DOES THE THERMAL WIND EXIST NEAR THE EARTH'S CORE BOUNDARY ?

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Temperature distribution in the Earth core determine many important processes such as convective motion, magnetic field generation, matter exchange between the core and the mantle, the thermal flux etc. This distribution depends on conditions on the core-mantle boundary and on the distribution of the thermal conductivity in the mantle. Seismic tomography shows that large horizontal tempetature and compositional gradients exists at the core-mantle boundary [1, 2]. The simple assumption that these inhomogeneities are extended into the top of the core contradicts to the common opinion that the horizontal temperature gradient (the thermal wind), wipes them out in a short time [3]. Hovever, this conclusion has been obtained without taking into account that the core volume is closed and the motion, if it started, can lead to a small redistribution of composition that stops this motion.

The common estimate of the thermal wind velocity at the top of the core is based on the Navier-Stockes equation. The curl of this equation admits a steady solution of the form

$$2\nabla \times \rho\Omega \times \mathbf{u} = \nabla\rho \times \mathbf{g} \tag{1}$$

where Ω is the angular velocity, g is the gravitational acceleration, and ρ is the density. One can see that the state with zero velocity, $\mathbf{u} = 0$, can exist only if $\nabla\rho \parallel \mathbf{g}$ and, hence, any horizontal gradient of the density, if it exists, creates a thermal wind. However,the horizontal temperature inhomogeneities may exists if the medium is chemically inhomogeneous. In this case

$$\rho = \rho_0(1 - \beta T) \qquad \nabla\rho = \nabla\rho_0 - T\nabla\rho_0\beta - \beta\rho_0\nabla T \tag{2}$$

where β is the coefficient of thermal expansion that depends on the distribution of molecular weight μ in the medium, and T is the deviation of the temperature away from some mean core adiabat. In the absence of thermal perturbations $\rho = \rho_0$ and $\nabla\rho_0 \times \mathbf{g} = 0$. β depends on the molecular weight, μ, of the medium. One can see from equation (2) that in the case of nonhomogeneous μ the horizontal temperature gradient may exist in the core near the mantle boundary. It does not create the thermal wind because the temperature gradient may be compensated by the gradient of composition. The state with $u = 0$ corresponds to the following μ distribution

$$\nabla\mu = (\frac{\partial\mu}{\partial\beta})\nabla\beta, \qquad \frac{\nabla\beta}{\beta} = -\frac{\nabla T}{T} \tag{3}$$

This distribution seems rather special but it is naturally selected. The steady state with zero velocity is a state with the minimum production of entropy. Any other state tends to this state if possible.

Consider the time evolution of the temperature and composition in a volume where the temperature distribution on the boundary is stationary. According to equations (1) to (3) the thermal wind velocity is smaller in regions where the molecular weight distribution is closer to (3). It makes these regions more stable.

Equation of continuity in the case of incompressible fluid ($\nabla \cdot \mathbf{u} = 0$) has the form

$$\frac{\partial\rho}{\partial t} + \mathbf{u}\cdot\nabla\rho = -\rho_0 T(\frac{\partial\beta}{\partial t} + \mathbf{u}\cdot\nabla\beta) - \rho_0\beta(\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T) + \mathbf{u}\cdot\nabla\rho_0 = 0 \tag{4}$$

Using equation of heat propagation

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \nabla(D_T\nabla T) \tag{5}$$

we can rewrite (4) in the form

$$\frac{\partial\beta}{\partial t} + \mathbf{u}\cdot\nabla\beta = -\frac{\beta}{T}\nabla(D_T\nabla T) + \frac{\mathbf{u}\cdot\nabla\rho_0}{\rho_0 T} \tag{6}$$

Terms in the right hand side of (6) are small because the characteristic scale of density is small and the diffusion is slow. We can see that evolution of composition and temperature are described by similar equations and characteristic timescale is the same.

Taking into account that terms with $\nabla\rho_0$ are small compared to other terms in (1) and neglecting also terms of the order of u^2 we obtain

$$2\rho(\Omega\nabla)\mathbf{u} = \rho_0\beta g T(\frac{\nabla\beta}{\beta} \times \mathbf{g} + \frac{\nabla T}{T} \times \mathbf{g}) \tag{7}$$

We can see that the condition (3) is sufficient to have $\mathbf{u} = 0$. According to (6) and (7) the state with $\mathbf{u} = 0$ is stationary.

If there is a heterogeneous mixture of different substances (say Fe and Si) but not a homogeneous solution then transport by flux is essential. If the medium is mixed at the molecular scale, then only diffusion can lead to the molecular weight redistribution. However, the diffusion is too slow and is insignificant in the core. Gases in the Earth's atmosphere are mixed at the molecular scale and characteristic time of perturbations in the atmosphere is much shorter than the time necessary for the compositional redistribution.

There are direct seismic observations that apparently show the stable stratification and heterogeneous structure of the top core regions (see Lay, Young 1990). The heterogeneous structure may be supported by the core - mantle exchange of the matter and extremely slow rate of mixing by the diffusion.

There is another example of stable compositional inhomogeneities in astrophysical conditions. Chemical anomalies in the form of large patches where the abundance of some elements (including He) may by ten or more times greater or smaller than in the surrounding medium were observed on the surface of magnetic Ap stars. Such inhomogeneities themselves are absolutely hydrodynamically unstable , but may be self-consistently stabilized by the temperature or magnetic fields.

The horizontal temperature inhomogeneities may persist at the top of the core even if there exists a slow convective motion. The viscosity of the core is less than $10^5\ cm^2/s$ and apparently is $\sim 10^{-3}\ cm^2/s$ [5]. In this case the Reynolds number is large even for a small velocity and the motion may be turbulent at small scales. The turbulent diffusivity that also determines the thermoconductivity is of the order of $\chi \sim u_t l_t$, where u_t and l_t are the turbulent velocity and characteristic scale respectively. The turbulence may be driven by friction with the mantle boundary. Taking $u_t \approx 10^{-2}\ cm/s$ and $l_t \approx 10^5\ cm$ we obtain $\chi \approx 10^3\ cm^2/s$.

The flux with the velocity u passes the distance l_T of the size of the typical temperature inhomogeneity on the core-mantle boundary in the time $\tau \sim l_T/u_T$. It is about $3 \cdot 10^4\ yr$ if $l_T \approx 10^8\ cm$ and $u \approx 10^{-2}\ cm/s$. This time is enough to warm the flux throughout to a depth of about $l \approx \sqrt{4\chi t} \approx 60\ km$.

The molecular diffusivity for the liquid iron is $\chi_m \approx 1\ cm^2/s$. Thus only a few km can be warmed throughout by the molecular diffusion during $3 \cdot 10^4\ yr$.

Significant horizontal temperature inhomogeneities in regions with large electric conductivity may produce an electric current and magnetic field of much larger values than had been previously estimated in the literature.

References : [1] Stevenson D. J., (1987) Geophys. J. Roy. Astr. Soc. 88, 311 ; [2] Dziewonski A. M. , (1984) J. Geophys. Res. 89, 5929 ; [3] Morelly A., Dziewonski A. M. (1987) J. Geophys. Res. 95, 678 ; [4] Lay T., Young C. J. (1990) Geophys. Res. Lett. 17, 2004 ; [5] Zharkov V. N. (1983) Interior Structure of the Earth and Planet, Harwood Acad. Publ. Switzerland