TIDAL DISSIPATION IN MERCURY. Bruce G. Bills[1,2], [1]SIO/UCSD, La Jolla, CA 92093, [2]NASA/GSFC, Greenbelt, MD 20771, bills@core2.gsfc.nasa.gov

Introduction: Tidal dissipation within Mercury may be adequate to maintain a fluid core against conductive and convective heat loss. Spatial and temporal variations in the dissipation rate are quite extreme due to the 3:2 spin-orbit resonance and large orbital eccentricity fluctuations. The largest rates of dissipation occur at times of maximum eccentricity. Details of the spatial pattern of dissipation depend rather sensitively on density, rigidity, and viscosity variations within the planet, which are largely unknown. Previous analyses of this problem have restricted the expansion of the tidal potential to terms linear in eccentricity, which is not adequate for Mercury.

Background: The role of tidal dissipation, as a prominent heat source in synchronously rotating satellites, has been widely appreciated for over 20 years [1,2,3]. The purpose of the present study is to begin analysis of the pattern of dissipation that occurs in Mercury. While the role of tidal dissipation within Mercury has been previously studied, both in terms of driving the planet towards a 3:2 spin-orbit resonance [4,5,6] and as a potential contributor to the global thermal budget [7], the spatial and temporal patterns of present day dissipation have not been explored at the same level of detail as for the synchronous satellite case.

Two important features distinguish the dissipation situation at Mercury from that encountered on Io or Europa, for example. First is the somewhat more complicated geometry associated with the 3:2 spin-orbit resonance. For example, the tidal bulge on Mercury would continue to move around the body, even if the orbital eccentricity were zero. The second important feature of Mercury is the very large and highly variable orbital eccentricity. The present eccentricity value is quite large (e=0.2056), but within the past few million years it has varied from less than half to nearly twice the present value [8]. As a result, linear analyses, which work very well for Io, are quite inadequate for Mercury.

Tidal Potential: The gradient of the gravitational field of the Sun in the vicinity of Mercury gives rise to forces which deform the planetary body. If we assume that the obliquity of Mercury is very small, so the sub-solar point is always on the equator, then the degree two term in the tidal potential can be written in the form

$$\Phi_2(r,\theta,\phi,\ t) = n^2 r^2 \sum_{j=0}^{2} F_j(\theta,\phi)\ H_j(t)$$

where n is orbital mean motion, (r,θ,ϕ) are spherical coordinates, centered in Mercury, F_j are spherical harmonic functions

$$2\,F_0 = P_{2,0}(\kappa)$$
$$4F_1 = P_{2,2}(\kappa)\cos(2\phi)$$
$$4F_2 = P_{2,2}(\kappa)\sin(2\phi)$$

with κ = sin(θ), and the temporal factors are expressed as Fourier series in orbital mean anomaly M = n t;

$$H_0(t) = \sum_{p=0}^{\infty} h_p^0(e)\ \cos(p\ M)$$

$$H_1(t) = \sum_{p=0}^{\infty} h_p^1(e)\ \cos(p\ M)$$

$$H_2(t) = \sum_{p=0}^{\infty} h_p^2(e)\ \sin(p\ M)$$

Dissipation Rate: The mean rate of tidal energy dissipation beneath a surface point on Mercury can be written as the product of the tide raising potential Φ_2 times the induced potential Ψ_2 [9,10]

$$\frac{dE(\theta,\phi)}{dt} = \frac{1}{4\pi G R}\left\langle \Phi_2(\theta,\phi)\frac{d\Psi_2(\theta,\phi)}{dt}\right\rangle$$

where the angle bracket indicates averaging over the tidal forcing period. The induced potential Ψ_2 is proportional to the imposed potential Φ_2, and is obtained from it via convolution with the complex Love number k_2, which is a function of internal structure (density, rigidity, viscosity) and forcing frequency. As the internal structure of Mercury is only very poorly known, the effective Love numbers are quite uncertain.

