A METHOD TO DETERMINE ASTEROID POLES

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Introduction. The determination of spin axis and shape is well known to be of fundamental importance for studies about the rotational and physical properties of asteroids. In particular, knowledge that the pole coordinate distribution is random or not could indicate the probable non-Maxwellian distribution of asteroid spin axes (see, e.g., [1]), while the distribution in terms of size and shape could place important constraints on the theories about the collisional history of some individual asteroids, of asteroid families, and of the asteroid population as a whole (see, e.g., [2], [3], [4], [5]). Many kinds of methods have been developped to determine pole coordinates. One is to use the epochs of maximum light in the observed photometric lightcurves and to calculate the number of synodic cycles between them. The synodic period changes with time in a way depending on the coordinates of the north pole, which so can be calculated. This method has been expressed in many ways, but the collective term "Epoch method" (E method) is used for all the pole determination methods that, like Photometric Astrometry, are based on observed epochs in the lightcurves. These methods are not based on any specified model of the asteroid, so from them you can derive no shape information. On the contrary, sidereal period and sense of rotation can be derived from these methods.

Another way to determine pole coordinates is to use the amplitudes and/or the magnitudes of the observed photometric lightcurves: if only the amplitudes are used this is an A method, if only the magnitudes this is a M method, and if you use both of them this is a AM method. The fundamental principle common to all these methods is the assertion of an amplitude-aspect angle and a magnitude-aspect angle relationship, based on some model of asteroid, usually a triaxial ellipsoidal model. With these methods the shape (model dependent) and poles can be derived, but the sidereal period and the sense of rotation cannot be found. The A, M and AM methods can be applied only to objects with very regular lightcurves .

Each of these methods alone can provide only a part of the required information, and composite methods, realized by putting E, A and M techniques together, can be very useful both to give complementary information, and to give complementary constraint on pole coordinates. Many EA and EAM methods have been developped (see, e.g., [6], [7], [8]) but never these methods were really used together : every method is used separately, and not with a fit technique, but with trial poles and shape parameters as input. Only Michalowski and Velichko [9] used the E and the A information together, by linking the E and the A part as elements of a nonlinear system of equations. In this paper a EA method is presented, from which it is possible to obtain the solution with no trial poles, but with a simultaneous least square fit on both the E and A part.

Results for rotational and shape parameters have been obtained for 18 asteroids : the value of the obtained parameters are generally in close agreement with those by other authors.

Photometric Astrometry. The changing relative geometry of the Sun, Earth and the asteroid (see, e.g., [6]) as they move in their orbits causes variations in the observed synodic period of rotation. The actual size and sign of these variations depend on the orientation of the spin axis and the sense of rotation, thus in principle the coordinates of the pole can be deduced by studying the changes in the synodic period.

The "Photometric Astrometry" method ([10] ; [11]) was the first Epoch method to take this into proper account. A time shift of half the difference in asterocentric longitude of the Earth and the Sun is added to the number of rotational cycles. Synodic intervals between subsequent epochs are converted to sidereal periods, and the north pole giving the lowest mean deviation from the sidereal period is chosen.

The classical formula of the Photometric Astrometry ([11] ; [12])

$$P_{sid} = \frac{\Delta T_c}{\Delta N \pm ((\Delta L_e/360°) + (\Delta \delta/360°) + \Delta n)} \qquad (1)$$

has been used in this paper in a slightly changed version taken from Michalowski [13] and used also in Michalowski and Velichko [9]. Let "s" be a parameter denoting the sense of rotation (s=1 for prograde and s=-1 for retrograde rotation), and if we use the frame reference of the phase angle bisector (i.e., bisector of the angle between the asteroid-Sun and asteroid-Earth directions), the expression ($\Delta L_e/360° + \Delta\delta/360°$) can be replaced with ($\Delta L/360°$), where L is the asterocentric longitude of the "sub phase angle bisector point", and so eq. (1) becomes

$$P_{sid} = \frac{\Delta T_c}{\Delta N + s((\Delta L/360°) + \Delta n)} \qquad (1a)$$

In this way, with simple calculations, it's possible to write eq. (1a) as

$$\Delta N + s((\Delta L/360°) + \Delta n) - (\Delta T_c/P_{sid}) = 0 \qquad (2)$$

If you have several lightcurves from different oppositions, you can choose pairs of them and substituting in eq. (2) the expressions for the parameters of this equation given in Michalowski [13] a set of nonlinear equations is obtained depending only on P_{sid} , λ_p and β_p

$$f_l(P_{sid}, \lambda_p, \beta_p) = 0 \qquad l=1,\ldots,k \qquad (3)$$

where "k" denotes the number of pairs of epochs. In this way the Photometric Astrometry consists in finding the solution of a set of nonlinear equations for the unknown parameters.

