

**A PRECISE LUNAR PHOTOMETRIC FUNCTION**; A. S. McEwen, U.S. Geological Survey, 2255 N. Gemini Dr., Flagstaff, AZ 86001.

### Overview

The Clementine multispectral dataset will enable compositional mapping of the entire lunar surface at a resolution of ~100-200 m, but a highly accurate photometric normalization is needed to achieve challenging scientific objectives such as mapping petrographic or elemental compositions [e.g., 1,2]. The goal of this work is to normalize the Clementine data to an accuracy of 1% for the UVVIS images (0.415, 0.75, 0.9, 0.95, and 1.0  $\mu\text{m}$ ) and 2% for NIR images (1.1, 1.25, 1.5, 2.0, 2.6, and 2.78  $\mu\text{m}$ ), consistent with radiometric calibration goals. The data will be normalized to  $R_{30}$ , the reflectance expected at an incidence angle ( $i$ ) and phase angle ( $\alpha$ ) of 30° and emission angle ( $e$ ) of 0°, matching the photometric geometry of lunar samples measured at the reflectance laboratory (RELAB) at Brown University [3]. The focus here is on the precision of the normalization, not the putative physical significance of the photometric function parameters. The 2% precision achieved is significantly better than the ~10% precision of a previous normalization [4].

### Datasets

Our primary dataset for deriving the photometric function is a set of calibrated and coregistered Galileo SSI images of the Moon extending from 19.5° to 129° phase angle and bandpasses with effective wave-lengths of 0.41, 0.56, 0.65, 0.76, 0.89, and 0.99  $\mu\text{m}$  [4, 5]. The Galileo data are supplemented by the low-phase (0°-4°) Clementine data analyzed by Buratti et al. [6] and the disk-integrated models of Helfenstein et al. [7]. To date I have processed and analyzed the Galileo data only at 0.56 and 0.76  $\mu\text{m}$ ; these data provide better signal:noise than the Galileo images at 0.41, 0.89, and 0.99  $\mu\text{m}$ . I had hoped to use the disk-integrated multispectral phase curves of Lane and Irvine [8] to extend the results to longer and shorter wavelengths, but their data are excessively noisy (as revealed by color ratios) and contain a systematic bias due to the varying proportions of major albedo and color units (mare, highlands, and bright craters) sampled at each phase angle. Well-resolved and calibrated multispectral images of the Moon at many phase angles from 2° to >90° are needed but not yet available [9]. No color phase observations of the Moon are available beyond 1.06  $\mu\text{m}$ .

### Choice of Photometric Equations

The first issue to address is whether or not the  $i$ -,  $e$ -dependence of the photometric function (or "limb-darkening") varies with albedo or surface unit, such as mare vs highland terrains. The photometric function of Hapke [10] is of the general form:  $R(i,e,\alpha) = w u_0/4(u+u_0) [P(\alpha) + H(u,w)H(u_0,w)]$ , where  $w$  is single-scattering albedo,  $u = \cos(e)$ ,  $u_0 = \cos(i)$ ,  $P(\alpha)$  is the single-scattering phase function, and  $H(u,w)H(u_0,w)$  is Hapke's multiple-scattering "H-function". The  $i$ -,  $e$ -dependence of the photometric normalization at any particular phase angle is determined by the relative importance of the H-function, which is weighted by  $w$  and  $P(\alpha)$ . This equation requires or assumes that dark materials or strongly backscattering materials will exhibit little "limb-

darkening" or  $i$ -,  $e$ -dependence at low phase angle, whereas bright or forward-scattering materials will show greater relative darkening toward the limb or terminator. However, experience with other atmosphereless bodies has shown that the limb darkening may be remarkably insensitive to different albedo units [11,12]. We tested for terrain dependence via 3 different limb-darkening parameters, one based on Hapke's H-functions, one on Minnaert's function, and one on the Lunar-Lambert function (described below). By using two coregistered images with nearly the same phase angle but with very different subspacecraft and subsolar positions, we can solve for the limb-darkening parameter of each pixel, except along a line of no solution. The results from Galileo images at 19.5° and 19.9° phase angle are very uniform for all 3 parameters. A few locations such as the maria in Grimaldi show slightly different results, but there is no general difference in limb-darkening behavior between maria, highlands, and bright Copernican craters. This result indicates that the limb-darkening function must principally vary only with  $i$ ,  $e$ , and  $\alpha$ , not with albedo.

