CAN GRAVITATIONAL INSTABILITY FORM PLANETESIMALS?

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It is widely believed that planetesimals formed by gravitational instability after solid particles settled into a thin layer in the central plane of the solar nebula. The usual assumption is that instability occurred as soon as this layer reached a critical density. This condition is necessary, but not sufficient. In the presence of nebular gas, drag-induced velocity dispersion may inhibit gravitational instability even at higher densities. The scale of possible instabilities is restricted, and direct collapse to solid bodies is prevented. The dominant formation mechanism is collisional coagulation.

The critical density (1) is $\delta^* = 3\Omega^2/2\pi G$ (Ω = Kepler frequency, G = gravitational constant). This is essentially the condition that solar tides and Keplerian shear do not disrupt a self-gravitating condensation. A layer of small (~cm-sized) particles coupled to the gas cannot attain δ^*. Analytic arguments (2) and numerical simulations (3) show that shear between the particle layer and pressure-supported gas generates turbulence that keeps the layer stirred up, until particles grow large enough to decouple from the turbulence (>m-sized). At this point, the layer can attain the critical density, but gravitational instability is not assured.

The particle velocity dispersion provides another constraint. The dispersion relation for thin rotating disks (Eq. 27 of Goldreich and Ward [4]) allows density perturbations to grow if $k^2c^2 - 2\pi G\sigma k + \Omega^2 < 0$ where c is the velocity dispersion and σ is the surface density. k is the wavenumber of a density perturbation of length scale $\lambda = 2\pi/k$. There is no solution for k, i.e., density perturbations will not grow, if c is larger than the critical value $c^* = \pi G\sigma/\Omega$. The critical velocity is analogous to an "escape velocity" for a particle to be gravitationally bound to an ensemble of particles (5). Goldreich and Ward (4) assumed collisions and gas drag would damp relative velocities, so they set $c = 0$; this implies that all density perturbations smaller than $\lambda_c = 4\pi^2 G\sigma/\Omega^2$ would be unstable. Angular momentum of the disk allows collapse to solid bodies only on somewhat smaller scales (their Eq. 39), producing the often-quoted km-sized planetesimals.

If particle velocities were isotropic, the layer would have a thickness $h \sim c/\Omega$, and density δ^* would imply $c \approx c^*$, i.e., both conditions would be met simultaneously. However, velocities are actually strongly anisotropic. Gas drag damps random velocities, but causes systematic radial motions due to orbital decay. For realistic size distributions this component dominates the velocity dispersion, which is comparable to the mean radial velocity . A sufficient condition for $c < c^*$ is that most of the mass is in bodies that have $dr/dt < c^*$. This occurs at sizes ~10-100 m in a low-mass nebula. In numerical simulations (6) a layer of particles of this size typically has a density several times δ^*. Gravitational instability can occur when this condition is met.

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In (7) I assumed gravitational collapse would follow the Goldreich-Ward model once the velocity dispersion became less than c^*; setting $c = 0$ predicts ~km-scale planetesimals made of "building blocks" tens of meters in size. However, the actual outcome is different due to the nonzero velocity dispersion. As particles grow to the critical size by collisional coagulation, the radial velocity dispersion slowly decreases until $c = c^*$. At that point, the dispersion relation implies that only perturbations of scale $\lambda_* = 2\pi^2 G\sigma/\Omega^2$ are unstable; larger and smaller ones are stable. The onset of instability forms flattened, gravitationally bound condensations of mass $\sigma\lambda_*^2$. These condensations typically contain enough mass to form bodies tens to hundreds of km in size. However, these do <u>not</u> collapse, because their angular momentum is too large due to the rotation of the swarm. The velocity dispersion remains significant because collisional damping of 10-100 m sized bodies is ineffective, and gas drag tends to maintain the velocity dispersion, rather than damp it. Thus, smaller-scale density perturbations are not unstable, and there is no collapse to solid km-scale planetesimals. Individual bodies within the condensations are not affected by the large-scale instability, and continue to grow by collisions. As the median size increases, the mutual gravitational perturbations of the particles become more important, increasing relative velocities. This increases the thickness and decreases the density of the condensations. If the random velocities are described by Safronov number $\theta = V_e^2/2c^2$, where $V_e = 2(GM/d)^{1/2}$ is the escape velocity of a particle of diameter d, the condensations reach density δ^* at median particle diameter $(4\pi G\theta/3\rho_s)^{1/2}\sigma/\Omega$, where ρ_s = particle density. In a typical low-mass nebula, this occurs at sizes of a few km, insensitive to heliocentric distance. At this point, solar tides can disperse the condensations. Thus, gravitational instability of a particle layer is transient and self-limiting; the main mechanism of planetesimal growth is collisional coagulation.

References: (1) V.S. Safronov, *Icarus* **94**, 260 (1991); (2) S. Weidenschilling, *Icarus* **44**, 172 (1980); (3) J. Cuzzi *et al.*, *Icarus* **106**, 102 (1993); (4) P. Goldreich & W. Ward, *Ap. J.* **183**, 1051 (1973); (5) W. Ward, in *Frontiers of Astrophysics*, E. Avrett, Ed., Harvard U. Press (1976), pp. 1-40; (6) S. Weidenschilling, in preparation (1995); (7) S. Weidenschilling, *Nature* **368**, 721 (1994).