DYNAMICAL FRICTION IN KEPLERIAN DISKS. Wm. R. Ward, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109 and San Juan Capistrano Research Institute, San Juan Capistrano, CA 92675.

Recent null attempts to detect Jupiter-sized planets about nearby solar type stars have led to questions concerning the frequency of planetary systems (1, 2). We stress that resonant interactions between a protoplanet and disk give rise to strong dynamical friction (i.e., disk tides) that causes orbital decay. A simple particle model is employed to illuminate the physical origin of the net negative torque from tidal interactions with a Keplerian disk. It may be that the ultimate survival of a planetary system is less likely that commonly believed.

The first order solution for a particle perturbed by a protoplanet, M, can be written (3), $r(t) = r + \Sigma_{m=1}^{\infty} Re\{R_m e^{i(\omega_m t - m\theta)}\}$, $\theta(t) = \theta + \Omega t + \Sigma_{m=1}^{\infty} Re\{\Theta_m e^{i(\omega_m t - m\theta)}\}$, where $R_m = \Psi_m / rD \approx -(i/2) r\Theta_m$. Here, $\Psi_m = -rd\phi_m/dr + 2qm\phi_m$ is the forcing function and $\phi_m = -(GM/r_s) b_{1/2}^m(r/r_s)$, $(m>2)$, represents the amplitude of the mth order term in the Fourier decomposed disturbing function with frequency $\omega_m = m\Omega_s$, where Ω_s is the mean motion of the secondary at orbital distance, r_s, $D \equiv \kappa^2 - (\omega_m - m\Omega)^2 \ll \Omega^2$ is the "frequency distance" from resonance, $\kappa = \Omega$ is the epicycle frequency, $b_{1/2}^m(\alpha)$ is the Laplace coefficient, and $q \equiv sgn(r - r_s)$. The solution is singular at a Lindblad resonance, $D = 0$; but the singularity can be removed by the addition of a small imaginary part into the frequency, $\omega_m \to \omega_m - i\epsilon$ (4, 5). Physically, such a frequency component can arise from damping, and its presence introduces the azimuthal asymmetry necessary for a torque to exist (5,6). The specific torque exerted on a particle from the mth order term is $T_p = -m\phi_m(r)\sin(\omega_m t - m\theta)$. When expanded to first order and averaged over an orbit, the torque reads

$$\langle T_p \rangle = -\frac{1}{2} m\Psi_m Im\{R_m/r\} = q\frac{m\Psi_m^2}{r^2}\left[\frac{\epsilon\Omega}{D^2 + 4\epsilon^2\Omega^2}\right]. \quad \textbf{(1)}$$

The torque density exerted on the disk in the resonance vicinity is $dT/dr \equiv 2\pi r\sigma\langle T_p \rangle$, where σ is the disk's surface density. Expanding D about the Lindblad resonance position while holding other quantities constant and integrating throughout the disk, yields the standard torque formula (7),

$$T_m = \int_{disk} \frac{dT}{dr} dr = -\frac{m\pi^2\sigma\Psi_m^2}{rdD/dr} = q\frac{\pi^2\sigma\Psi_m^2}{3\Omega_s\Omega} \quad \textbf{(2)}$$

where the derivative has been evaluated at resonance, viz., $rdD/dr = -3qm\Omega_s\Omega$. The reaction torque on the protoplanet is equal but opposite to T_m, so that outer resonances cause orbit decay while inner ones cause orbit expansion. Comparing torques of like order, outer torques are strengthened by the slower orbital frequency of the resonant particles.

Next we examine the forcing functions, $\Psi_{m,q}^2$. To a good approximation, $b_{1/2}^m(\alpha) \approx (2/\pi)K_0(\Lambda)/\sqrt{\alpha}$, where K_ν denotes a modified Bessel function of argument $\Lambda = m|\alpha - 1|/\sqrt{\alpha}$, with $\alpha \equiv r/r_s$. The forcing function can then be rewritten as $\Psi_{m,q} = -q(2/\pi)(GM_s/r_s)m\psi_{m,q}$, where

$$\psi_{m,q} = \frac{1}{2}\left(1 + \frac{1}{\alpha}\right)K_1(\Lambda) + \left(2m|\alpha^{3/2} - 1| + \frac{q}{2m}\right)\frac{K_0(\Lambda)}{\sqrt{\alpha}} \quad \textbf{(3)}$$

Figure 1 shows $\psi_{m,q}^2$ as a function of $m|\alpha - 1|$ for $m = 5$. Note that in the vicinity of a Lindblad resonance, where $m(\alpha - 1) = m[(1 + q/m)^{2/3} - 1] \approx 2q/3 - 1/9m$, the outer forcing function is larger. In addition, outer

resonances lie slightly closer to the perturber than inner ones (8). Writing $\Lambda \approx 2/3 - q/3m$, eqn (3) can be expanded to give $\psi_{m,q} \sim \psi_o + (q/m)\delta\psi$, where $\psi_o = K_1(2/3) + 2K_0(2/3)$ and $\delta\psi = [5K_1(2/3) + K_0(2/3)]/6$. The fractional difference due to the forcing function between outer and inner resonances is thus $\Delta\Psi_m^2/\langle\Psi_m^2\rangle \sim (4/m)\delta\psi/\psi_o \approx 5/3m$. Since the corresponding fractional difference in orbital frequency is $\Delta\Omega/\Omega \approx \Delta r(d\Omega/dr)/\Omega \approx -2/m$ (where $\Delta r \sim 4r/3m$ is the radial spacing between outer and inner resonances of a given Lindblad pair), the overall fractional difference in the combination Ψ_m^2/Ω is of order $\sim 11/3m$. This asymmetry is intrinsic to the Keplerian disk.

