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The collision integrals for a population of planetesimals with mass m_1 due to gravitational interaction with population m_2 are evaluated by direct integration. The velocity evolution of bimodal collision-free gravitating system with $m_2 \ll m_1$ is considered for the cases when it is controlled a) by large bodies and b) by small bodies.

A triaxial Gaussian velocity distribution, usually admitted for a swarm of planetesimals rotating around the Sun along elliptical orbits with $e, i \ll 1$, seems to be a good approximation [1-3]. In the case under consideration the time between two consequent close encounters is much larger than the period of rotation around the Sun. The distribution function for a population m reads:

$$f(\mathbf{r}, \mathbf{v}) = \frac{N(R)}{(2\pi)^2\ \sigma_R\ \sigma_\varphi\ \sigma_z\ h} \exp\left[-\sum_i \frac{v_i^2}{2\sigma_i^2} - \frac{z^2}{2h^2} \right], \quad (1)$$

where $\mathbf{r}$ is the radius-vector in cylindrical coordinates R, φ, z centered on the Sun, $\mathbf{v}$ is the velocity relative to the circular Keplerian velocity $\mathbf{V}_K(R)$, $N(R)$ is the number of bodies in the vertical column with unit basis. We assume that the chaotic velocity dominates the shear velocity at close encounters and impulse approximation is valid. The changes of velocities are taken according to the two body problem. In result of one encounter

$$\Delta\ v_{1i}^2 = \frac{2\ m_2}{m_1 + m_2}\ v_{ci}\ \Delta\ V_i + \left(\frac{m_2}{m_1 + m_2} \right)^2 \Delta\ V_i^2\ , \quad (2)$$

where $\mathbf{v}_c$ is the velocity of the center of mass of the bodies and $\mathbf{V} = \mathbf{v}_1 - \mathbf{v}_2$ is their relative velocity. The first term in the right-hand side of Eq.(2) determines the energy exchange between the bodies m_1 and m_2, the second term redistributes the energy between the degrees of freedom. After integrating over all parameters of the encounters between m_1 and m_2 in a unit of time, these terms convert into the following two:

$$\frac{\partial\ \sigma_{1i}^2}{\partial\ t}\bigg|_{\mathbf{grav}} = \left(\sigma_{2i}^2 - \frac{m_1}{m_2}\ \sigma_{1i}^2 \right) \frac{2\ A_i}{\bar{\tau}_{12}} + \frac{\sigma_{1i}^2 + \sigma_{2i}^2}{\bar{\tau}_{12}}\ B_i\ , \quad (3)$$

$i = R, \varphi, z$, $\bar{\tau}_{12}$ is analogous (with correction for anisotropic velocity distribution) to the Chandrasekhar relaxation time,

$$\frac{1}{\bar{\tau}_{12}} = \frac{4\ G^2\ m_2^2\ N_2(R)\ \ln\Lambda}{\hat{\sigma}_R\ \hat{\sigma}_\varphi\ \hat{\sigma}_z\ \hat{h}}\ , \quad \hat{\sigma}_i^2 = \sigma_{1i}^2 + \sigma_{2i}^2\ , \quad \hat{h}^2 = h_1^2 + h_2^2\ ,$$

G is the gravitational constant, Λ is the ratio of the maximum impact parameter for encounters to the minimum one.

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The coefficients A_i and B_i are elliptic integrals,

$$A_i = \int_0^{\pi/2} \frac{\cos^2\vartheta \sin\vartheta \, d\vartheta}{a_i b_i}, \quad B_i = \int_0^{\pi/2} \frac{(1-3\cos^2\vartheta)\sin\vartheta \, d\vartheta}{a_i b_i},$$

$$a_i = (\cos^2\vartheta + \sin^2\vartheta \, \hat{\sigma}_i^2/\hat{\sigma}_j^2)^{1/2}, \quad b_i = (\cos^2\vartheta + \sin^2\vartheta \, \hat{\sigma}_i^2/\hat{\sigma}_k^2)^{1/2}.$$

Since the eccentricity and inclination do not change under the Sun's attraction, it follows from (3) that at $e,i \ll 1$

$$\frac{d\langle e_1^2\rangle}{dt}\Big|_{grav} = (\langle e_2^2\rangle - \frac{m_1}{m_2}\langle e_1^2\rangle)\frac{A_R+A_\varphi}{\bar{\tau}_{12}} + \frac{(\langle e_1^2\rangle + \langle e_2^2\rangle)(B_R+B_\varphi)}{2\,\bar{\tau}_{12}},$$

$$\frac{d\langle i_1^2\rangle}{dt}\Big|_{grav} = (\langle i_2^2\rangle - \frac{m_1}{m_2}\langle i_1^2\rangle)\frac{A_z}{\bar{\tau}_{12}} + \frac{(\langle i_1^2\rangle + \langle i_2^2\rangle)\,B_z}{2\,\bar{\tau}_{12}}. \quad (4)$$

The gravitation plays a dominant role in velocity evolution of planetesimals, so the consideration of a collision-free gravitating system is of interest. It follows from (4) that in a system of equal bodies the ratio $\langle i^2\rangle^{1/2}/\langle e^2\rangle^{1/2}=\beta$ tends to equilibrium value $\beta^*\simeq 0.55$ found from the condition $B_R+B_\varphi=B_z$. At $\beta=\beta^*$ the values $\langle e^2\rangle^{1/2}, \langle i^2\rangle^{1/2}$ grow proportional to $t^{1/4}$. From comparison of relaxation times in bimodal system we conclude that the velocity evolution is controlled by the larger bodies m_1 when $m_1\Sigma_1 \gg m_2\Sigma_2$. Σ denoting the surface density. All the solutions tend asymptotically to $\langle e_2^2\rangle = k\langle e_1^2\rangle$, $\langle i_2^2\rangle = k'\langle i_1^2\rangle$, where the constants $k\simeq 2$, $k'\simeq 1.8$ at $m_2 \ll m_1$. In the case $m_1\Sigma_1 \ll m_2\Sigma_2$ when velocity evolution is governed by small bodies $m_2 \ll m_1$ (the runaway growth situation), we find from (4) that $\langle e_1^2\rangle$, $\langle i_1^2\rangle$ will adjust to the quasi-equilibrium values

$$\langle e_1^2\rangle = \frac{m_2}{m_1}\langle e_2^2\rangle\Big(1+\frac{B_R+B_\varphi}{2(A_R+A_\varphi)}\Big) = 1.16\,\frac{m_2}{m_1}\langle e_2^2\rangle,$$

$$\langle i_1^2\rangle = \frac{m_2}{m}\langle i_2^2\rangle\Big(1+\frac{B_z}{2\,A_z}\Big) = 1.23\,\frac{m_2}{m_1}\langle i_2^2\rangle. \quad (5)$$

One can expect that at the final stage of planetary accumulation there was also tendention to chaotic energy equipartition between the growing planets and planetesimals [4].

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