse layer upward to the surface and horizontally toward the geyser, are comparable, and the horizontal distance is much larger. Thus, if the effective subsurface conductivity is not much larger than the conductivity of the greenhouse layer, the flux of energy toward the geyser will be very small. Numerical models confirm that the energy passed by ordinary conduction to a central geyser region by a surrounding collar operating as a greenhouse is always small compared with the energy absorbed in the geyser region itself. A "leaky greenhouse" geyser with or without a surrounding collector of solar energy could operate at the inferred power, but it would have to be several kilometers in diameter. I will therefore use a porosity of 0.1 and an effective grain size of 2 m in the following calculations. The corresponding permeability is $k_D \simeq 10^{-4}\,\mathrm{m^2}$, far in excess of the values reported for fractured terrestrial lava [5], but only slightly beyond the range of values commonly tabulated for sediments [3]. Furthermore, a possible mechanism may exist on Triton for maintaining a nitrogen layer of the required porosity and coarseness. The equilibrium surface temperature of 37 K is close to the transition temperature of 35.6 K between α and β N_2; the volume change associated with this transition is close to 10% [6]. Even a small departure from the global equilibrium temperature during the winter may therefore result in seasonal conversion of a near-surface N_2 layer to the denser phase, with thermal fracturing leading to $\phi \simeq 0.1$. Although reconversion of the nitrogen to the β phase *in situ* would reclose the pores, some jostling of the fragmented layer is likely, so that a porosity of 0.1 could well be maintained. The fracture spacing could be controlled by the thickness of the nitrogen layer, so that the meter-sized "grains" posited above would not be unreasonable for a layer some meters thick. Seasonal deposition of N_2 frost on top of the fractured layer could form the required gas-tight seal and greenhouse trap.

Steady-State Thermal Models Making a realistic model of a solar-driven geyser would be complicated. The model would have to be time dependent and nonlinear (because of gas-phase energy transport). The appropriate boundary conditions are also complex: (1) a mixed boundary condition on the "collector" region, where the nonlinear heat flux downward and the linear flux through the greenhouse layer must sum to the solar input, and (2) an even more involved temperature-energy relation where gas is extracted to run the geyser. A good deal can nevertheless be learned from much simpler models. I have calculated steady-state, finite-difference solutions to Poisson's equation with cylindrical symmetry for a variety of approximately appropriate boundary conditions. I approximate the "geyser" by a circular disk on the axis maintained at the ambient (potential) temperature; a real geyser can extract no more energy from the system than such a cold patch can without violating the second law of thermodynamics. On the "collector" (a larger annulus concentric with the geyser) I impose either Neumann (fixed heat

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flux) or Dirichtlet (fixed potential temperature) boundary conditions. These conditions, respectively, are the appropriate limits of a realistic mixed boundary condition for small or large collectors. Unlike the true boundary condition, which involves both linear conduction to the boundary and nonlinear conduction, the limiting forms involve only the nonlinear mechanism. Hence they allow simple scaling of a solution to the linear problem for the potential temperature to obtain the temperature field and fluxes for any given size of the real system. To achieve the steady-state solution, I used an explicit time-stepping scheme for the linear diffusion equation [7], with a multigridding scheme [8] to achieve convergence on a 513 by 513 grid in reasonable time, despite the small timesteps required for stability.

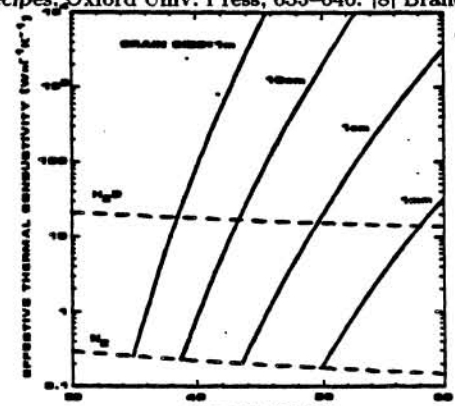
Figure 2 shows the potential temperature field for a Neumann boundary condition and no geyser; the peak potential temperature scales as the collector radius R_d: $\Delta\Theta_{max} \propto F_0 R_d/k$, where F_0 is the imposed flux. If the nonlinear conduction mechanism dominates (as assumed), the actual temperature then increases as the logarithm of R_d. Let F_0 be a relatively small fraction of the available solar flux, say $0.1\,\mathrm{W\,m^{-1}\,K^{-1}}$, the remainder being used to support a 4 K temperature drop across the greenhouse. The Neumann model then achieves a self-consistent central temperature of 41 K for $R_d \simeq 30$ km. Smaller collectors would have lower central temperatures (though not as low as the Neumann model predicts, because the available heat flux would increase somewhat), while larger collectors would resemble the Dirichtlet model and would not achieve temperatures much higher than 41 K.