So far we have ignored effects of the surface density. To compensate for the inherent torque asymmetry, a particle disk with a power law density ($\sigma \propto r^{-k}$) would require an index of $k \sim 11/4$. This is a much steeper gradient than typical models of circumstellar disks.

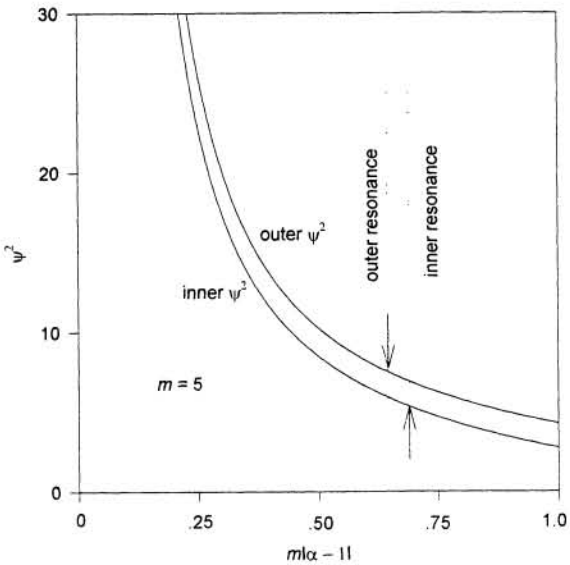

Figure 1. The values of ψ^2 in the inner and outer disk and the locations of the inner and outer resonances as a function of $m|\alpha - 1|$ for $m = 5$.

Furthermore, in a gaseous disk, a surface density gradient produces a radial pressure gradient that alters the rotational profile of the disk, causing an inward shift in all resonance sites. This turns out to largely buffer the effect of the density gradient so that the net torque becomes relatively insensitive to k (8, 9). Disk pressure also increases the natural oscillation frequency of the disk to $\sim \kappa\sqrt{1+(mc/r\Omega)^2}$, where c is the gas sound speed. This displaces high order resonances away from the perturber and greatly weakens the torque for $m > r\Omega/c$ (10, 11). The most important resonances thus have fractional differences equal to or greater than $\sim 11/3(h/r)$, where $h \sim c/\Omega \sim O(10^{-1}r)$ is the scale height of the disk. Consequently, the net torque can be a significant fraction of the average torque magnitude from one side. Finally, the gas response is wavelike, with the torque exerted over the spatial scale of the first wavelength. For most orders, this exceeds the adjacent resonance spacing so that the overall torque is rather smoothly distributed over the disk rather than highly localized in discrete annuli. This renders the gas disk torque less susceptible to saturation via the opening of gaps. Characteristic orbital decay times, $\tau \equiv r/\dot{r}$, are of order

$$\tau \sim C\left(\frac{M_\odot}{M}\right)\left(\frac{M_\odot}{\pi\sigma r^2}\right)\left(\frac{c}{r\Omega}\right)^2 P \qquad \textbf{(4)}$$

where P is the orbital period of the protoplanet and C is a constant of order unity. For a one solar mass primary, this reads $\tau \sim 6x10^5(M/M_\oplus)^{-1}(\sigma/10^3 gcm^{-2})^{-1}(T/10^2K)(r/AU)^{1/2}$ years, where $M_\oplus$ is an earth mass and T is the disk temperature.

References

(1) Walker, G. A. H., A. R. Walker, A. W. Irwin, A. M. Larson, S. L. S. Yang, and D. C. Richardson, *Icarus* **116**, 359 (1995). (2) Black, D. C., *Annu. Rev. Astron. Astrophys.* **33**, 359 (1995). (3) Goldreich P. and S. Tremaine, *Annu. Rev. Astron. Astrophys.* **20**, 249 (1982). (4) Meyer-Vernet, N. and B. Sicardy, *Icarus* **69**, 157 (1987). (5) Hahn, J. M., W. R. Ward, and T. W. Rettig, *Icarus* **117**, 25 (1995). (6) Greenberg, R. *Icarus* **53,** 207 (1983). (7) Goldreich, P. and S. Tremaine *Astrophys. J.* **233**, 857 (1979). (8) Ward, W. R. *Icarus* **67**, 164 (1986). (9) Korycansky, D. G. and J. B. Pollack, *Icarus* **102**, 105 (1993). (10) Goldreich, P. and S. Tremaine, *Astrophys. J.* **241**, 425 (1980). (11) Artymowicz, P. *Astrophys. J.***419**, 155 (1993).