THERMAL CONVECTION IN AN INITIALLY COMPOSITIONALLY STRATIFIED FLUID HEATED FROM BELOW: APPLICATION TO A MODEL FOR THE EVOLUTION OF THE MOON. K.M. Alley and E.M. Parmentier, Department of Geological Sciences, Brown University, Providence, RI 02912

Stable compositional stratification resulting from the formation and early differentiation of a planet may play a significant role in its thermal evolution. For a cooling planet, the cold thermal boundary layer at the surface becomes unstable, generating plumes that penetrate to a depth of neutral buoyancy [1]. This depth depends on the magnitude of stratification and how it evolves with time. These plumes create a mixed layer which thickens with time at a rate governed by the time for an unstable thermal boundary layer to form by heat conduction and by the sinking time of the plumes.

A mixed layer which thickens with time also results from heating at the base of a stably stratified fluid. One model of mare basalt formation supposes that dense illmenite and radioactive element enriched cumulates resulting from the final stages of magma ocean solidification sink to the center of the Moon. This forms a core which gradually heats the overlying mantle, leading to the development of a chemically mixed layer overlying the core. Melting at the top of this heated mixed layer may provide a source for mare basalts, producing melt at appropriate depths and time interval after solidification of the magma ocean [2]. A previous study [2] has presented a simple theoretical evolution of this mixed layer. The purpose of this study is to use numerical solutions of the equations governing buoyant viscous flow to investigate this simple model of lunar evolution by better quantifying the rate of development of a mixed layer. The numerical experiments are not intended as complete models of the Moon's interior but rather as a tool to further our understanding of it.

We have performed numerical experiments using finite difference approximations in 2D to solve the boundary value problem of constant heat flux into the base of an initially uniformly cold fluid with a stable linear compositional stratification. The growth of the mixed layer by plumes generated in the hot thermal boundary layer is shown in Figure 1 for the case with a thermal Rayleigh number ($Rt = \rho_o \alpha q g d^4 / k \mu \kappa$) of 10^7 and a ratio of compositional to thermal buoyancy ($R = \Delta\rho k / \rho_o \alpha q d$) of 0.10. (where q=heat flux into base of fluid, d=depth of fluid, k=thermal conductivity, κ=thermal diffusivity, $\Delta\rho$=initial compositional density difference across fluid, ρ_o=reference density, α=coefficient of thermal expansion, and μ=viscosity.) Advection of the non-diffusing compositional density is particularly important for accurately describing the mixing behavior. Comparison of the simple upwind differencing method, upwind differencing with Smolarkiewicz corrections [3], and the tracer particle method using initially 9 tracer particles per finite difference cell, shows that while the simple upwind method is clearly unacceptable, the Smolarkiewicz corrected upwind and tracer particle methods both give reasonable results. Although the Smolarkiewicz method is faster, the particle method results in less numerical diffusion.

To clarify the physical processes controlling the development of the chemically mixed layer, we have formulated simple analytical solutions for the rate of change of the thickness of the mixed layer. The basic formulation has been described previously [2], and comparisons of the analytic solutions to the numerical results are shown in Figure 2 for three different pairs of Rt and R. The initial thickening rate of the mixed layer depends on the ratio of thermal to compositional buoyancy but is independent of the thermal Rayleigh number based on the layer thickness, as should be expected when the thickness of the mixed layer is small compared to the fluid layer thickness. The analytical model, however, predicts a more rapidly thickening mixed layer, and plume rise velocities significantly greater than observed in the numerical solutions. We are currently reevaluating the simplification that plume rise time is governed by the velocity of a non-entraining, thermally buoyant plume rising through a well-mixed half-space.

References: [1] Alley K.M. et al. (1995) EOS Trans. AGU, 76, 633.
[2] Hess P.C. and Parmentier E.M. (1995) EPSL, 134, 501. [3] Smolarkiewicz P.K. (1984) J. Comput. Phys., 54, 325.