For a Neumann model with a geyser of radius $R_g \ll R_d$, the output power obeys $P_{out} \simeq \pi F_0 R_g R_d$, because the geyser collects a flux of $\sim k\Delta\Theta_{max}/R_g$ over an area proportional to R_g^2, with $\Delta\Theta_{max}$ as given above. Requiring $P_{out} \simeq 5\times10^6$ W with $R_d = 30$ km, we find that $R_g \simeq 550$ m. The size of the collector is not important above the limit given; a Dirichtlet model with an infinitely large collector at a temperature of 41 K yields the required P_{out} for $R_g \simeq 430$ m. (The slight discrepancy is due to an artifact of the numerical model: imposing a sharp discontinuity in temperature between collector and geyser leads to a weak divergence of P_{out} as the mesh is made finer.) These values of R_g are still in excess of the target value of 50 m estimated above, but they are an order of magnitude less than the $\gtrsim 3.5$ km required for a "leaky greenhouse" with no surrounding collector. In these estimates, R_g scales inversely with the assumed permeability k_D.

Discussion The steady-state models discussed above cannot be taken literally, though I argue that they may be a reasonable guide to the scaling of the geyser mechanism. As Figure 2 shows, the steady-state thermal field is roughly hemispherical. The time required to heat the subsurface of Triton to a depth comparable to R_d would, however, far exceed one Tritonian season. Forty Earth years of heating at $0.1\,\mathrm{W\,m^2}$ would suffice to warm only the uppermost ~ 100 m of the body by 4 K. The output power to a geyser of given size may nonetheless be comparable to that given above. This may be seen by considering a finite layer of highly permeable nitrogen underlain by impermeable nitrogen or water ice, either of which is effectively a perfect insulator by comparison. Assume the thin layer, with thickness $Z \lesssim 100$ m, has achieved a thermal steady state. The area over which the geyser receives energy from the collector is now reduced to something proportional to $R_g Z$, but the potential temperature drop $\Delta\Theta_{max}$ now occurs across a distance of the order of Z, so that the power is roughly the same. In addition, the collector size required to achieve a temperature of 41 K is reduced as Z is made small. Similar results may well obtain if the geyser taps a developing thermal boundary layer within a thicker permeable region. (Note that, although a thick nitrogen layer on Triton is likely ruled out on rheologic grounds [9], a highly permeable water-ice layer of arbitrary depth could serve as the geyser reservoir, provided the pore spaces were permeated by nitrogen.)

The models presented here are, at best, only marginally able to explain the observed features of Triton's geysers. The calculated geyser radius is greater than that estimated from the plumes themselves, the 30-km collector radius needed to attain 41 K is considerably larger than the dark features observed at the bases of the active plumes [1], and the posited permeability of the reservoir layer is extremely high. The solutions to these problems may well lie in consideration of explicitly time dependent geyser models. Such a model, in which a geyser taps a gas reservoir some time after it has started to charge, affords the most obvious way of obtaining a finite plume lifetime (less than the length of the Tritonian summer, after which even a quasi-static geyser would shut off). In this context, it is suggestive that the thermal-diffusion distance calculated using the effective conductivity posited above and a period of 5 years ranges from ~ 40 m (at 37 K) to ~ 300 m (at 41 K)—precisely the range of geyser radii obtained in the two estimates above. Furthermore, if the estimated power of 5×10^6 W corresponds to a peak rather than to a steady-state output, the requirements for permeability and/or geyser radius may be relaxed somewhat. I am currently investigating time-dependent geyser models to attempt to quantify these assertions. Meanwhile, the absence of recognized active plumes on Triton north of the subsolar latitude remains a powerful stimulus for the investigation of insolation-powered geyser mechanisms.

References Cited [1] Smith, *et al.* (1989), *Science*, **246**., 1422–1449. [2] Weast, R. C. (ed.) (1975) *Handbook of Chemistry and Physics*. 56th ed., E14. [3] Turcotte, D. L., and Gerald Schubert (1982) *Geodynamics*, Wiley, 381–384. [4] Hobbs, P. V. (1974) *Ice Physics*, Clarendon Press, 357–361. [5] Carr, M. H. (1979) *J. Geophys. Res.*, **84**, 2995–3007. [6] Hampel, C. A., and G. G. Hawley (1973) *Encyclopedia of Chemistry*, Van Nostrand, 454–455. [7] Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling (1986) *Numerical Recipes*, Oxford Univ. Press, 635–640. [8] Brandt, Achi (1974) *Math. Comp.*, **31**, 331–390. [9] Kirk, R. L. (1990), this volume.



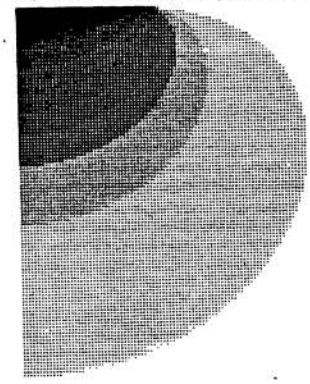


Figure Captions Figure 1. Effective thermal conductivity as a function of temperature for permeable N_2 with grain sizes of 1 mm, 1 cm, 10 cm, and 1 m (solid curves), impervious N_2 (lower dashed curve, scaled from [2] with T^{-1} dependence), and impervious H_2O (upper dashed curve, interpolated from [4]). Figure 2. Potential temperature field for a steady-state model of a dark spot (geyser "collector") on Triton with elevated temperature (black) on a surface disk and ambient temperature (white) on the other boundaries. Distinct gray shades represent 16 equal subintervals of total temperature range. Left edge is axis of cylindrical symmetry.