GENERALIZED COAGULATION EQUATION AND MASS-SPECTRUM OF PROTOPLANET

Vityazev A., Kukharenko Yu., Bashkirov A., and Pechernikova G.
United Institute of Physics of the Earth, Moscow 123810, Russia

The classic Smoluchowski equation is valid only in the thermodynamic limit, when a number of the particles N in the volume V is great, so that the concentration of particles $n = N/V$ is finite. However during coagulation n can become as small as one likes. Formal application of the Smoluchowski equation to the case of small values of leads to the difficulties. They lie in the fact that its steady-state solution is identical with zero and all the moments, beginning from the second one, diverge. It is due to an unlimitation and a positive definity of the quadratic form (n,An), where A is the kernel of the Smoluchowski integral operator.

We derived the generalized equation of coagulation which differed from the Smoluchowski one by the presence of the renormalized function $f_{12}(m,m',t)=n(t)(n(t)-\frac{1}{V})\rho_{12}(m,m',t)$ Here m and m' are masses of two coagulating particles and $\rho_{12}(m,m',t)$ is the pair probability function. In this case the renormalized kernel is not positive definite and the equation has the stable steady-state non- zero solution and has the finite moments. At the final stage the relaxation goes on to this solution according to the exponential low. The generalized equation of coagulation makes possible to describe the most interesting stage of relaxation.

For three kernels 1)$A=a$, 2)$A=b(m+m')$, 3)$A=cmm'$, where a,b,c are positive constants, we obtained the concentration $n(t)$ and the second momentum $M_2(t)$ of the mass distribution of particles in the forms

1) $n(t)=\dfrac{n_0}{Vn_0-V(n_0-\frac{1}{V})e^{-\frac{\alpha}{2V}t}}$, $M_2(t) = M_2(0)+2M^2V[1-\frac{1}{Vn_0}][1-e^{-\frac{\alpha t}{2V}}]$

($n_0 \equiv n(t=0)$, M is the total mass of particles in the unit volume)

2) $n(t) = \frac{1}{V} + (n_0 - \frac{1}{V})e^{-bMt}$, $M_2(t) = M_2(0)\left[\dfrac{n_0}{\frac{1}{V} + (n_0 - \frac{1}{V})e^{-bMt}}\right]^2$

3) the solution represented in the implicit form

$$n(t)=\frac{1}{V}+(n_0-\frac{1}{V})e^{Vn_0-Vn(t)-\frac{c}{2}M^2t}, \quad \frac{1}{M_2(t)}=\frac{1}{M_2(0)}+\frac{2}{M^2}[n(t)-n_0]\varepsilon\ln\frac{M_2(0)}{M_2(t)}$$

where ε is infinitezemal positive number putting to zero after relaxation to the steady state.

For coagulation kernel $A \propto (m^\alpha + m'^\alpha)$ we found the intermediate asymptotic solution in the form $n(m,t)=n(t)m^{-q}$, with $q=1+\alpha/2$.

It is well known that the largest bodies have in average smaller eccentricities and inclinations. So in the region of mass m far more the mean mass the effective values of α are smaller than commonly used 4/3. In other words mass-spectrum of largest bodies is more steeper and (in the absence of gas accretion) is determined by surface density distribution of solid material. Our results showed that collisions of large bodies in a forming planetary systems are more frequent and more energetical than previously assumed.

This investigation is performed under support of Russian fond of fundamental researches N93-02-17171.