RESONANCE PASSAGE VIA COLLISIONS

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When multiple particles are present at a Lindblad resonance, those librating are susceptible to collisions and gravitational scattering that may eject them into circulating orbits. Should an impact/scattering event jump a particle's Jacobi integral across the separatrix that divides libration from circulation, the particle is dislodged from resonance. Below we quantify the minimum impulse necessary to knock a particle from resonance. As a simple illustration of our results, we consider the impact event that may have formed the Pluto–Charon system, and show that a Charon–sized impactor would have ejected Pluto from the 3:2 resonance with Neptune over a large portion of Pluto's orbit.

First we characterize the pre–encounter system in the context of the circular restricted three–body problem: a massless test particle orbits exterior to a perturber having a disturbing function dominated by a single term of the form $F(a,e)\cos\phi$ associated with a $j+1:j$ resonance. Here, a and e are the particle's semi–major axis and eccentricity, and the resonance angle is $\phi = j\lambda_p - (j+1)\lambda + \tilde{\omega}$, where λ_p and λ are the perturber's and test particle's mean longitude, and $\tilde{\omega}$ is the particle's perihelion longitude. This system admits two constants of the motion—the usual Jacobi integral $J(a,e,\phi)$, and $\beta(a,e) = a[j-(j+1)\sqrt{1-e^2}]^2$, where a is in units of the resonance radius. Replacing a appearing in J (subsequently written in dimensionless form) with the parameter β and expanding to forth order in e and first order in the perturber's mass μ (in solar mass units) yields $J(e,\phi;\beta) \simeq Ae^4 - Be^2 + Ce\cos\phi + D$, which has constant coefficients $A \simeq 3(j+1)^2/4$, $B(\beta) \simeq 3(j+1)(1-\beta)/2$, $C = 8j\mu/5$, and $D \simeq 3 + 2/j$, all written to lowest order in the small quantity $|1-\beta| \ll 1$.

A graphical analysis of the $J(e,\phi)$ level curves [1] reveals the range of J values for which a particle at resonance librates rather than circulates. A particle with zero free eccentricity has a forced eccentricity that minimizes J, so $\phi = \pi$ and $e_- \simeq \sqrt{B/2A} + \mathcal{O}(C/B)$. The particle's level curve is thus a point in a $J(e,\phi)$ phase space diagram, denoted as $J(e_-,\pi;\beta) \equiv J_-$. A particle on the level curve separating a librating orbit from a circulating one (the separatrix) now has some additional free eccentricity, and exhibits a local minima in $J(e)$ when $\phi = 0$, denoted as $J(e_+,0;\beta) \equiv J_+$, where $e_+ \simeq e_-$.

An impactor striking a librating particle may be treated as an instantaneous, non–adiabatic change in the particle's orbit elements: $e_- \to e_- + \delta e$, $\beta \to \beta + \delta\beta$, and $\tilde{\omega} \to \tilde{\omega} + \delta\tilde{\omega}$. The particle's post–encounter Jacobi integral is, to second order, $J' \simeq J_- + [4A(\delta e)^2 - \Delta B]e_-^2 - 2\delta e\Delta Be_- + e_-C(\delta\tilde{\omega})^2/2$. Similarly, the new separatrix level is $J'_+ \simeq J_+ + [4A(\delta e)^2 - \Delta B]e_+^2 - 2\delta e_+\Delta Be_+$, where $\Delta B = -3(j+1)\delta\beta/2$ and $\delta e_+/e_+ = \Delta B/2B$. An encounter that knocks the particle out of libration is one that promotes J above the separatrix of the perturbed state, *ie.*, $J' > J'_+$. Noting that $J_+ - J_- \simeq 2Ce_- \gg e_-C(\delta\tilde{\omega})^2$ and $\delta\beta \simeq \delta a/a - 2(j+1)e\delta e$, we find that all the remaining perturbations in δe and $\delta\beta$ vanish to second order, and the criterion $J' > J'_+$ greatly simplifies to

$$|\delta a| > 8a\sqrt{j\mu e/15}. \qquad (1)$$

Generally, an impact acting in the radial direction will most strongly affect the particle's eccentricity, but since the above expression is insensitive to δe, we conclude that a librating particle is much more stable against purely radial impacts. This has been demonstrated in a numerical experiment by R. Malhotra [2], who also derived the above expression by considering tangential impacts. One of the aims of this communication is to show that Eq. (1) is in fact more general.

