TWO-PARAMETER EMPIRICAL PHOTOMETRIC FUNCTION IN ANALYSIS OF EARTH-BASED OBSERVATIONS OF THE MOON. Yu. I. Velikodsky, L. A. Akimov, and V. V. Korokhin. Astronomical Observatory of Kharkov National University. Sumskaya St., 35, Kharkov, 61022, Ukraine. E-mail: velikodsky@astron.kharkov.ua.

Abstract: Improved empirical Akimov's formula is presented. This photometric function gives a good fit to Earth-based and Clementine observational data. An influence of albedo and relief on the disk brightness distribution is discussed.

Introduction: When optical photometry is used for prognosis of composition and structure of the lunar surface, one must bring different observational data to the same photometric conditions. To do this one should use a photometric function F(α,φ,λ), that describes changes of surface reflectance with variations of the phase angle α, photometric latitude φ and longitude λ. The photometric coordinates φ and λ can easily be transformed to traditional angles of incidence (i) and emergence (ε): cosi=cosγ·cos(λ-α); cosε=cosφ·cosλ. We suppose that the best function F(α,φ,λ) to be used is presented in [1]:

$$F(\alpha,\varphi,\lambda) = \exp(-\mu\alpha)\cdot\Psi(\alpha,\varphi,\lambda)\,, \qquad (1)$$

where

$$\Psi(\alpha,\varphi,\lambda) = \frac{\cos(\alpha/2)}{\left(1-(\sin(\alpha/2))^{1+q(\alpha)}\right)\cos\lambda}\cdot(\cos\varphi)^{q(\alpha)}\times \times\left(\left(\cos(\lambda-\alpha/2)\right)^{1+q(\alpha)}-\left(\sin(\alpha/2)\right)^{1+q(\alpha)}\right) \qquad (2)$$

is brightness distribution over the lunar disk for specified α relative to mirror point (for which φ=0, λ= α /2); q(α) – "the smoothness factor"; μ – "the effective roughness coefficient". Formula (1) does not take into account the opposition effect that appears at α <20° as multiplier depending on phase angle (see, e.g., [2]). And at α >20° the photometric function (1) depends on two parameters: μ and q. In [1] it was shown that for the Moon at α <60° dependence q(α) is linear: q= να /π (ν – some coefficient), and at larger phase angles q increases more quickly than by a linear law. The same expression (q=να /π) was used in [3]. There it was showed that function (1) can secure the good accuracy (1-3%) at photometric calibrations of Clementine images (at least, at 30°<α<80°). And here we attempted to study the dependence q(α) at larger phase angles using Earth-based data and to determine an influence of albedo and relief on the parameter q. Note that at values of q, which are typical for the Moon, a longitude distribution of brightness at large phase angles changes very slowly with variations q, but a latitude distribution changes essentially. Therefore, to study behavior of q at different phases we used the latitude dependence of brightness:

$$\Psi_{lat}(\alpha,\varphi) = (\cos\varphi)^{q(\alpha)}\,. \qquad (3)$$

We plotted q(α) for lunar highlands at α <135°, compared latitude dependence of highlands in two spectral bands, and compared latitude dependence of maria and highlands.

Observations and results of processing: The observations were performed with onedimensional CCD-detector on the 70-cm reflector and, in part, on the 20-cm refractor of KhAO. Two spectral bands were used: red (0.75 μm) and blue (0.47 μm), phase angle was 2°-135°. Also photographic images of the Moon at α=142° in red (0.63 μm) and UV (0.38 μm) rays were used. Image processing was performed with universal program complex <IRIS> (http://members.xoom.com/cyteg).

Smoothness factor q(α) calculation. For analysis of the latitude dependence a ratio of brightness at given phase angle α to brightness near the full moon phase (α_0) was determined. For small phase angles (α_0≈2°) the brightness distribution Ψ(α_0, φ, λ)≈1, and therefore, distributions of the following value were obtained with that division:

$$b(\alpha,\varphi,\lambda) = f(\alpha,\alpha_0)\cdot\Psi(\alpha,\varphi,\lambda)\,. \qquad (4)$$

Here f(α,α_0) – a phase ratio of albedo reduced to the mirror point. A distribution of the b(α,φ,λ) along any photometric meridian is described by expression (3) accurate to variations of f(α.α_0) over the lunar surface. One can obtain q(α) by fitting the b(α,φ,λ) distribution along latitude φ by expression (3).