Next, we applied the limb-darkening corrections via all three functions to see which provided the best "flattening." All three functions do a good job, but the Lunar-Lambert function in the form proposed in [11] is slightly better. (The Lunar-Lambert function in the form proposed in [13] requires albedo-dependent limb-darkening, like Hapke's function, and cannot be used.) The Lunar-Lambert normalization function used here is:  $X_L(i,e,\alpha) = [L(\alpha)u_0/(u+u_0) + (1-L(\alpha))u_0]$ . The variation of the limb-darkening parameter  $L$  with  $\alpha$  was fit to a 3rd-order polynomial:  $L(\alpha) = 1.0 + A\alpha + B\alpha^2 + C\alpha^3$ . We set the first term to 1.0 to match the observation of no limb-darkening at  $\alpha = 0^\circ$  [14].

Our choice of equations to describe the phase function was heavily influenced by the work of Helfenstein et al. [7]:  $F(\alpha) = B(\alpha,h,B_0) [(1-F)P(\alpha,g_1) + F P(\alpha,g_2)]$ ,  $B(\alpha,h,B_0) = 1 + B_0/(1 + \tan(\alpha/2)/h)$ ,  $P(\alpha,g) = (1-g^2)/(1+g^2+2g\cos(\alpha)^{1.5})$ , where  $F$ ,  $g$ ,  $B_0$  and  $h$  are parameters determined by fits to a dataset. The backscatter function,  $B(\alpha,h,B_0)$ , is poorly constrained by the Galileo data as the lowest phase angle is 19.5°, so I chose to use the parameters determined in [7] for the shadow-hiding model, with  $h = 0.048$ . Helfenstein et al. determined  $B_0$  at 9 wavelengths ( $\lambda$ ) from the color phase data of Lane and Irvine [8]; a polynomial fit to these results yields:  $B_0 = 19.9 - 59.6\lambda + 59.9\lambda^2 - 20.1\lambda^3$ . I chose to utilize these fits in spite of obvious inconsistencies or noise in the color ratios of the Lane and Irvine observations because a better dataset does not exist or is not generally available. The backscatter function is most important at low phase angles so there is only a small systematic bias in disk-integrated observations from Earth due to sampling different proportions of diverse albedo units. Hopefully the remaining color discrepancies are random and have not significantly biased the backscatter function fits. To describe the backscatter function at  $\alpha$  less than 3°, I fit the following linear functions to the Clementine-based results of Buratti et al. [6]:  $F(\alpha) = 1.0 + X_B \alpha$ ,  $X_B = X_{B0} + X_{B1}\lambda$ , with  $X_{B0} = -0.0817$  and  $X_{B1} = 0.0081$ .  $F(\alpha)$  from the previous

## LUNAR PHOTOMETRIC FUNCTION: McEwen, A.S.

equations are normalized at  $\alpha$  equals  $3^\circ$ . The parameter  $g_1$  is expected to vary with albedo [15], so we modeled it via:  $g_1 = D + E R_{30}$ . Final normalization to the RELAB geometry is given by:  $R_{30} = R(i,e,\alpha) [X_L(30^\circ,0^\circ,30^\circ)/X_L(i,e,\alpha)] [F(30^\circ)/F(\alpha)]$ .