A change in the particle's semi-major axis, δa, is related to radial and tangential velocity impulses δv_r and δv_θ, acting over a short time interval Δt, through the Lagrange equation $\delta a = \dot{a}\Delta t \simeq 2[e\delta v_r \sin f + \delta v_\theta(1 + e\cos f)]/\Omega$, where f is the particle's true anomaly and Ω is its mean motion. As a simple application, consider a librating particle of mass M_p and eccentricity e struck by an impactor of mass m_i in a circular orbit. The impactor has a relative approach velocity $\Delta\mathbf{v} \simeq -eV(\sin f\hat{\mathbf{r}} + \cos f\hat{\theta}/2)$, where $\hat{\mathbf{r}}$ and $\hat{\theta}$ are unit vectors in the radial and tangential directions. Further, assume the impact is sufficiently inelastic such that all the target and impactor mass remains bound as a binary pair, such as might occur during moon formation via a giant impact. Conservation of linear momentum requires the binary to recoil with velocity $\delta\mathbf{v} = m_i\Delta\mathbf{v}/(M_p + m_i)$. Inserting this expression into the above Lagrange equation which is then inserted into Eq. (1) yields the impactor threshold mass m_i that dislodges the target mass M_p from libration,

$$\frac{m_i}{M_p + m_i} > \frac{\sqrt{16j\mu/15e}}{|e\sin^2 f + \cos f(1 + e\cos f)/2|}, \tag{2}$$

as a function of the target particle's phase f. This result should be regarded a conservative estimate, for it assumes the target had no free eccentricity, whereas one with nonzero free eccentricity has J closer to the separatrix and requires a smaller impactor to dislodge it. Also note that while Eq. (1) may suggest a higher eccentricity protects a particle from the impact, its increased velocity relative to an impactor instead makes it easier to dislodge, [Eq. (2)].

To illustrate this point, consider a hypothetical impact that may have formed the Pluto–Charon pair while at the 3 : 2 resonance with Neptune. Figure 1 shows the threshold impactor mass, Eq. (2), that knocks Pluto out of libration at various phase f. The curves reveal that an eccentric target is most susceptible to being dislodged at perihelion and aphelion, where impacts occur with a largely tangential impulse, whereas impacts at $f = \pi/2$ and $3\pi/2$ result in an ineffective radial impulse. The figure also indicates that Pluto, having an eccentricity ~ 0.25, is dislodged from libration over a large fraction of its orbit by an impactor having a mass $\leq$ Charon's mass. This suggests that if Pluto was swept into resonance due to radial migration of Neptune [3], any Charon–forming impact more likely occurred before Pluto's eccentricity was pumped up to its present value.

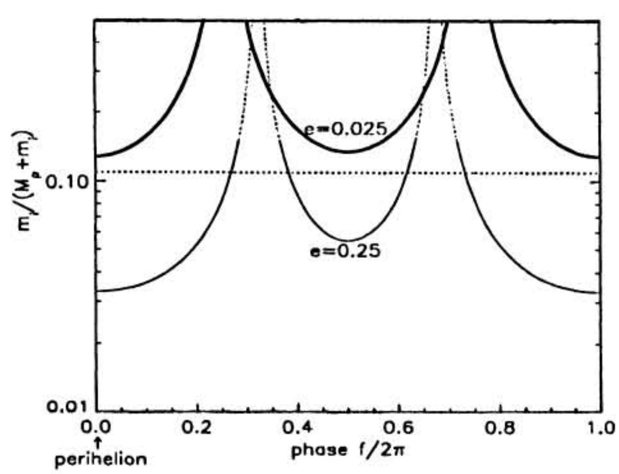

Figure 1. The minimum mass ratio $m_i/(M_p + m_i)$ that ejects the target from resonance as a function of its true anomaly f, for selected values of e. The dotted line indicates the Charon–Pluto value.

References

[1] Peale S. (1986) *Satellites* (J. Burns and M. Mathews, Eds.)

[2] Malhotra R. (1993) *Icarus*, **106**, 264.

[3] Malhotra R. (1993) *Nature*, **365**, 819.