Equilibrium Models of Mass Distribution and Collisional Lifetimes of Asteroids. David R. Williams and George Wetherill, DTM, Carnegie Institution of Washington, 5241 Broad Branch Road N.W., Washington DC 20015.

An understanding of the steady state distribution expected in the present day asteroid belt is important to our understanding of the collisional evolution of the asteroids and their physical properties. We have extended earlier work to show that, in the absence of gravity, a simple power law distribution as a function of mass with constant exponent will give an equilibrium distribution of asteroids for all bodies much smaller than the largest asteroids. This result holds for realistic fragmentation mechanisms and is independent of the physical properties of the asteroids. Inclusion of the effects of gravity on disruption and fragmentation of asteroids precludes an analytic solution to this problem, and rules out a simple power law distribution. We are currently calculating numerical solutions in order to determine the expected steady state mass distribution in the asteroid belt.

In order to understand the relationship of the asteroids and meteorites to the early solar system, we must understand their collisional and thermal histories. For example, we need to know whether a given asteroid is likely to be a pristine relic of the initial formation of the asteroids or a collisional fragment of a larger body, i.e. we need to know the collisional lifetime of a given asteroid. The collisional lifetime of an asteroid is a function of the number of asteroids with which it can collide. Because smaller asteroids greatly outnumber larger ones, the majority of collisions an asteroid experiences will be with bodies smaller than itself. At collisional velocities of approximately 5 km/sec, small asteroids will be able to catastrophically disrupt bodies many times their size [1]. Unfortunately, even for the present asteroid belt the number of asteroids is not well known for those with diameters less than 30 to 40 km. Therefore, while the collisional lifetimes are very sensitive to the number of smaller asteroids, the number of smaller asteroids is not well constrained. In addition, the physical properties of even the present asteroids are not well known, and there is much more uncertainty regarding the properties of primordial asteroidal bodies. It is therefore important to understand the extent to which the collisional evolution is actually dependent on physical properties.

One approach to this problem is to assume that smaller asteroids have reached a steady state distribution over the age of the solar system, so that the number of bodies of a given mass lost due to collisions is equal to the number of fragments supplied by collisions involving larger asteroids. (Obviously, this assumption breaks down at the largest asteroids, because there are no larger bodies to resupply them.) This approach was taken by Dohnanyi [2,3], who derived an integro-differential equation for the change in number of particles as a function of particle mass. The mass distribution is represented as a power law function of the form $N(m)dm = Am^{-\alpha})dm$, where N equals the number of particles of mass m to m + dm, A and α are constants. Collisions can be one of two types. For small projectiles, cratering occurs, with the cratering fragments being lost to space. For collisions involving projectiles above a certain cutoff mass, catastrophic disruption is assumed to occur, in which the target body is fragmented and dispersed. The projectile, defined as the smaller of any two bodies involved in a collision, is completely fragmented in both types of collisions. Dohnanyi [2,3] assumes the cutoff mass is directly proportional to the mass of the target body. He further assumed that the largest fragment resulting from any collision was directly proportional to the size of the impactor [2] or to the size of the target [3]. If physical properties are assumed to be independent of mass, an analytic solution can be found for bodies much smaller than the largest body. Dohnanyi [2,3] obtained the important result that a steady state is found for $\alpha = 11/6$, independent of the material properties assumed, when cratering collisions are ignored and only catastrophic collisions are taken into account.

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We have extended Dohnanyi's formulation to show that $\alpha = 11/6$ is an exact solution when cratering collisions are included, and also for the case in which catastrophic collisions are neglected and cratering collisions are assumed to represent the sole mechanism of mass distribution. Therefore, the equilibium values are not affected by the relative importance of cratering collisions versus catastrophic collisions in the asteroid belt. We have reformulated the equation using a more realistic model for fragmentation due to catastrophic collisions, in which largest fragment size (M_b) is a function of both target mass (M_t) and projectile mass (M_p) with the form $M_b = \lambda\ M_t^{1+\beta}\ M_p^{-\beta}$, where λ and β are constant. Note that fragment size is inversely proportional to projectile mass, because larger projectiles impart more energy into the target and tend to break it into smaller pieces. The steady state value, $\alpha = 11/6$, holds for this case as well, showing this solution is quite insensitive to details of the fragmentation mechanism.

One important factor which is not taken into account in these solutions and which invalidates the self-similarity assumption of the asteroid physical properties is gravity. Larger asteroids, by nature of their higher gravity, are more resistant to catastrophic distuption and to loss of fragments. We have formulated a particle flux equation which takes into account the effect of self-gravity on the cutoff collision size for catastrophic disruption, the fragment size distribution, and the loss of fragments during cratering collisions. The projectile mass required to cause a catastrophic collision is given by $M_p = (c_1 + c_2\ M_t^{0.5})\ M_t$, where c_1 and c_2 are constants. The value of c_1 is taken from [2] and the value of c_2 is determined using pi-scaling theory [4]. The largest fragment size is also a function of the target mass. We assume that the smallest possible projectile mass which can result in a catastrophic collision will leave a largest fragment one-half the mass of the original target. For cratering collisions, the largest possible projectile has a mass just below the cutoff mass, and will eject a mass of material just under one-half the mass of the target, by definition. The largest fragment is assumed to be a constant proportion of the mass ejected from the crater. These equations do not permit an analytic solution, but it is obvious that a power law with a constant exponent will not result in a steady state distribution, due to the mass dependent effects of gravity. Gravity would be expected to prolong the collisional lifetimes of the largest asteroids, which in turn would result in less lower mass fragments being produced. Gravity becomes less important than internal strength at diameters of about 1 to 10 km, so we would expect to see some deficit of particles near this mass range. At smaller sizes, the distribution should approach a Dohnanyi-type power law as the effects of gravity become less important. We are currently solving these equations numerically to find steady state solutions for the asteroid belt. We will compare these solutions to the actual observations of the asteroids, and discuss their implications and the stability of the solutions to the assumptions involving physical parameters and fragmentation mechanisms.

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