HORIZONTAL STRESSES INDUCED BY VERTICAL PROCESSES IN PLANETARY LITHOSPHERES; W. B. Banerdt, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109

Understanding the state of stress in the elastic lithosphere is of fundamental importance for planetary geophysics, as it is the link between the observed geologic structures on the surface and the processes which form and modify these structures. As such it can provide valuable constraints for the difficult problem of determining interior structure and processes. On the Earth, most large–scale, organized deformation can be related to lateral tectonics associated with plate dynamics; however, the tectonics on many extra–terrestrial bodies (such as the Moon, Mars, and most of the outer–planet satellites) appears to be primarily vertical in nature, and the horizontal stresses induced by vertical motions and loads are expected to dominate the deformation of their lithospheres [e.g., 1–3]. The largest stress contributions from vertical loading come from the flexure of the lithosphere, which induces both bending moments and membrane stresses. We are concerned here only with non–flexural changes in the state of stress induced by processes such as sedimentary and volcanic deposition, erosional denudation, and changes in the thermal gradient that induce uplift or subsidence. This analysis is important both for evaluating stresses for specific regions in which the vertical stress history can be estimated, as well as for applying the proper loading conditions to global stress models. It is also of interest for providing a reference state of stress for interpreting stress measurements in the crust of the Earth [e.g., 4].

A great deal of confusion exists in the literature about the effects of vertical changes in the lithosphere on its horizontal state of stress (σ_h). Much of this confusion can be traced to an uncertainty in the type of lateral boundary condition applied to the lithospheric column. Generally, a lateral constraint condition has been assumed [e.g., 5-7] in which the horizontal displacement is assumed to vanish due to the "resistance" of the surrounding rock. McGarr [8] pointed out fundamental logical inconsistencies in the lateral constraint assumption, and argued on both theoretical and observational grounds that this situation is "thoroughly improbable". In its place, he advocated a fixed–stress boundary condition, in which the state of stress in the region outside the area involved in the vertical changes is unaffected. This is physically equivalent to placing a fixed–displacement boundary at an infinite distance. Actual boundary conditions for a real lithosphere almost certainly lie between the lateral constraint and fixed stress cases. Thus these two end members can be used to place bounds on the magnitude of deviatoric stresses induced in the lithosphere by vertical processes. However, previous derivations of σ_h for these situations [7,8] contain errors resulting from incorrect physical assumptions.

Lateral Constraint. In this case changes in the vertical stress results in an additional horizontal stress due to a combination of elastic lateral stress accommodation ("Poisson stress"), isostatic subsidence or uplift on a sphere, and thermal re–equilibration. An implicit assumption in previous derivations is that the three contributions to the stress are independent and can be computed separately and simply superposed to arrive at an expression for the stress [7]. Such is not the case. For example, the horizontal expansion induced by vertical compression acts to help support the lithosphere in a spherical geometry, reducing the vertical displacement and hence the compression due to isostatic subsidence on a sphere. As the three mechanisms are not independent, they must be solved simultaneously as an elastic problem in a spherical geometry. Whereas for the Earth the difference between the two approaches is negligible, the error in using superposition can be quite large for smaller planets, with a strong dependence on assumed lithosphere thickness (see Fig.1).

Fixed Stress. In order to calculate stresses in the loaded region, McGarr [8] required that horizontal forces balance and solved for the resulting stresses [e.g., 9]. He further assumed that changes in the overburden thickness are equivalent to changes in the elastic lithosphere thickness, changes in the temperature of the lithosphere affect the lithosphere thickness through thermal expansion alone, and thermal erosion and underplating of the lithosphere can be modelled as thickness changes alone, independent of other thermal effects.

The three latter assumptions are inconsistent with current understanding of the relationship between temperature and the structure of the lithosphere. For a given geologic material, the effective thickness of the elastic lithosphere is determined by the depth to a critical isotherm, which is in turn a function only of the heat flux or thermal gradient. The addition or removal of overburden will not result in any change to the total lithosphere thickness once thermal re–equilibration has occurred. In addition, the lithosphere is, in general, composed of two major layers: a crustal layer and an elastic mantle layer, both of which overlie ductile mantle. Changes (e.g., erosion, deposition) near the surface involve material with the density of crustal rocks

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whereas the compensating changes at the base of the lithosphere (due to vertical migration of the critical isotherm) involve the mantle, which has a higher density. Thus the integrated vertical stress will decrease and the buoyancy of the column will increase. These corrections produce significant changes in the horizontal stress results. For a nominal set of parameters typical sedimentary basins on the Earth, the corrected expression results in a ratio of horizontal to vertical stress of about –0.2, versus +0.3 for the original derivation [8]. Note that if the base of the elastic lithosphere is within the crust, there is no net change in the mechanical state after addition or removal of material at the surface, and the induced horizontal stress is zero.

