RESONANCE TRAPPING DUE TO NEBULA DISK TORQUES;
J.M. Hahn, University of Notre Dame, and W.R. Ward, Jet Propulsion Laboratory

A protoplanet embedded in the solar nebula launches spiral density waves from its Lindblad resonances in the gas disk, and its gravitational attraction for these disturbances results in a mutual torque exerted between the protoplanet and the disk. Consequently the orbit of a sufficiently massive protoplanet may decay on a timescale shorter than the nebula lifetime [1], and this mechanism is most significant during the formation of the cores of the giant planets. Due to their increased mobility, migrating protoplanets may have been able to accrete large swaths of the disk and/or encounter other protoplanets [1,2]. Thus disk torques may have played an important role in determining the formation history and orbit spacings of the giant planets. An interesting phenomenon also associated with orbit decay is *resonance trapping*, whereby a large body is able to halt further orbit decay of smaller bodies at commensurability resonances. Examples of this effect include the trapping of planetesimals experiencing aerodynamic gas drag [3,4] and dust suffering Poynting–Robertson drag [5]. Below we address the cosmogonic implications of resonance trapping of planetary embryos experiencing orbit decay due to nebula disk torques. The following employs an approach similar to Malhotra's (1993) discussion of the gas drag trapping problem.

The disk torque exerted on a body causes its semi–major axis a and eccentricity e to decay at rates $\dot{a}_d = -a/\tau_a$ and $\dot{e}_d = -e/\tau_e$ with timescales [6]

$$\tau_a = (\frac{h}{a})^2(C_a\mu\mu_d\Omega)^{-1} \quad \text{and} \quad \tau_e = (\frac{h}{a})^4(C_e\mu\mu_d\Omega)^{-1}, \tag{1}$$

where h is the nebula scale height, μ is the object's mass in solar units, $\mu_d = \pi\sigma a^2/M_\odot$ is the normalized disk mass with σ the gas disk surface density, and Ω is the mean motion. The $C_{a,e}$ constants depend on the details of the nebula model, but typical values are $C_a \sim \mathcal{O}(1)$ and $C_e \sim \mathcal{O}(0.1)$. In the jovian zone of the solar nebula we assume $\sigma \sim 400$ gm/cm^2 and $h/a \sim 0.07$, so the restriction that the orbit decay timescale τ_a is shorter than the nebula lifetime $\sim \mathcal{O}(10^7)$ years requires the object to have a mass at least $\mathcal{O}(0.1)$ Earth–masses. Since $\tau_e \ll \tau_a$, migrating bodies generally have low eccentricities.

This form of orbit migration is halted once a sufficiently massive protoplanet is able to open an annular gap in the nebula about its orbit and shut off its tidal interaction with the gas disk [7,8]. This discussion will assume a first–formed protoplanet has stabilized its orbit in this manner. Now consider an embryo spiraling toward a protoplanet due to disk torques. As it approaches the protoplanet's m^{th} outer Lindblad resonance, the variation in the embryo's orbit elements is given by the Lagrange equations

$$\dot{a} \simeq \frac{8}{5}m(m+1)e\mu_p a\Omega\sin\phi + \dot{a}_d \quad \text{and} \quad \dot{e} \simeq \frac{4}{5}m\mu_p\Omega\sin\phi + \dot{e}_d, \tag{2}$$

where $\mu_p \gg \mu$ is the the mass of the trapping protoplanet in solar units, ϕ is the resonance angle, and only the m^{th} Fourier component of the protoplanet's perturbation (the first right–hand terms) are retained to first order in e. The equilibrium orbit of a trapped body is obtained by setting $\dot{a}$ and $\dot{e}$ to zero, yielding

$$\sin\phi_{\text{eq}} = \frac{5\mu_d}{8m}(\frac{\mu}{\mu_p})(\frac{h}{a})^{-3}\sqrt{\frac{2C_aC_e}{m+1}} \quad \text{and} \quad e_{\text{eq}} = \frac{h}{a}\sqrt{\frac{C_a/C_e}{2(m+1)}}. \tag{3}$$

For the nebula parameters assumed here, $e_{\text{eq}} \simeq 0.16/\sqrt{m+1}$. The criterion for resonance

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trapping is estimated from the relation $|\sin\phi_{eq}| \leq 1$, which yields the mass ratio $\mu/\mu_p \lesssim 0.3m^{3/2}$ when equation (3) is evaluated with the assumed nebula parameters. For an object much smaller than the protoplanet, this requirement is easily satisfied and trapping likely. Though only Lindblad resonances are considered here, trapping is also possible at sites where Lindblad and corotation resonances overlap [9,10].

Resonance trapping may provide a mechanism by which a first-formed protoplanet can localize mass and stimulate additional planet formation at exterior resonances. It has been suggested that Saturn, which lies near a 5/2 resonance with Jupiter, may have been assembled from planetesimals delivered to resonances by gas drag [9]. But once planetesimals have grown larger than about 10 km, gas drag effects becomes too slow to significantly contribute to the planet formation process. Instead we suggest that mass delivery and resonance trapping become much more important during the later stage of the disk's evolution when much of the disk debris has been collected into larger runaway growths. As described above, disk torques will trap these bodies at a proto-Jupiter's exterior resonances where they might combine to form larger planets. However it remains uncertain whether trapping ultimately encourages Saturn formation at resonance, since mutual encounters can also scatter bodies through the trapping barrier [4], and warrents further investigation.

It is also possible that nebula disk torques played a role in determining the structure of the planetary system orbiting the pulsar PSR B1257+12 [11]. The two largest known planets orbit the neutron star near a 3/2 resonance, and it is natural to ask how these planets arrived in this configuration. Of the many scenarios describing planet-pulsar formation, the more plausible hypotheses generally invoke the dynamical disruption or evaporation of a stellar companion orbiting a luminous neutron star [12]. The companion's destruction produces a disk from which planets presumably condense. It is suggested here that an initial planet may have encouraged subsequent planet formation at exterior resonances by trapping debris suffering nebula disk torques.

[1] Ward W.R. (1989) *Ap.J. Letters*, 345, L99.
[2] Ward W.R. and Hahn J.M. (1995) *Ap.J. Letters*, 440, L25.
[3] Weidenschilling S.J. and Davis D.R. (1985) *Icarus*, 62, 16.
[4] Malhotra R. (1993) *Icarus*, 106, 264.
[5] Roques F. et al. (1994) *Icarus*, 108, 37.
[6] Ward W.R. (1993) *Icarus*, 106, 274.
[7] Ward W.R. and Hourigan K. (1989) *Ap.J.*, 347, 490.
[8] Lin D. and Papaloizou J. (1993) *Protostars and Planets III*, p. 749.
[9] Beaugé C. et al. (1994) *Mon. Not. R. Astron. Soc.*, 270, 21.
[10] Kary D.M. and Lissaur J.J. (1995) *Icarus*, 117, 1.
[11] Wolszczan A. (1994) *Science*, 264, 538.
[12] Podsiadlowski P. (1993) *Planets around Pulsars*, p. 149.