## Results

Eleven adjustable parameters are used in this model (A, B, C, h,  $B_0$ , D, E, F,  $g_2$ ,  $X_{B0}$ ,  $X_{B1}$ ), so it will not win any awards for elegance. I fixed the values of h,  $B_0$ ,  $X_{B0}$ , and  $X_{B1}$  as described above and determined the other 7 parameters by fits to the Galileo dataset. The Galileo dataset consists of 10 reprojected images (or 2-3 image mosaics with very similar photometric angles) in each bandpass with phase angles of  $19.5^\circ$ ,  $20^\circ$ ,  $32^\circ$ ,  $40^\circ$ ,  $49^\circ$ ,  $57^\circ$ ,  $68^\circ$ ,  $81^\circ$ ,  $91^\circ$ , and  $101^\circ$ . The parameters were adjusted to minimize the sum over all pixels of the absolute value of  $1.0 - R(i,e,57^\circ)/R(i,e,\alpha)$ , and normalized by the number of overlapping pixels in each image to give equal weight to each phase angle. The  $57^\circ$  phase mosaic was selected for ratioing as it is an intermediate phase angle and it covers most of the overlap area with the other images. The results for the 0.56 and 0.76  $\mu\text{m}$  images were very close for parameters A, B, C, D, and F, and were constrained to identical values. Only parameters E and  $g_2$  appear to be strongly wavelength dependent. When  $L(\alpha)$  is fit [16] to the Hapke function model of [7], the result is very similar to that determined by the direct fitting of parameters A, B, and C. Final results are given in Table 1. The mean percent precision,  $[(1 - R(i,e,57^\circ)/R(i,e,\alpha))*100]$ , is about 2% for each bandpass, which falls a bit short of our goal, but may be limited by the Galileo signal:noise ratio and by the effects of imperfect image-to-image pixel registrations (in spite of registration to better than 0.05 pixel at the  $1^\circ/\text{pixel}$  scale utilized for the fitting). Whole image ratios deviate by up to 0.5%. A critical test of the results is to compare the 0.76/0.56  $\mu\text{m}$  color ratio at each phase angle following normalization to  $R_{30}$ . The color ratios agree to within 1%. There are no systematic deviations as a function of photometric angle, except for increased noise near the terminators.

The use of non-zero values of D (albedo-dependent  $g_1$  values) clearly improves the results, whereas use of  $D=0$  results in systematic discrepancies for relatively bright and dark surface units at low phase angles (less than  $50^\circ$ ). Note

that the positive value for D indicates a positive correlation between  $g_1$  and albedo [cf. 15]. The normalized low-phase images show no systematic deviations as a function of major general terrain units (mare, highland, or bright craters), but two specific regions appear slightly unusual: the ejecta deposits of Orientale and Mare Humorum. Perhaps these regions have unusual textural properties. Systematic discrepancies as a function of albedo do appear at high phase angles (greater than  $80^\circ$ ). In ratios to  $R_{30}$ -corrected images from lower phase inputs, the maria are relatively bright and bright Copernican craters are relatively dark. This result indicates that  $g_2$  is also albedo-dependent; darker materials have a stronger forward-scattering lobe than do brighter materials. In the highest phase Galileo images ( $\alpha > 100^\circ$ ) some Copernican craters are relatively dark, and some maria are as bright as some highlands. As a result, there is no way to normalize observations acquired only at high phase ( $\alpha > 100^\circ$ ) to lower phase albedos, because any particular high-phase reflectance could correspond to either a bright material with a weak forward-scattering lobe or to a dark material with a strong forward-scattering lobe. Fortunately the Clementine multispectral mapping was acquired entirely at phase angles less than  $90^\circ$ .

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Table 1. Photometric Parameters

$\lambda$ ( $\mu\text{m}$ )	A	B	C	D	E	F	$g_2$
0.56	-0.019	$0.242 \times 10^{-3}$	$-1.46 \times 10^{-6}$	0.14	-0.262	0.55	0.440
0.76	-0.019	$0.242 \times 10^{-3}$	$-1.46 \times 10^{-6}$	0.14	-0.331	0.55	0.255