A STOCHASTIC APPROACH TO THE EMPLACEMENT OF PAHOEHOE LAVA FLOWS;

***Stephen M. Baloga**, Proxemy Research Inc., Laytonsville MD 20882; and **David A. Crown**, Dept. of Geology and Planetary Science, University of Pittsburgh, Pittsburgh, PA 15260*

A pahoehoe flow field is emplaced by lava transport and the formation of hundreds to thousands of small, relatively discrete and discernable units that we will refer to as "toes". Detailed field measurements of several hundred toes in Hawaii and the Snake River Plain indicate that the mean volume (slightly less than 1 m^3) and dimensions of such toes are independent of the toe location on a particular flow lobe [1]. Remarkably, preliminary statistical analysis of the data obtained to date indicates that there is no significant difference in toe dimensions of pahoehoe lava between Hawaii and Idaho.

Two features distinguish the emplacement of a pahoehoe flow from most other types of geologic mass movements, such as aa lava flows, lahars, sediment-laden floods, or debris flows. First, the formation of a toe is a local process subject only to the local physics and ambient conditions. Second, the formation of a toe is influenced by a host of random factors, such as the local topography, the existence of previously emplaced neighboring toes, or local lava supply constraints. Observations of active toe formation and emplaced flow fields suggest that it is impossible to scale the governing processes for a single toe to the large-scale advance and evolution of the flow itself without considering the random effects that influence each toe.

Yet, on a spatial scale that makes each toe small, pahoehoe flows behave much like other mass movements. The plan form of a pahoehoe flow is difficult to distinguish from an aa flow. Although the orientation of toe formation occurs in random directions, the flow itself generally propagates according to the large-scale downslope direction, filling in and spreading through topographic lows as they are encountered, and advances faster on steeper slopes.

With a stochastic approach, parcels of lava are treated as random walkers where each toe may serve as a branching or budding site for the formation of one or more subsequent toes. The local influences on an individual toe are treated as unpredictable random variables. The advance and evolution of the flow field, however, is determined by the averaging the random walking motion over the ensemble of possibilities. Thus, one cannot predict the detailed behavior of a particular toe, but, the overall physical properties of the flow field (e.g., depth, width, advance rate) are predicted by the stochastic theory.

The random walk of a parcel of magma is specified by four probabilities, p_d, p_u, p_l, p_r, for motion downslope, upslope, or laterally, and rules for the interaction of one parcel with another. There are many different types of random walks; the appropriate formalism for pahoehoe lava flows cannot be established without detailed phenomenological field study. For relatively small pahoehoe flow fields composed exclusively of toes, or lobes within large flow fields dominated by branching between toes, we have considered two modes of emplacement, "face-breaking propagation" and "uncorrelated Brownian motion". With face-breaking propagation, only the toes at the margin of a flow may serve as sites for subsequent breakouts of new toes. With the uncorrelated Brownian random walk, any existing toe may serve as the site for a new budding event.

When enough parcels are present, the stochastic model will have a continuum limit that describes the shape and evolution of the flow at a scale that makes the individual toes small. The flow field shape for these two types of random walks is governed by the equations:

$$\frac{\partial P}{\partial t}+V_x\frac{\partial P}{\partial x}+V_y\frac{\partial P}{\partial y}=0 \qquad \text{(Face-breaking)}$$

$$\frac{\partial P}{\partial t}+V_x\frac{\partial P}{\partial x}+V_y\frac{\partial P}{\partial y}=D_x\frac{\partial^2 P}{\partial x^2}+D_y\frac{\partial^2 P}{\partial y^2} \qquad \text{(Brownian)}$$

STOCHASTIC EMPLACEMENT: S.M. Baloga and D.A. Crown

where P is the depth of the deposit and

$$V_x = (p_d - p_u)\frac{\delta}{\Delta}, \quad D_x = \frac{p_d + p_u}{2}\frac{\delta^2}{\Delta}$$

represent the mean downstream advance velocity and diffusive spreading coefficient, respectively. Lateral parameters V_y and D_y are defined similarly. The length δ and frequency Δ of an individual budding event are parameters that can be determined by field study.

The predicted distribution of toes within a pahoehoe flow field is very different for these two modes of emplacement. The face-breaking mode of emplacement produces a simple, relatively flat-topped flow with a planimetric aspect ratio of V_x/V_y. This type of flow propagates radially from the vent, has a sharp front and margins, and expands at the mean rate of budding. The depth and extent of the flow are modulated only by the local topography and any changes in the rate of magma supply at the vent. In contrast, the Brownian mode of emplacement features a central ridge that decays more slowly with a Gaussian character toward the margins and the front. This governing equation features a flow rate that is proportional to the local gradient of the flow, like the effect of a pressure term in deterministic fluid dynamic models. Parts of such flows will tend to move away from other neighboring parts that are thicker.

To compare these elementary stochastic models with field cases, we have focused on small well-defined and isolated pahoehoe lobes composed of discrete toes on the surface. In this regime, the lobes studied to date exhibit a shape characteristic of a biased, uncorrelated Brownian random walk in qualitative agreement with the predictions of the second model above. At all Hawaiian sites identified in [1], the deposits feature a central thickening with a gradual thinning toward the margins and cross-sectional profiles characteristic of a dependence on the local gradient of the deposit. These observations hold even though these deposits were emplaced on markedly different underlying slopes (up to 15 degrees) and surface roughnesses (aa versus pahoehoe).

Pahoehoe flow fields exhibit a wide variety of morphologies and structures. In the limited emplacement regime of isolated lobes composed exclusively of discrete and distinguishable toes, the phenomenology is consistent with a biased, but uncorrelated random walk. The obvious next steps of this study are the relation of the probabilistic parameters of the random walk to more conventional physical variables (e.g., effusion rate, temperature, viscosity) and a delineation of the degree of influence of the random factors. Pahoehoe flows with longer durations and higher flow rates than the lobes studied here often form channels. This phenomenon will require extension of the stochastic model to embrace the so-called correlated random walk [3] that defied theoretical exposition for half a decade.

References: [1] Crown, D.A. and S.M. Baloga, this issue. [2] Zauderer, E., Partial differential equations of applied mathematics, Chap. 1, John Wiley, New York, 1983. [3] Goldstein, S. Q. J. Mech. Appl. Math. 4, pp. 129-156, 1951.