ISOTROPY OF GRAVITY AND TOPOGRAPHY ON THE EARTH, MOON, MARS AND VENUS

Bruce G. Bills and Frank G. Lemoine, Laboratory for Terrestrial Physics, NASA GSFC, Greenbelt, MD

A simple and obvious hypothesis concerning the terrestrial planets is that there should be no preferred directionality or orientation to the (non-hydrostatic) topographic and/or gravitational equipotential slopes. Our objective is to quantify that notion and use current models of gravity and topography of the Earth [1,2], Moon [3], Mars [4,5] and Venus [6] to test the hypothesis. Our analysis involves several steps. We first define a metric for isotropy, which supposes equality of mean square values for north-south slopes with east-west slopes. We then apply the metric separately to topography and gravity models for each planet. We also apply the same metric to the cross-variance of gravity and topography together. Finally, we attempt to assess which deviations from isotropy are real features of the planets and which are merely artifacts due to errors in the existing models.

A rotating fluid planet will attain a state of hydrostatic equilibrium in which the gravity and topography will be completely characterized by even zonal harmonics [7]; there will be significant north-south slopes and no east-west slopes. However, for a solid planet, subjected to impact cratering, mantle convection, surface erosion and myriad other processes, it seems *a priori* likely that the topography and gravity will exhibit no significant preferred directionality or orientation. If the non-hydrostatic gravitational potential $\Phi(r,\theta,\varphi)$ and equipotentially referenced topography $H(\theta,\varphi)$ are expanded in terms of spherical harmonic series,

$$\Phi(r,\theta,\varphi) = \mu/R \, \Sigma \, G_{nm} \, (r/R)^{-(n+1)} \, \Lambda_{nm}(\theta,\varphi) \qquad\qquad H(\theta,\varphi) = R \, \Sigma \, H_{nm} \, \Lambda_{nm}(\theta,\varphi)$$

we can easily construct estimates of the mean square of the geoid heights or topographic heights at each harmonic degree n and order m by squaring the magnitudes of the (complex) coefficients and then summing as appropriate:

$$V_{nm}(G,G) = |G_{nm}|^2 \qquad\qquad V_n(G,G) = \Sigma \, V_{nm}(G,G)$$

and similarly for topography. In addition to the auto-covariances of H and G, we can similarly define the cross-covariance of G versus H. The mean square north-south geoid slopes $S^{\theta}_n(G)$ and mean square east-west geoid slopes $S^{\varphi}_n(G)$ at each harmonic degree are simply [8]

$$S^{\theta}_n(G,G) = \Sigma \, [n\,(n+1) - (2n+1)\,m/2] \, V_{nm}(G,G) \qquad S^{\varphi}_n(G,G) = \Sigma \, [\,(2n+1)\,m/2] \, V_{nm}(G,G)$$

and likewise for topography alone, or for the topography*gravity cross variance. If the gravity (or topography, or their product) were in fact isotropic, then the ratio

$$F_n(G) = S^{\theta}_n(G) \, / \, S^{\varphi}_n(G)$$

of north-south slope variance to east-west slope variance would equal unity. It should be obvious that, in principle at least, the topography and gravity could both be isotropic (anisotropic) and yet the cross variance could be anisotropic (isotropic).

Figures 1, 2 and 3 illustrate the slope variance ratio for the topography, gravity, and the cross variance, respectively, for the Moon ($n \leq 70$) and Mars ($n \leq 50$). In the models presented, the high degree lunar variances of both gravity and topography are anisotropic, with more N-S than E-W variation. Mars topography is quite isotropic (except at n=2) and the gravity is rather anisotropic, with less N-S than E-W variation. In both cases, this is more likely a failing of the models than an actual feature of the planets.

References

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