MODELLING A CANDIDATE LAVA FOUNTAIN FISSURE ERUPTION ON IO. J. W. Head[1], L. Wilson[2] and the Galileo SSI Team; [2]Dept. of Geological Sciences, Brown University, Providence, RI 02912 (James_Head_III@brown.edu); [2]Environ. Sci. Dept., Lancaster Univ., Lancaster LA1 4YQ, UK (L.Wilson@lancaster.ac.uk);

On the basis of the models for the ascent and eruption of magma developed for a variety of planets (e.g., [1]) we can assess the recent data for a candidate lava fountain fissure eruption on Io [2]. Assume that the vent consists of a line source (a fissure) of length Y, about 25 km. There is a dark pyroclastic deposit which appears related to the vent and which extends out to a distance X of about 30 km. If this deposit is formed by small pyroclasts, small enough that their terminal velocity through the gas leaving the vent is small (which is likely to be the case for magmatic eruptions into a vacuum based on the observed size distributions and inferred eruption dynamics of the lunar glass beads sampled by the Apollo missions [3,4], then the gas speed, U, needed to allow them to reach range X is given by the ballistic formula

$$X = (U^2 \sin 2\theta)/g \qquad (1)$$

where g is the acceleration due to gravity, ~1.8 m s^{-2}, and θ is the maximum angle from the vertical of any clast leaving the vent. The minimum value of U corresponds to using θ = 45°, implying U equal to at least 232 m s^{-1}. If θ were much less, say 10°, the estimate of U would increase to at least 397 m s^{-1}. If θ were as small as 5° U would be at least 558 m s^{-1}. This latter value is not at all improbable: the way the mixture of gas and partially entrained pyroclasts spreads out laterally above a vent in an eruption into a vacuum is determined by the Mach number of the flow in the vent, which will almost certainly be choked rather than pressure balanced [5,6]. We adopt U = 500 m s^{-1}, which corresponds to θ = 6.25°, for later illustrations.

The simplest model of the eruption is one in which we assume that the distribution of pyroclast sizes is unimodal with a mean diameter φ. Then if the volume flux from the vent is V, the total number of pyroclasts erupted per unit time is N where

$$N = V/[(4/3)\ \pi (\varphi^3/8)] = (6\ V)/(\pi \varphi^3) \qquad (2)$$

In this model, the incandescent part of the lava fountain would represent the region within which the number density of pyroclasts was so large that they shielded one another from radiating heat, so remaining nearly isothermal [7]. Assume that the shielding is perfect out to some radial distance R and then decreases to zero over a radial distance Λ. Analysis of the image, after plausible corrections for charge bleeding, suggests that R is about 10 pixels, i.e. about 2 km and that Λ is somewhat smaller than one pixel, say 150 m. As long as R is much less than X (which, with R = ~2 km and X = ~ 30 km is true here) the speed of the pyroclasts will not change much from their launch speed U as they cover the distance R, and so they travel essentially radially out to this distance. We can then derive an expression for Λ as follows. We first find the number density of pyroclasts at the radial distance R. In some small time interval Δt the number launched from the vent is (N Δt) = (6 V Δt)/(π φ^3); because the erupted volume flux and the clast velocities are constant, this same number of clasts will travel from R to (R + ΔR) where ΔR = (U Δt). The volume though which they pass is (π R ΔR Y) = (π R U Δt Y). Thus the number per unit volume at R is (6 V Δt)/(π φ^3)/(π R U Δt Y), i.e. [{6 (V/Y)}/(π^2 φ^3 R U)], and the average separation of the pyroclasts, S, is the cube root of the volume per particle, i.e. $[(\pi^2 \varphi^3 R U)/\{6 (V/Y)\}]^{1/3}$. Consider one layer of pyroclasts, normal to the direction to the vent, with the average spacing S. The area of sky obscured by a single pyroclast is [(π/4) φ^2] and so the fraction of the area masked by one layer is [(πφ^2)/(4 S^2)]. Hence the number of layers for near-complete obscuration is [(4 S^2)/(π φ^2)] and since these layers are separated by the average distance S, the linear distance in the cloud of pyroclasts over which obscuration becomes near-complete, Λ, is

$$\Lambda = S\ [(4\ S2)/(\pi\varphi 2)] = [(4\ S3)/(\pi\varphi 2)] = [(2\ \pi\varphi R\ U)/\{3\ (V/Y)\}] (3)$$

Wilson & Head [8] showed that a correction factor, G, is needed in the above formula to allow for the fact that in general a range of grainsizes, rather than a single grainsize, will be present amongst the pyroclasts. If the size range varies from a factor of ~10 around the mean value, then G is close to 2.