In this way values of the smoothness factor q of highlands for α=30°-135° were obtained (fig. 1, points). At that for calculating the parameter q we used points on the lunar surface with albedo smaller than certain value (~0.12) corresponding to a level between maria and highlands. Values of q were calculated for a set of photometric longitudes with a step along λ being about one image pixel, and then were averaged with all λ. Variations of q along the longitude was 0.03-0.1, standard deviation of measured values from given by function (3) ones was 5-10%. First of all, it is caused by that the phase ratio f(α,α_0) in (4) is not a constant for all surface of highlands. Smoothness factor q, in turn, also can vary over the highland surface.

We noticed that the dependence q(α) up to a constant multiplier is very close to phase dependence of exponent at cos φ in the theoretical Akimov's formula that was derived for extremely rough surface [1] and, in a different way, for fractal-like surface [2]. In this formula $q(\alpha)=\alpha/(\pi-\alpha)$. And we fitted dependence $q(\alpha)$ by equation

$$q(\alpha)=\nu\alpha/(\pi-\alpha)\,, \quad (5)$$

where ν – some coefficient. It turned out, that for highlands ν=0.53±0.07.

In a different way, we fitted the brightness distribution along φ and λ directly by function (2). It gave almost the same value: ν=0.51±0.04 (fig. 1, curve). It says that using of only latitude dependence is well-posed.

Spectral dependence of q. In visible range of spectrum albedo of the lunar surface essentially changes with the wavelength. Therefore, measurements in different spectral bands permit one to study an influence of albedo on the latitude dependence.

A distribution of a color index was approximate by expression $C(\varphi)=C_0\cdot(\cos\varphi)^{\Delta q}$, where Δq is a difference of smoothness factor values for two wavelengths. It was obtained that for lunar highlands at α=105° the color index C_1(0.75/0.47) does not changes essentially with the latitude (Δq=–0.03±0.02), and at α=142° the color index C_2(0.63/0.38) increases with the latitude (Δq=–0.12±0.06), i.e. brightness falling toward poles in the UV area is larger than in the red one, although the lunar albedo, on the contrary, in the ultraviolet is smaller and multiple scattering should lead to a reverse effect.

The latitude dependence of maria and highlands. Maria and highlands differ not only with albedo and also with relief, because highlands are mach more mature. It may lead to distinction in the latitude dependence. Values of smoothness factor q for maria and highlands at two phase angles was found. At α 45° values are q=0.11±0.07 for highlands and q=–0.06±0.07 for maria. At α=133° – q=1.51±0.04 for highlands and q=0.91±0.08 for maria. For maria the smoothness factor is smaller, although at small phase angles (α=45°) the difference is of the order of error. A negative value of q for maria at α=45° is not significant and is accounted for greater, than on highlands, heterogeneity of mare surface with phase dependence parameters.

Conclusion: 1. We showed that the empirical photometric function (1) with parameter q(α) given in the form of (5) with a good accuracy fits observational data at α<135° and can be used for photometric calibration of lunar images.

2. Results of our observations show that the parameter q(α) practically does not change with variations of albedo. At the same time for highlands the value of q is larger than for maria. But highlands have not only larger albedo and also more powerful relief than maria. Therefore, variations of q(α) is accounted, mainly, for influence of the relief, and the coefficient ν in (5) can characterize the relief power.

3. Some decrease of the parameter q with the wavelength (on limit of accuracy) can mean that all relief scales including sizes comparable with the wavelength form the latitude distribution of brightness.

References: [1] Akimov L. A. (1988) Kinematika i fisika nebesnykh tel, 4, No 1, 3-10. [2] Shkuratov Yu. G. et al. (1999) Icarus, 141, 132-155. [3] Velikodsky Yu. I. et al. (1999) LPSC XXX, Abstract #1039.

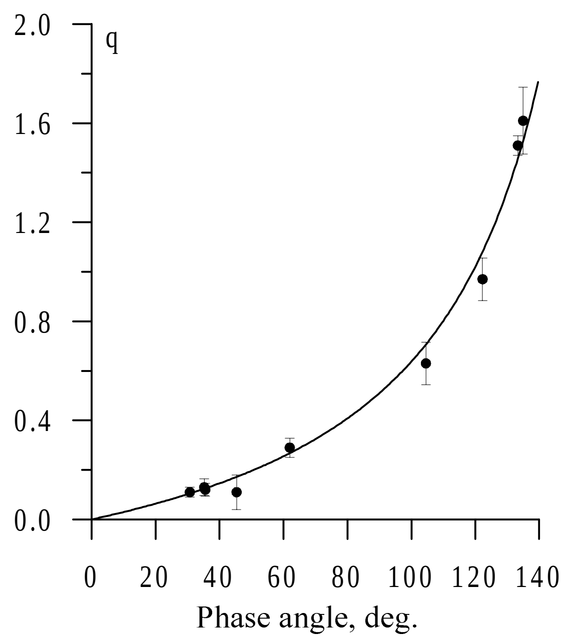

Fig. 1