Type II Migration and Giant Planet Survival. Wm. R. Ward, Dept. of Space Studies, Southwest Research Institute, Boulder, CO 80302

Type II migration, in which a newly formed large planet opens a gap in its precursor circumstellar nebula and subsequently evolves with it, has been implicated as a delivery mechanism responsible for close stellar companions [1]. Large scale migration is possible in a viscously spreading disk of surface density $\sigma(r,t)$ when most of it is sacrificed to the primary in order to promote a small portion of the disk to much higher angular momentum orbits. Embedded planets generally follow its evolution unless their own angular momentum is comparable to that of the disk. The fraction of the starting disk mass, $M_d = 2\pi\int r\sigma(r,0)\,dr$, that is consumed by the star depends on the distance at which material escapes the disk's outer boundary. If the disk is allowed to expand indefinitely, virtually all of the disk will fall into the primary in order to send a vanishingly small portion to infinity. For such a case, it is difficult to explain the survival of *any* giant planets, including Jupiter and Saturn. Realistically, however, there are processes that could truncate a disk at a finite distance, r_d. Recent numerical modeling has illustrated that planets can survive in this case [2,3]. We show here that much of these results can be understood by simple conservation arguments.

The amount of mass M_o lost through the disk's outer boundary is simply that necessary to remove all of the disk's original angular momentum,

$$L_d = 2\pi\int r\sigma\sqrt{GM_\star r}\,dr \quad ; \quad M_o = L_d/\sqrt{GM_\star r_d}$$

where $M_\star$ is the primary's mass. For a power law profile, $\sigma \propto r^{-k}$ ($k < 2$) and initial outer rim r_o,

$$L_d = C_k M_d\sqrt{GM_\star r_o} \quad ; \quad M_o/M_d = C_k\sqrt{r_o/r_d}$$

where $C_k \equiv (2-k)/(5/2-k)$. Thus, the fraction of the disk accreted by the star is $M_{acc}/M_d = 1 - C_k\sqrt{r_o/r_d}$, and the initial boundary between inward and outward destined material is then

$$r_c = r_o[1 - C_k\sqrt{r_o/r_d}]^{1/(2-k)}$$

Suppose, as an important special case, that the original radius of the disk is the same as the truncation radius, $r_o = r_d$; then the critical radius is $r_c = r_d/(5-2k)^{1/(2-k)}$ and is shown in Figure 1. We see in this case that a significant fraction of the disk exits the system through the outer boundary.

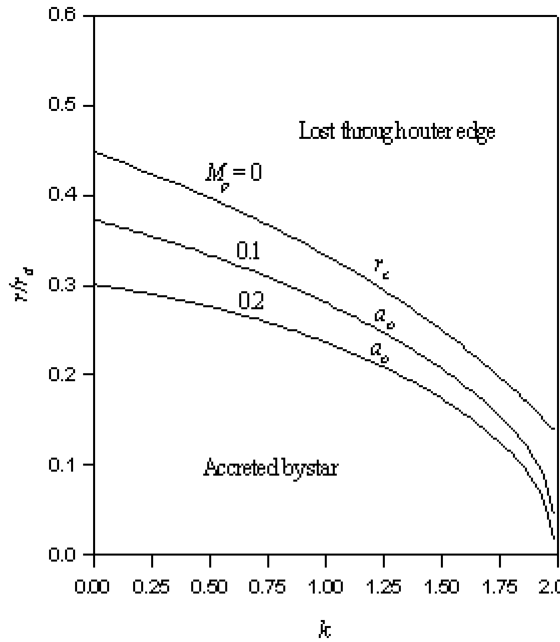

Figure 1: The critical radius r_c/r_d when $r_d = r_o$, $M_p = 0$ as a function of the surface density power index k. Also shown are the minimum initial semimajor axes for survival for protoplanets of mass $M_p/M_d = 0.1,\ 0.2$

Now add to the disk a planet of mass M_p with an initial orbit radius a_o. The disk mass external to the planet is $M_{ext} = M_d[1 - (a_o/r_o)^{2-k}]$; if the planet remains in the disk ($0 < a < r_d$), it will act as an angular momentum repository:

$$M_p\sqrt{GM_\star a} = L_d + M_p\sqrt{GM_\star a_o} - M_{ext}\sqrt{GM_\star r_d}$$

The final position of the planet when the disk's mass is exhausted can be written in terms of the critical radius as

$$\frac{a}{r_o} = \left((\frac{a_o}{r_o})^{1/2} + [(\frac{a_o}{r_d})^{2-k} - (\frac{r_c}{r_d})^{2-k}](\frac{r_d}{r_o})^{1/2}\frac{M_d}{M_p} \right)^2$$

It is clear that a planet initially located at r_c will not ultimately migrate, although it can undergo some transitional motion as the disk profile adjusts from its initial state. A planet inside (outside) the critical radius will migrate inward (outward). Setting $a = 0$ defines the minimum initial semi-axis $a_{\min}$ for which a planet will survive. For small planets $M_p \ll M_d$, this approaches r_c, for large planets $M_p \gg M_d$ it approaches zero. Curves for $M_p/M_d = 0.1,\ 0.2$ are included in Figure 1.