In a purely elastic body, the imposed and induced potential are in phase, and there is no dissipation. For a viscoelastic body, the induced potential will lag behind the imposed potential, and the dissipation rate will be proportional to the imaginary part of the complex Love number. Written in this form, the global average dissipation rate is

$$\frac{dE}{dt} = \frac{n^5 R^5}{2G}\sum_{j=0}^{2}\left\langle F_j^2\right\rangle \sum_{p=1}^{\infty} p\ \mathrm{Im}[k_2(pn)]\left(h_p^j(e)\right)^2$$

and the scale factor is

$$\frac{n^5 R^5}{2G} = 2.49\times 10^{11}\ W$$

corresponding to a surface heat flow of 3.3 mW m^{-2}. For comparison, estimates of current heat flow from primordial and radioactive sources are typically 20 mW m^{-2} [7]. This suggests that tidal heating can be important contribution to the global energy budget.

Love Numbers: The largest source of uncertainty in calculating the tidal dissipation rate for Mercury is the lack of information about internal structure. Even the density structure is only rather poorly constrained, and the rigidity and viscosity values are quite unconstrained. As a starting point it is instructive to consider the response of a homogeneous Maxwell viscoelastic body, forced at an arbitrary frequency f. The corresponding tidal Love number for degree 2 components in the potential is [11]

$$k_2(\tau, \mu, \eta, f) = \frac{3\tau}{2\tau + 19(\mu + i\eta f)}$$

where

$$\tau = \rho\, g\, R = 49.0\ MPa$$

is an effective gravitational rigidity, μ is elastic rigidity, and η is viscosity. In this simple model, the imaginary part of the complex Love number is small at both very high and very low forcing frequencies, and reaches a maximum, equal to ½ the purely elastic Love number, at a frequency of

$$f_{\max} = \frac{2\tau + 19\mu}{19\eta}$$

If the orbit were circular, the only forcing frequency would be the orbital frequency n. As the eccentricity increases, higher harmonics become more important, both in the tidal potential and in the dissipation rate. For example, if the internal structure of Mercury is such that the orbital frequency matches the peak in dissipation ($n = f_{max}$), then the second harmonic contribution to dissipation exceeds the fundamental frequency contribution for all eccentricity values above 0.27. Both the global average, and the spatial pattern of dissipation depend on eccentricity.

In a more realistic model, with high rigidity and viscosity in the outer layers, and a relatively weak deep interior, the local dissipation rate will be an even stronger function of location and time, as the orbital eccentricity varies. It is at least conceivable that, during times of high eccentricity, the rate of tidal dissipation in the deep interior may be sufficient to drive convective instability in a fluid core. In that case, Mercury might exhibit an episodic dynamo.

Future orbital missions to Mercury will attempt to measure the amplitude and phase of forced librations as a means of detecting a putative fluid core [12,13,14]. If there is substantial tidal dissipation occurring within Mercury, then the mean orientation of the axis of least inertia at times of periapse will be displaced by a tidal torque, similar to what is seen for the Moon [15]. Measurement of that angle would provide important constraints on this poorly understood aspect of Mercury.

References: [1] Peale, S.J. et al. (1979) *Science*, 203,892-894. [2] Ross, M. and G. Schubert (1987) *Nature*, 325, 132-134. [3] Ojakangas, G.W. and D.J. Stevenson (1989), *Icarus*, 81, 220-241. [4] Goldreich, P. and S.J. Peale (1966), *Astron. J.* 71, 425-438. [5] Burns, J.A. (1976), *Icarus*, 28, 453-458. [6] Peale, S.J. and A.P. Boss (1977), *J. Geophys. Res*., 82, 743-749. [7] Schubert, G. et al. (1988), *Mercury*, U.A. Press, 429-460. [8] Laskar, J. (1988), *Astron. Astrophys*., 198, 341-362. [9] Zschau, J. (1986), *Tidal Friction and the Earth's Rotation*, Springer-Verlag, 62-94. [10] Platzman, G.W. (1984), *Rev. Geophys*., 22, 73-84. [11] Munk, W.H., and G.J.F. MacDonald (1960), *The Rotation of the Earth*, Cambridge Univ. Press. [12] Peale, S.J. (1972) *Icarus*, 17, 168-173. [13] Peale, S.J. (1976) *Nature*, 262, 765-766. [14] Peale, S.J. (1981) *Icarus*, 48, 143-145. [15] Williams, J.G. et al. (2001), *J. Geophys. Res*., 106, 27,933-27,968.