With this addition, the above formula becomes

$$(V/Y) = [(2\ \pi)/3]\ [(\varphi R\ U)/(G\ \Lambda)] \qquad (4)$$

Inserting the relevant values inferred from the image, R = 2000 m, U = say 500 m s^{-1} and Λ = 150 m, and taking G = 2, we can find the volume eruption rate per unit length of the fissure, (V/Y) as a function of φ. Some values are shown in Table 1. If the typical sizes of pyroclasts on Io are similar to those on the Moon, in the range 100 μm to 1 mm, (V/Y) is in the range 0.7 to 7 m^3 s^{-1} m^{-1}. Larger fluxes are implied by larger pyroclast sizes but, as φ increases, the vertical component of the clast velocity becomes significantly less than U as the terminal velocity of the clast becomes an appreciable fraction of U, and so the value of (V/Y) becomes progressively less than that predicted by the formula.

There is an alternative model. In this, the current activity is regarded as being confined entirely to the incandescent lava fountain and the dark mantling deposit is regarded as being either coincidental or the product of an earlier eruptive event in the same area. We do not think that this is a likely scenario, but explore it for the sake of completeness. The eruption velocity U must now be obtained from the observed lateral extent of the lava fountain, R = ~2000 m, via

$$U^2 = (g\ R)/\sin 2\theta \qquad (5)$$

and the distance within the cloud of pyroclasts over which obscuration becomes near-complete, Λ, is given by the formula derived in Wilson & Keil [9] as

$$\Lambda = (4\ \theta\ \varphi\ R\ U)/[9\ G\ (V/Y) \sin 2\theta] \qquad (6)$$

leading to

$$(V/Y) = [(4\ \theta)/(9 \sin 2\theta)][(\varphi R\ U)/(G\ \Lambda)] \qquad (7)$$

In this case, the values of U corresponding to θ = 45°, 10° and 5° are 60, 102 and 144 m s^{-1}, and we adopt U = 129 m s^{-1}. at θ = 6.25°, as in the earlier example, for illustration. Using the values R = 2000 m and Λ = 150 m as before we then find the variation of (V/Y) with φ shown in Table 2.

For mean pyroclast sizes in the range 100 μm to 1 mm, (V/Y) is in the range 0.02 to 0.2 $m^3 s^{-1} m^{-1}$, the values being systematically about 35 times smaller than in the first model.

It is interesting to compare these two sets of values of (V/Y) with the eruption rates typical of basaltic fissure eruptions on Earth. An eruption on the south rim of the main caldera of Kilauea volcano in Hawai'i which fed a lava stream down an old rainwater gully was analyzed by Heslop et al. [10] to derive the integrated volume flux from an ~30 m long active fissure as about 200 $m^3 s^{-1}$, implying (V/Y) = ~7 $m^3 s^{-1} m^{-1}$. It is probably fortuitous that this value corresponds closely with the predicted eruption rate from our preferred model for Io. However, we can explore the plausibility of the values deduced by considering the rise of magma in dikes from reservoirs in the crust and mantle of Io.