Geometric Constraint. The above discussion regarding the fixed stress boundary condition applies only to two–dimensional structures, such as a long ridge or syncline. However, many planetary loading problems involve quasi–circular features like impact basins or volcanic constructs. In this situation the proper boundary condition is less ambiguous, as lateral constraint is furnished by the resistance (due to the circumferential, or "hoop" stress) of the surrounding material to radial expansion or contraction. The ratio of horizontal to vertical stress in this case is given by $\nu/2$, in the absence of thermal or spherical effects. For comparison, this ratio would be $\nu/(1-\nu)$ for lateral constraint, 0 for a fixed stress, and 1 for hydrostatic conditions (Fig.2). In addition, a radially decreasing horizontal stress is induced in the region surrounding the load, in contrast to the other cases discussed previously.

It should be noted that these stresses are not included in thin shell or thin plate elastic models, such as those typically used to infer lithosphere thickness from mascon or volcanic loads [e.g., 2, 10-11], due to the approximations inherent in the formulation. Thus they should be added to those solutions in order to more accurately estimate the state of stress. Whereas the radial positions of stress peaks will not be influenced, the relative magnitudes of the principal stresses, which determines the style of faulting, will change, possibly affecting the interpretation of the models.

References: [1] Melosh, *Proc. LPSC 9th* 3513, 1978; [2] Solomon & Head, *JGR* 84:1667, 1979; [3] Banerdt et al, in *Mars* 249, 1992; [4] McGarr & Gay, *Ann. Rev. Earth Planet. Sci.* 6:405, 1978; [5] Price, *Fault & Joint Development in Brittle & Semi-Brittle Rock*; [6] Voigt & St. Pierre, *Adv. Rock Mech., Proc. 3rd Cong. Int. Soc. Rock Mech., IIA*:580, 1974; [7] Haxby & Turcotte, *Geol.* 4:181, 1976; [8] McGarr, *JGR* 93:13609, 1988; [9] Artyushkov, *JGR* 78:7675, 1973; [10] Banerdt, *JGR* 91:403, 1986; [11] Comer et al, *Rev. Geophys.* 23:61, 1985.

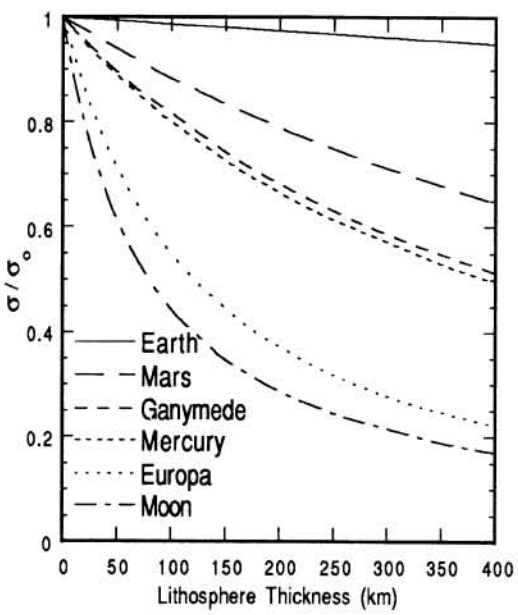

Fig. 1. Ratio of horizontal stress calculated from a fully spherical elastic solution to that calculated by superposition of individual contributions, as a function of lithosphere thickness, for various planets and satellites. Nominal values for elastic constants, thermal expansion coefficients, and thermal gradients have been assumed. Values deviating significantly from 1 indicate that the derivation of [7] gives a poor approximation to the stress.

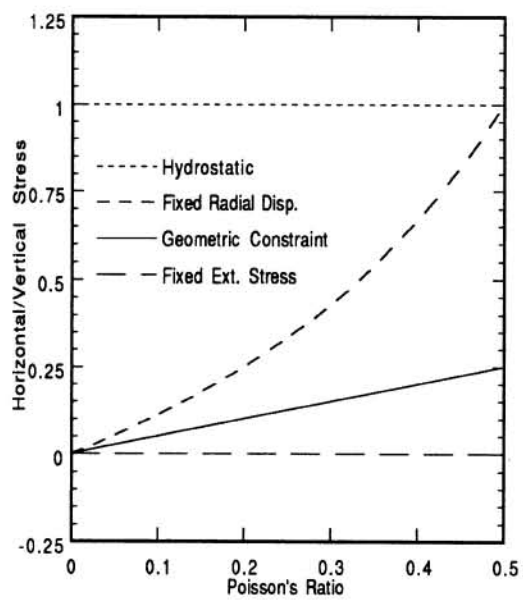

Fig. 2. Ratio of the elastically induced horizontal stress to the vertical load stress for various boundary conditions.