

THE ACCURACY OF THE LONG WAVELENGTH SHAPE OF MARS; *David E. Smith¹ and Maria T. Zuber^{2,1}*, ¹Laboratory for Terrestrial Physics, NASA/ Goddard Space Flight Center, Greenbelt MD 20771; ²Dept. of Earth, Atmospheric, and Planetary Sciences, MIT, Cambridge, MA 02139-4307.

We have recently completed a re-analysis of the Mariner 9 and Viking 1 & 2 occultation data and produced a new model for the long wavelength topography of Mars [1]. To derive this model we re-computed the orbits of the spacecraft using a new gravity field model [2] for Mars, employed the latest planetary ephemerides and planetary dynamics models, re-estimated the atmospheric refraction corrections, and referenced the topography to an improved geoid. We believe the overall accuracy of the topographic model to be approximately 500 meters.

The principal sources of error in the analysis of the occultation data to determine the planetary radius include the spacecraft position, the timing of the loss of signal, and the location of the grazing ray at the time of occultation due to ephemeris errors, geodetic positioning uncertainties, and local topography. Depending on the orbital radius of the spacecraft at the time of occultation, the geometry of the spacecraft and planet, and the topography of the limb, these errors have amounted to several kilometers in planetary radii as determined in previous studies [3,4,5].

The uncertainty in the spacecraft orbit is frequently the largest source of error in calculating the radius. The original orbits used by the investigators for the Mariner 9 and Viking occultation measurements had radial accuracies of 1-3 km. Our orbital analysis utilized the GEODYN/SOLVE system of programs, which numerically integrate the spacecraft Cartesian state and force model partial derivatives by employing a high order predictor-corrector model. The force modeling includes a spherical harmonic representation of the planet's gravity field, as well as point mass representations for the Sun, Earth, and other planets. Solar radiation pressure, atmospheric drag on the spacecraft, tidal parameters, planetary rotation, measurement and timing biases, and tracking station coordinates are estimated along with spacecraft orbits. This analysis made use of improved planetary ephemerides (DE234), reference models (J2000), and parameters of the planetary coordinate system. By comparing successive orbits we believe we have improved the radial accuracy of spacecraft orbits with respect to Mars' center of mass by nearly an order of magnitude, from the kilometer level to the ~100-300 m.

Planetary ephemeris errors place the planet in the wrong place (by kilometers) at the time of the occultation (circa 1971) and although the spacecraft position is with respect to the planet, and to first order the errors cancel, some small fraction remains. Also, since the 1970's the knowledge of the rotation of Mars has improved and thus the position of the occultation point can be improved, principally in longitude, to approximately 500 m.

Atmospheric refraction causes the ray path of the radio signal to bend around the planet and change the apparent radius. For Mars, the magnitude of this refraction by the 6-mbar atmosphere is on average slightly less than 0.01° and contributes ~150 meters of error in the radius of the planet for every 1000 km of spacecraft distance from the occultation point. Since the Mariner 9 and Viking spacecraft altitudes varied from 300 km to 30,000 km, this was a major potential error source. To correct the Viking data we used refraction angles provided by G. Lindal, obtained from the original data. For the Mariner 9 data we estimated the atmospheric refraction angle (α) from $\alpha = 8.429 \times 10^{-2} P T^{-3/2}$, where P and T are atmospheric pressure and temperature, respectively. This expression, provided by D. Hinson of Stanford University, assumes a constant atmospheric temperature and is estimated to be valid to ~10% for small refraction angles. The biggest refraction corrections occurred at high latitudes, where the spacecraft were typically more distant from Mars. Our atmospheric refraction corrections to the radius were typically of order 1 to 2 km but the largest was 12 km.

Table 1 is an occultation error budget derived from the uncertainties in the orbital analysis, a 0.1 second error (assumed) in the timing of the occultation, 10% of the atmospheric refraction correction, and allowing 100 meters for the effects of planetary ephemeris errors. All these errors have been projected onto the planetary radius and Table 1 shows that the single largest source of

error is the orbit at approximately 400 m with atmospheric refraction the next largest at about 300 m followed by occultation timing and planetary ephemeris errors. The total rss error of a typical occultation is estimated to be a little over 500 m. The error model for the atmospheric refraction was ~10% of the total correction. The topography is the difference between the radius to the surface and the radius of the geoid along the same radius vector. Thus, the error in topography is the rss of the radius error and the geoid error. Our estimate [2] of the error in the geoid is between 15 meters and 50 meters, depending on location and is generally much smaller than the planetary radius error. As a consequence we believe the average error in our topographic model of Mars is approximately 500 meters with some locations being as large as 1.8 km and others as small as 300 meters. An analysis of the covariance matrix indicates errors larger than 700 or 800 meters only in the region of Syrtis and Isidis and results from sparsity of data in that region.

The occultation data were also compared with Earth-based radar data acquired during the 1971-82 period. Radar data provide an independent estimate of the radius of the planet at lower latitudes and are sensitive to different errors. The primary source of error for the radar data is the planetary ephemeris [6] and this error, combined with a lesser error induced by solar plasma effects, indicate a vertical accuracy of these measurements of ~300 m, which is comparable to that of the occultation data. However, the radar measurement is an average for an area of several square kilometers and the occultation measurement is a point location. Thus, a systematic difference between the radii derived from the two datasets might be expected, and in a comparison of the topography derived from the radar data versus the occultation data a systematic difference in radius was detected with a mean of 730 ± 173 m, with the radar-derived radii being smaller than the occultation radii. The sign of this difference is consistent with the radar data providing a measure of the planetary radius to the floors of valleys and the occultation data providing a measure of the planetary radius to the higher topographic locations, such as mountains.

ERROR SOURCE	RMS (m)	MEAN (m)	MAX (m)
Spacecraft Orbit	371	292	1754
Atmospheric Refraction	272	176	1222
Occultation Timing	176	162	276
Planetary Ephemeris	100	100	100
TOTAL (RSS)	502	391	1778

Table 1. Error estimate for occultation radii measurements

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