A dike containing buoyant melt with a density ρ will extend upward through country rocks of density β by cracking the rocks ahead of it as long as the stress intensity K^+ at its upper tip exceeds or is just equal to a material property of the country rocks called the apparent fracture toughness, K_f [11]. The dike will be forced to pinch shut at its lower tip if the stress intensity there, K^-, decreases to zero. When $K^+ = K_f$ and $K^- = 0$, it is easy to show, using the definition of the stress intensity [11], that the driving pressure of the dike, P_0, defined as the internal magma pressure minus the external compressive stress at the dike centre, and the half-height of the dike, A, are given by

$$A = [K_f /(g\ \Delta\rho)]^{2/3} \quad (8)$$

and

$$P_0 = [(g\ \Delta\rho\ K_f^2)/8]^{1/3} \quad (9)$$

where $\Delta\rho = (\beta - \rho)$. The mean width of the dike is W given by

$$W = [\pi (1 - \nu)\ P_0\ A]/(2\ \mu) \quad (10)$$

where ν and μ are the Poisson's ratio and shear modulus for the host rocks, appropriate values at shallow crustal depths being 0.25 and 3 GPa, respectively. If we assume in the present case that dikes must be continuous through the crust to supply the very long-lived eruptions typical of Io then we might expect A = 5 to 15 km (i.e. crustal thicknesses in the 10 to 30 km range). If the crust and magma are of similar composition, Δρ is likely to be 5% to 10% of ρ, say 200 kg m^{-3}. However, there are likely to be layers of SO_2 frost, solid sulphur or liquid SO_2 aquifers intercalated with silicate eruptives in the crust of Io, particularly near the surface, and the average value of Δρ may be much smaller than this. K_f appears to be about 100 MPa $m^{1/2}$ in basaltic systems, and equation (8) would be satisfied with Δρ = 200 kg m^{-3} if A were 4.26 km, in which case P_0 would be 0.77 MPa. The average dike width would then be 1.3 m. The corresponding values for Δρ = 100 and 50 kg m^{-3} are 6.76 and 10.73 km; 0.61 and 0.48 MPa; and 1.6 and 2.0 m, respectively. The mean rise speeds, u, of magma in such dikes are found by balancing the buoyancy force against wall friction. If the magma motion is laminar, u is given by

$$u = (W^2\ g\ \Delta\rho)/(12\ \eta) \quad (11)$$

where η is the magma viscosity, which, on the basis of temperature observations that imply ultramafic compositions for magmas erupted on Io [12], we take as 1 Pa s. If the magma motion is turbulent, u is given by

$$u = [(2\ W\ g\ \Delta\rho)/(0.01\ \rho)]^{1/2} \quad (12)$$

where we take the magma density, ρ, to be 3000 kg m^{-3}. Assuming first that the magma motion is laminar, we find that for Δρ = 200, 100 and 50 kg m^{-3}, u will be 51, 38 and 30 m s^{-1}, respectively. Evaluation of the corresponding Reynolds numbers (defined as Re = [(2 u W ρ)/η] gives values of 4.0 x 10^5, 3.6 x 10^5 and 3.6 x 10^5, respectively, all clearly not consistent with laminar magma motion. Using the formula for turbulent motion we have u equal to 5.6, 4.4 and 3.5 m s^{-1}, respectively, with Re equal to 4.4 x 10^4, 4.2 x 10^4 and 4.1 x 10^4, respectively, all internally consistent with turbulent motion implying that this set of velocities is the correct one to use. Finally, since the volume flux per unit length in a fissure is given by (V/Y) = (u W), we can find the three fluxes corresponding to Δρ = 200, 100 and 50 kg m^{-3} to be 7.3, 7.0 and 6.9 $m^3 s^{-1} m^{-1}$, respectively. All of these are again extremely close to the value inferred from the eruption dynamics for a lunar-like pyroclast size distribution.

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Table 1. Values of the volume eruption rate per unit length of the fissure, (V/Y) as a function of φ, the mean pyroclast diameter, in the case where the dark mantle deposit is assumed to be part of the present eruption.

φ	(V/Y)/($m^3 s^{-1} m^{-1}$)
10 μm	0.07
100 μm	0.7
1 mm	7
10 mm	70
100 mm	700
1 m	7000

Table 2. Values of the volume eruption rate per unit length of the fissure, (V/Y) as a function of φ the mean pyroclast diameter, in the case where the dark mantle deposit is assumed *not* to be part of the present eruption.

φ	(V/Y)/($m^3 s^{-1} m^{-1}$)
10 μm	0.0019
100 μm	0.019
1 mm	0.19
10 mm	1.9
100 mm	19
1 m	190