A NEW APPROACH TO EVALUATE COLLISION PROBABILITIES AMONG ASTEROIDS, COMETS, AND KUIPER BELT OBJECTS.

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Introduction: A new method to evaluate the long-term collision probabilities between orbiting objects has been developed. It is designed to work with any orbital evolution model to estimate the collision probabilities of a system (asteroids, comets, Kuiper Belt objects, or planetesimals) as the system evolves in time. Contrary to Öpik's classical method based on uniform sampling in space, this method is based on uniform sampling in time. Since orbital elements are updated frequently rather than based on some assumptions, this dynamical approach is more general than Öpik's. When applied to an Öpik system – a system of objects with fixed semimajor axes (*a*), eccentricities (*e*), and inclinations (*i*), and randomly distributed right ascensions of the ascending node (Ω), arguments of pericenter (ω), and mean longitudes (λ), this method yields results that are consistent with those obtained using Öpik's method.

The Classical Approach: The classical way to evaluate the collision probabilities between asteroids, comets, and planetesimals was pioneered by Öpik [1]. This method was later generalized by Wetherill [2], Kessler [3], and Greenberg [4] to allow for non-circular orbits for objects involved and to handle singularity problems associated with certain close encounter geometries. The fundamental assumption of these approaches is that the *a*, *e*, *i* of each object are fixed while its Ω, ω, λ are distributed randomly between 0 and 2π. Based on this assumption, all possible close encounter geometries between any two orbiting objects can be sampled randomly or uniformly to obtain the long-term average collision probability between the two. This method has been applied to estimate collisions between asteroids and comets [2, 4–7], collisions between Jovian satellites [3], and collisions between artificial satellites [8]. However, this assumption is not valid for objects with fast changing *(a, e, i)*'s or non-uniform precessing rates or objects in mean motion resonance with each other. Examples include asteroids near secular resonances, comets, and objects under strong dissipative forces.

Concept of the Cube Approach: With the help of modern computers, it is now possible to perform numerical simulations of the orbital evolution of an N-body system. Therefore, there is a need for a collision model that can work with an orbital evolution simulation to allow for source and sink terms of the objects involved as well as to utilize their updated orbital elements as they evolve in time. The "Cube" approach is designed to accomplish just that. Its objective is to estimate the long-term collision probabilities by means of uniform sampling of the system in time. Mathematically the number of collisions between two objects, *i* and *j*, over a long period of time (between *t_begin* and *t_end*) can be expressed as:

$$N_{tot} = \int_{t\_begin}^{t\_end} P_{i,j}(t)\, dt = \int_{s=0}^{s=L} \int_{t_s}^{t_{s+1}} P_{i,j}(t)\, dt\, ds, \qquad (1)$$

where $P_{i,j}$ is the collision rate, L is the number of time intervals between *t_begin* and *t_end*, t_s and t_{s+1} represent the beginning and the end of a given time interval *s*. If t_{s+1} - t_s is sufficiently short such that the collision characteristics between the two do not change significantly, $P_{i,j}$ can be considered constant during that period and be taken out from the second integral:

$$N_{tot} = \int_{s=0}^{s=L} [t_{s+1} - t_s] \times P_{i,j}(s)\, ds. \qquad (2)$$

Procedures of the Cube Approach: To evaluate $P_{i,j}$, we further assume that, at each snapshot (*i.e.*, at the end of each time interval), a collision is possible only when two objects are within a small volume element. We intend to search for collisions between objects that could collide during a time period roughly equal to the time required for an object to cross a volume element and this time period has to be much shorter than $t_{s+1} - t_s$. A modified finite element model can be designed for the identification. At each snapshot the 3-dimensional space is divided into many cubes. The cube dimension is characterized by the short-period planetary perturbations on the positions of the objects. The position and velocity of each object are calculated based on their orbital elements at that instance. The cube inside which each object is located can be identified in a straightforward manner. When the two objects are within the same cube, the collision rate, $P_{i,j}$, is given by:

$$P_{i,j} = s_i s_j V_{imp} \sigma\, dU, \qquad (3)$$

where s_i and s_j are the spatial densities of object *i* and *j* in the cube, respectively, V_{imp} is the relative velocity between the two, σ is the collision cross-sectional area, and dU is the volume of the cube. Equation (3) is based on the same spatial density approach developed by Kessler [3]. In other words, on a microscopic scale (*i.e.*, within a cube), the kinetic theory of gas is applied to evaluate the collision probability. Contrary to uniform sampling of the space where collisions are possible, this approach uses uniform sampling of the system in time with updated orbital elements every time step. No assumptions regarding secular variations in objects' orbital elements are required. Note that the only objects selected for collision evaluation are those within the same cube. For those identified as the only object in a cube, they are not processed any further. This is a fast and efficient way of performing pair-wise comparisons. The computation time of this approach increases with N, rather than N^2, for an N-body system. This approach can be coupled with any numerical simulations of the orbital evolution of objects (asteroids, comets, Kuiper Belt objects, planetesimals) to evaluate the collision probability as the system evolves. At every integration time step one needs to identify multi-object cubes and use equation (3) to determine the collision rate of each pair of objects. Then a Monte Carlo method can be applied to determine whether or not a collision actually occurs.

Equation (3) is assumed to represent the average collision characteristics between two snapshots. As a standard statistical sampling technique, more snapshots, or smaller time step between snapshots, are preferred. The identification of objects with non-zero collision rate and the computation time of equation (3) should be small compared with the orbital integration time of an N-body system. Therefore, the integration time step should be used to obtain the best collision estimates. In addition, the cubes have to be small enough to capture the characteristics of the true collision nature of the system. In general, a dimension of 1% or less of the average semimajor axis of objects in the system is sufficient. In the extreme case where the cube is as big as the whole system, this approach mimics a simple particle-in-a-box collision model.

Comparisons with the Classical Approach: To validate and verify the Cube approach, we have applied the method to previously published collisions between asteroids and comets [2, 4, 5, 7] and collisions between Jovian satellites [3]. All objects in the test cases are assumed to have fixed (a, e, i)'s and randomly distributed $(\Omega, \omega, \lambda)$'s. The intrinsic collision probabilities (10^{-18} km^{-2} yr^{-1}) between a hypothetical asteroid "Astrid" (a=2.75 AU, e=0.2727, i=15.8°) and several asteroids and comets are listed in Table 1. Although columns A-E are all based on the same (Öpik's) approach, there are some variations in collision probability. The differences could be due to how the algorithm is implemented and how the singularity problem is handled or ignored [7]. Overall the Cube predictions are consistent with those calculated by various authors using Öpik's method.

Table 1. Intrinsic collision probabilities (10^{-18} km^{-2}yr^{-1}) between a hypothetical asteroid "Astrid" and several asteroids and comets.

Object	A	B	C	D	E	Cube
1948EA	3.10	2.49	3.03	3.19	3.20	3.23
Apollo	4.22	3.24	3.46	3.48	3.60	3.77
Adonis	4.13	3.92	4.24	4.53	4.53	4.79
1950DA	3.90	3.13	3.61	3.64	3.76	3.65
Encke	3.49	2.91	3.25	3.43	3.43	3.64
Brorsen	0.94	0.81	0.90	0.95	0.95	1.01

A – Wetherill (1967) [2]. B – Greenberg (1982) [4]. C, D – Namiki and Binzel (1991) [5]. E – Bottke and Greenberg (1993) [7].

Kessler generalized Öpik's method using the "spatial density" concept and applied the derived equations to collisions between four Jovian satellites [3]. The collision rates (10^{-10} yr^{-1}) in his 1981 paper are listed in Table 2. Results from a recent recalculation, using the same equations but on a current computer, are also listed. Again, the Cube approach agrees well with Kessler's method.

Table 2. Collision rates (10^{-10} yr^{-1}) between four Jovian satellites.

Collision pair	Kessler[1]	Kessler[2]	Cube
Himalia-Elara	4.3	4.1	4.2
Himalia-Lysithea	2.8	3.4	3.5
Himalia-Leda	3.1	3.0	3.2
Elara-Lysithea	0.52	0.51	0.52
Elara-Leda	0.57	0.57	0.56
Lysithea-Leda	0.039	0.038	0.038

[1]Kessler (1981) [3]. [2]A recent calculation [9].

References:

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