Amplitude-Magnitude Method. The fundamental principle common to all A and M methods is assertion of an amplitude-aspect or magnitude-aspect relationship, based on some model of the asteroid. Furthermore, a correction for the changing solar phase angle must be made. A triaxial ellipsoidal model is mostly used in these methods (for a wider discussion see,

e.g., Magnusson [6]).
In the present work it is used only the amplitude equation used in [6] (but the aspect angle ϕ is the more usual Earth-referred aspect angle)

$$A=1.25\log\left[\frac{(b/c)^2\cos^2(\phi)+\sin^2(\phi)}{(b/c)^2\cos^2(\phi)+(b/a)^2\sin^2(\phi)}\right]+\beta_A\alpha \qquad (4)$$

where A is the observed amplitude, ϕ is the aspect angle, α the phase angle, β_A the linear phase correction coefficient. For every lightcurve an expression is obtained

$$f_l(\lambda_p,\beta_p,b/a,b/c,\beta_A) = 0 \qquad l=1,\dots,m \qquad (5)$$

where m is the number of Amplitude-aspect relations.
Linking the two methods. From Photometric Astrometry it is obtained

$$f_l(P_{sid},\lambda_p,\beta_p) = 0 \qquad l=1,\dots,k \qquad (3)$$

and from the amplitude-aspect relationship

$$f_l(\lambda_p,\beta_p,b/a,b/c,\beta_A) = 0 \qquad l=1,\dots,m \qquad (5)$$

If all these expressions are put together you obtain

$$f_l(P_{sid},\lambda_p,\beta_p,b/a,b/c,\beta_A) = 0 \qquad l=1,\dots,(k+m) \qquad (6)$$

like in Michalowski and Velichko [9]. Now the parameters are obtained in a way different from that used in [9].
All these expressions (7) cannot be used all together if they are in this shape. Since the epoch and amplitude parts of the f_l are measured in different units, a procedure of standardization of the variables is needed. This is done simply by dividing each f_l by the standard deviation s of all the f_l of the same kind. This operation has to be made separately for each part (epochs and amplitudes). The new resulting variables are in the so called standard normal form

$$z_i=\frac{f_i}{s} \qquad (7)$$

so they constitute a vector $F=(z_1,\dots,z_{k+m})$ and the chosen parameters are those ones composing the vector $X=(P_{sid},\lambda_p,\beta_p,b/a,b/c,\beta_A)$ that minimizes the square norm of the vector F

$$\|F\|^2=\sum_{i=1}^{k+m} z_i^2 \qquad (8)$$

In this way with a single simultaneous fit it is possible to obtain sidereal period, pole orientation, shape, amplitude-phase correction for an asteroid, and also the sense of rotation, making the fit first for s=1 and then for s=-1, and finding for which value of s there is a solution.
Results. Pole and shape solutions have been obtained with this method for 18 asteroids : 9 Metis, 15 Eunomia, 16 Psyche, 22 Kalliope, 43 Ariadne, 44 Nysa, 55 Pandora, 79 Eurynome, 87 Sylvia, 130 Elektra, 201 Penelope, 511 Davida, 532 Herculina, 584 Semiramis, 624 Hektor, 694 Ekard, 951 Gaspra and 1862 Apollo. For all of them it has been possible to obtain a complete solution, and for many of them (9 Metis, 15 Eunomia, 22 Kalliope, 43 Ariadne, 87 Sylvia, 130 Elektra, 201 Penelope, 511 Davida, 532 Herculina, 624 Hektor, 951 Gaspra and 1862 Apollo) it has been even possible to remove the solution symmetry ambiguities present in asteroid pole solutions (see, e.g., [14]). For some of them (9 Metis, 43 Ariadne, 87 Sylvia, 624 Hektor, 951 Gaspra and for some aspects 532 Herkulina and 1862 Apollo) this one is the first pole solution resolved from ambiguities. For sake of brevity it is not possible to describe the solutions for each individual asteroid in detail. The values of the parameters obtained for these asteroids are in close agreement with those by other authors.
Conclusions. The method presented in this paper allows to derive all the parameters (P_{sid}, λ_p, β_p, a/b, b/c, β_A and sense of rotation) to describe the rotational and shape properties of an asteroid. All the parameters are derived by linking together the well-established techniques of Photometric Astrometry and Amplitude-Aspect relationship, and not with trial solutions, but with one only simultaneous least square fit, bypassing the problems given by each method taken alone.
The method has been applied to 18 asteroids. The values of the parameters obtained for these asteroids are in close agreement with those by other authors, some ambiguities have been resolved for the first time, and that is strongly encouraging for further applications and developments of this mathematical procedure.
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