Collisional Dynamics of Ring Particles in the Gravitational Field of Saturn

Keiji Ohtsuki

Geophysical Institute, University of Tokyo, Tokyo 113, Japan

The orbital evolution of Saturn's rings due to inelastic collisions between particles has been widely studied so far (*e.g.*, [1]). In evaluating the effect of inelastic collisions, it is essential to know the precise values of relative impact velocity of colliding particles and the relative coordinate at the time of impact, because the change of orbits sensitively depend on them. However, in the previous studies, the Keplerian motion of particles were not taken into account correctly due to its complexity, and simple estimate was made with neglect of the gravitational field of Saturn.

We show examples of orbital change due to collisions in Fig.1; we take the coordinate on which Saturn and one of the colliding particles (which is at the origin of this coordinate system) is at rest. We show five orbits with slightly different orbital phase angles as shown in the upper magnified figure; the difference is so small that we can hardly distinguish the five orbits before collision. However, such small differences in phase angle lead to large difference in orbital shape after collision, as shown in this figure with bold curves.

In this work, adopting Hill's approximations [2], we take into account the gravitational field of Saturn and have derived analytic expressions of relative impact velocity $\mathbf{v} = (\dot{x}, \dot{y}, \dot{z})$ and relative coordinate $\mathbf{x} = (x, y, z)$ at the time of impact of two particles in purely Keplerian motion, where mutual gravitational attraction between particles is neglected. With these quantities we can directly calculate the change of orbital elements. Taking into account the flux of particles induced by the Keplerian shear and averaging over phase angles, we obtain average variations $< de^2/dt >$ and $< di^2/dt >$ for given e and i; with our scaling, time and length are scaled by Ω_K^{-1} (Ω_K being the Keplerian angular frequency) and $a_0 h$ (a_0 being the reference semimajor axis), respectively, with $h = (m_1 + m_2/3M_S)^{1/3}$, and e and i are normalized by h. The results for equal mass ring particles show that $< di^2/dt >$ takes positive or negative values to lead to the state $e \simeq 2i$ and the sign of $< de^2/dt >$ strongly depends on the restitution coefficient, ε. In two-dimensional case we obtain an approximate expression of $< de^2/dt >$ as

SATURN'S RINGS

Ohtsuki, K.

$$< de^2/dt > \simeq 1.32(1+\varepsilon)r_p e^3(\varepsilon - 0.56). \quad (1)$$

This shows that as far as $\varepsilon > 0.56$, planetesimals can get energy of random motion from the Saturn's gravitational field, in spite of inelastic collisions. However, for such strongly dissipative collisions as $\varepsilon < 0.56$, the gain from this field cannot compensate the loss due to inelastic collisions and the average random energy decreases. We also obtain similar expressions for three-dimensional case and find that they also sensitively depend on ε.

On the other hand, the restitution coefficient of icy materials for the low impact velocity ($0.015 < v(cm/s) < 5.1$) relevant to Saturn's rings is studied experimentally by Bridges *et al.* [3]; they express it with impact velocity v as

$$\varepsilon(v) = 0.32 \times (\frac{v}{cm/s})^{-0.234}. \quad (2)$$

Since the coefficient has negative power-dependence in v, average eccentricity of ring particles is enhanced with low v where ε is close to 1.0, while it is damped with high v where ε becomes small and collisional dissipation is effective. Then we expect there exists some equilibrium state where average eccentricity does not change. We calculate the change of orbital elements using Eq.(2) and find that equilibrium value of eccentricity for Saturn's rings is around 10^{-7}, which indicates that the thickness of the ring layer is about 10 m. The above result well matches the observational data by Voyager 2.

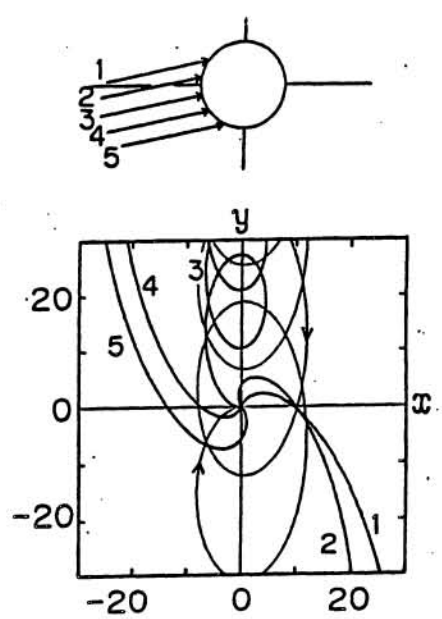

Fig.1. Examples of orbital changes due to inelastic collisions.

References

[1] Goldreich and Tremaine (1978). *Icarus* **34**, 227.

[2] Nakazawa *et al.* (1989). *Astron. Astrophys.* **220**, 293.

[3] Bridges *et al.* (1984). *Nature* **309**, 333.