

THEORETICAL PREDICTIONS FOR FRAGMENT SIZE DISTRIBUTIONS

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Cratering impacts produce fragments obeying a power law $N(L) \propto L^{-b}$, where N is the total number of fragments larger than L . Catastrophic impacts into finite targets are better characterized by two power-law exponents, $b_1 > b_2$, governing large and small fragments respectively [1,2]. This transition in slope, and indeed the quantitative form of $N(L)$, can be explained in terms of a simple geometrical model based on shock wave decay. The model not only provides a physical basis for understanding the slope transition observed in impact experiments: it also yields quantitative predictions for $N(L)$ that are in good agreement with the data over a broad range of fragment sizes.

1. Analysis

An expanding shock front emanates from the impact with a strain rate decay exponent n , which we at present assume to be constant. Strain rates in the vicinity of the projectile are $\sim v_i/2a$, where v_i and a are the projectile velocity and radius, such that at a distance r from an impact

$$\dot{\epsilon}_r \approx (v_i/2a)(r/a)^{-n}. \quad (1)$$

Concentric shells centered at the impact point (Fig.1) have a differential volume $dV = \pi r^2(2-r/R)$, where R is the target radius; within these shells the strain rate $\dot{\epsilon}_r$, and hence the dominant fragment size L , are constant. The differential number of fragments produced in a shell is therefore $dN = dV/L^3$. Grady and Kipp's expression for $L(\dot{\epsilon})$ [3], together with Eq.1, relates L and r , although any peaked distribution function relating fragment sizes monotonically to strainrate will suffice to predict a slope transition. The cumulative number of fragments larger than L is

$$N(L) = \int_{r(L)}^{2R} dN(r') \quad (2)$$

For $r(L) \ll R$ (or for a cratering impact in general) we can approximate $dV = 2\pi r^2$ and integrate to infinity, leading to a small-fragment power law with $b_2 = 3 - 3/n - 9/mn$, where the Weibull constant $m = 9.5$ for basalt. In MKS units for basalt Eq.2 becomes

$$N(L) = \begin{cases} 2.8(a v_i)^{3/2} L^{-1.03} & n = 2 \\ 2.0 a^2 v_i L^{-1.68} & n = 3 \end{cases} \quad (3)$$

The fragment *mass* distribution exponents are 0.34 and 0.56, respectively – in agreement with observation [1,2]. Larger impactors and higher impact velocities produce more fragments. The low-mass slope from an impact by the Fujiwara group [5] implies that $n = 2.33$, although this value does not provide the best fit of $N(L)$ to the data.

2. The Slope Transition

At $r = 4R/3$ the shell volume dV no longer expands with r^2 , but begins to decrease, causing a steeper power law. The slope transition size is derived for basalt:

$$L_{st} = \begin{cases} 0.67 (a v_i)^{-0.76} R^{1.52} & n = 2 \\ 0.83 (a^2 v_i)^{-0.76} R^{2.28} & n = 3 \end{cases} \quad (4)$$

More energetic impacts lead to smaller transition sizes, which is observed [4]. We compare this prediction to the Fujiwara data by inserting the appropriate values for a , v_i and R : Eq.4 predicts $6.3 \times 10^{-4} \text{m} < L_{st} < 4.9 \times 10^{-3} \text{m}$ for $2 < n < 3$; the observed transition size is $L_{st} = 2.1 \times 10^{-3} \text{m}$.

3. Predicting the Large Fragments

The largest single fragment occurs antipodal to the impact in this analysis. Eq.1 predicts a strain rate at $r=2R$ as a function of n ; Grady-Kipp theory then predicts a most probable fragment size. We can thereby solve for n by matching this most probable size to the observed largest fragment. For the Fujiwara target $n=4.22$ by this method. The half-space assumption is no longer valid; we must solve the integral without approximation. The final form is cumbersome, but when the material parameters for basalt ($m=9.5, k=10^{33} \text{m}^{-3}, c_g=1.5 \text{km/s}$) together with the impact parameters ($R=3 \text{cm}, a=0.7 \text{cm}, v_i=3.2 \text{km/s}$) are substituted, agreement between theory and experiment is very close except for the very largest and smallest fragments (Fig.2). The slope transition is more subtle than in the experiment, and occurs at a larger size, although mass loss at small sizes in the Fujiwara target might enhance the abruptness of the measured transition.

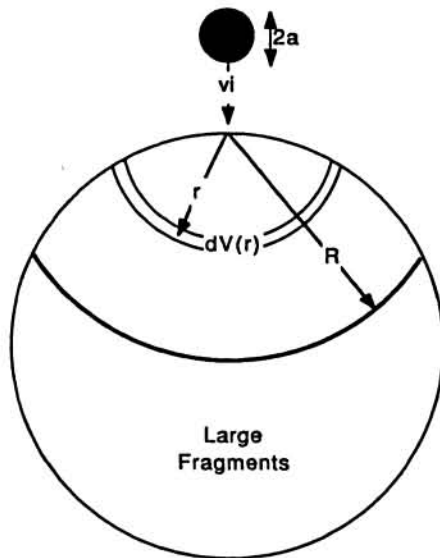
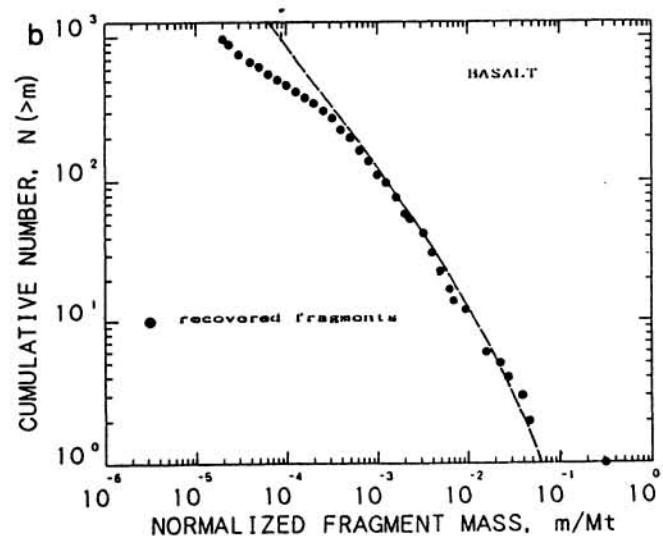


Figure 1. The impact geometry assumed in the equations. The shock is taken to be symmetrical about the impact point, encompassing a differential volume $dV(r)$ which at first increases with r^2 but eventually encounters the target boundaries.

Figure 2. A comparison of laboratory data [5] with predictions of the theory. The strain rate decay constant n is determined by requiring $L(r=2R)$ to be equal to the observed largest fragment size; there are no other free parameters.



REFERENCES: [1] Takagi, Mizutani & Kawakami, *Icarus* 59: 462-477 (1984). [2] Davis & Ryan, *Icarus* 83: 156-182 (1990). [3] Grady & Kipp, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* 17: 147-157 (1980). [4] Fujiwara, Kamimoto & Tsukamoto, *Icarus* 31: 277-288 (1977). [5] Nakamura and Fujiwara, *Icarus* 92: 132-146 (1991).