Shu *et al.* [3] have suggested that UV radiation from the early Sun could have truncated the disk at $r_d \sim O(10)AU$ by photoevaporation. Hollenbach *et al.* [4] review a number of processes that could remove the disk. Prior to this, a disk would spread and approach a similarity solution, which for a power law viscosity $\nu(r) \propto r^n$ takes the form [6]

$$\sigma(r) = \frac{\sigma_*}{t_*^{5/(4-2n)}}\left(\frac{r_d(t_*)}{r}\right)^n \exp - [r/r_d(t_*)]^{2-n}$$

where $t_* \equiv 1 + 3(2-n)^2(\nu_o t/r_o^2)$ is a time-like quantity, $\nu_o = \nu(r_o)$, and σ_* is a constant depending on initial conditions. The exponential 'edge' of the disk expands as $r_d \propto t_*^{1/(2-n)}$; inside this distance the surface density varies as r^{-n}. There is a flux reversal distance that is a constant fraction of the edge, *viz.*, $r_F = r_d/(4-2n)^{1/(2-n)}$. Inside r_F disk material flows inward. If the disk spreads to the point that the planet forming region is interior to r_F, all planets are in jeopardy.

To estimate the viscosity index n we adopt an alpha prescription of the form $\nu = \alpha c^2/\Omega \propto T r^{3/2}$ where T is the disk temperature. Balancing radiation losses per unit area of the disk proportional to T^4 with the energy dissipated due to viscous stresses proportional to $\sigma\nu\Omega^2$ leads to $n \approx 3/4$ inside r_d. Figure 2 shows the evolutionary profiles of an $n = 3/4$ disk; the flux reversal point is indicated by the open circle. (The real profile would differ somewhat because $n \neq 3/4$ external to r_d.) If the onset of a high UV flux opens a gap in the disk at some fixed distance $r_d' < r_d(t)$, the initial state of the interior portion of the disk is a power law with $k = n = 3/4$. The critical distance separating inward bound from outward bound material is thus $r_c = 0.37 r_d'$. For example, this would lie near the current orbit of Jupiter for $r_d' = 14\ AU$. In general, the ratio of the critical distance after truncation to the flux reversal point of the disk prior to truncation is

$$\frac{r_c}{r_F} = \frac{r_d'}{r_d(t_*)}\left(\frac{4-2n}{5-2n}\right)^{1/(2-n)}$$

which for $n = 3/4$ reads $0.76 r_d'/r_d(\tau)$. As a result, a protoplanet that was inside r_F and undergoing type II decay before the disk was truncated, could find itself outside the critical point once photodissociation created a much nearer edge and thereby not be forced into the star.

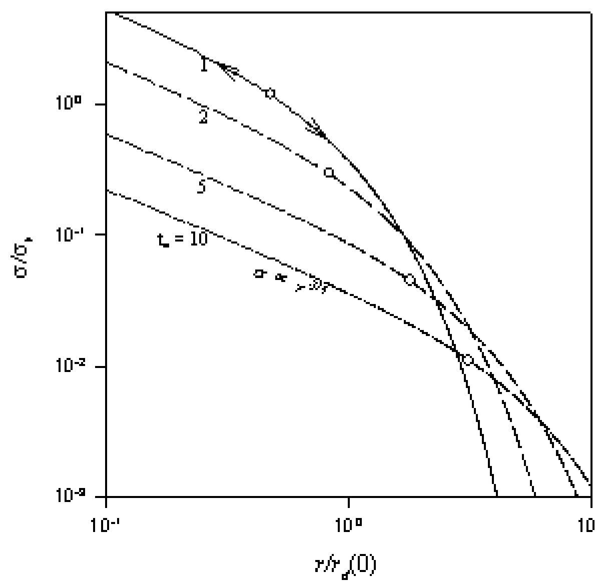

Figure 2. Evolutionary profiles $\sigma(r)/\sigma_*$ of similarity solution for $n = 3/4$. Inner disk approaches $\sigma \propto r^{-3/4}$ profile. Disk edge r_d and flux reversal distance r_F (open circles) migrate outward with time.

This work was supported by NASA's Planetary Geology and Geophysics Program.

[1] Lin, D. N. C., P. Bodenheimer & D. C. Richardson (1996) *Nature* **380**, 606-607. [2] Trilling, D. E., Lunine, J. I., & Benz, W. (2002) *Astron. & Astrophys,* **394**, 241-251. [3]Armitage, P. J., Livio, M., Lubow, S. H. & Pringle, J. E. (2002) *M..N.R..A.S.* **334**, 248-256. [4] Shu, F. H., D. Johnstone & D. Hollenbach (1993) *Icarus* **106**, 92-101. [5] Hollenbach, D. J., H. W. Yorke & D. Johnstone (2000) In *Protostars & Planets IV* , (Eds.V Mannings, A. P Boss & S. S. Russell) Univ. Arizona Press:Tucson, AZ, pp 401-428. [6] Lynden-Bell, D. & J. E. and Pringle (1974) *M.N.R.A.S.* **168**, 603-637.