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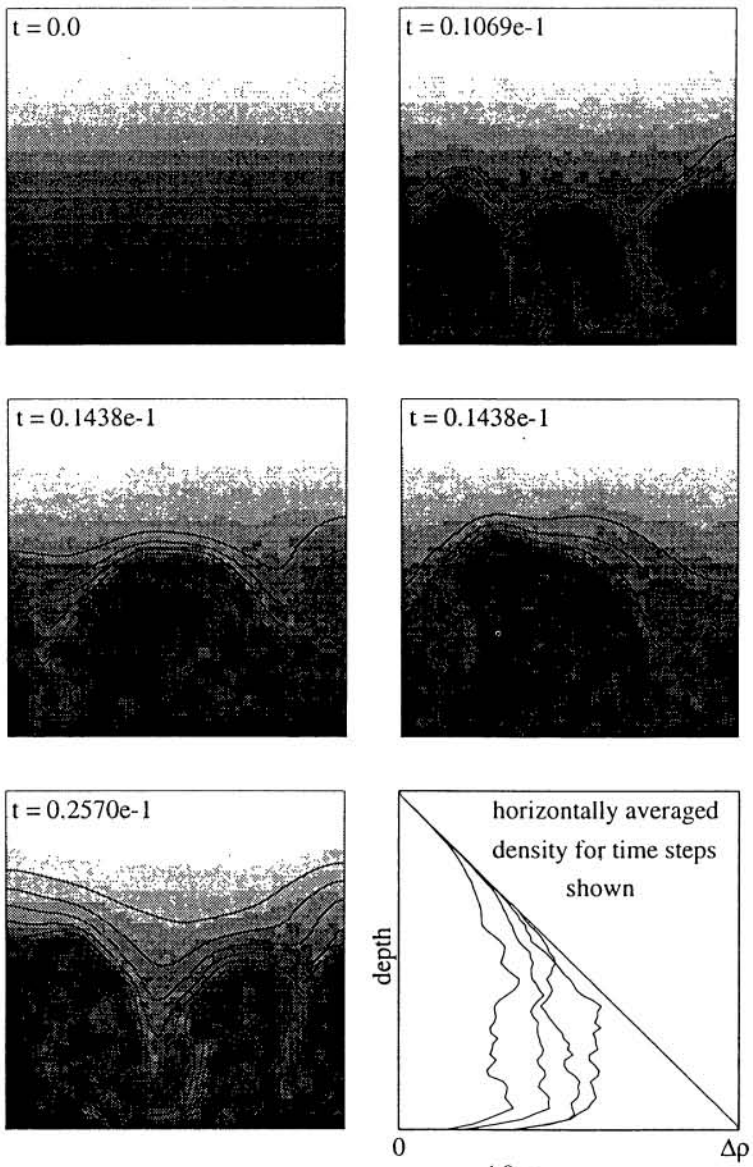

Figure 1. Numerical experiment showing the temporal evolution of temperature and compositional density in an initially stable stratified fluid of uniform temperature heated from below, and the corresponding horizontally averaged profiles of total (thermal and compositional) density. Contour lines represent temperature and shading represents compositional density ranging from 0. (white) to Δρ (black).

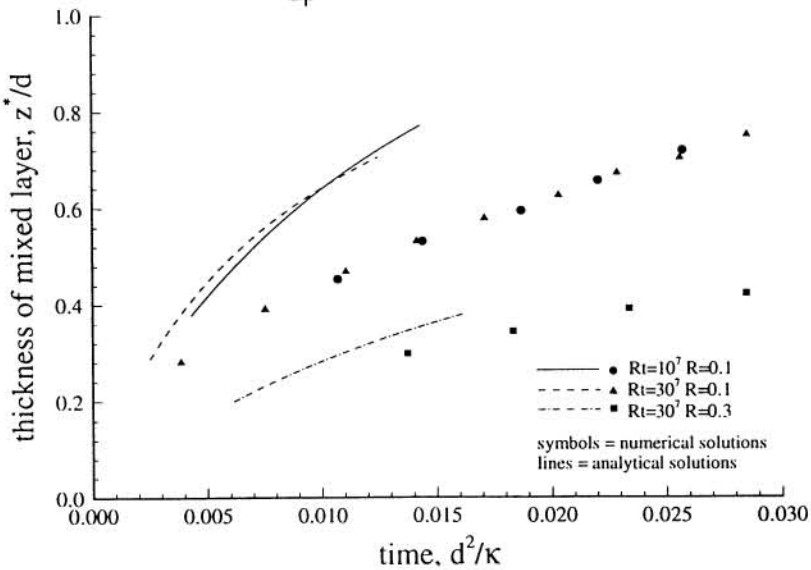

Figure 2. Comparison of analytical and numerical results for three pairs of Rt and R.